소프트웨어 오류 자동 수정 기법

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Korea University

Dec 4, 2018

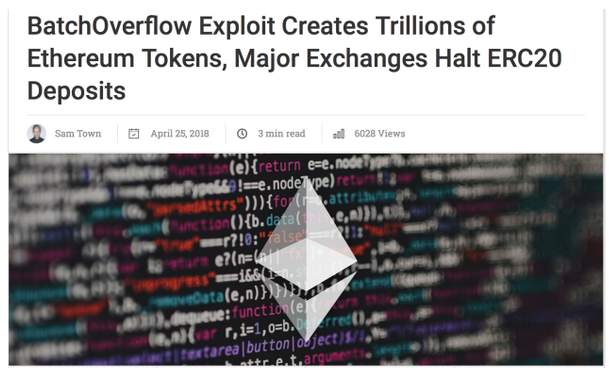
(co-work with Junhee Lee and Seongjoon Hong)

소프트웨어 오류 문제

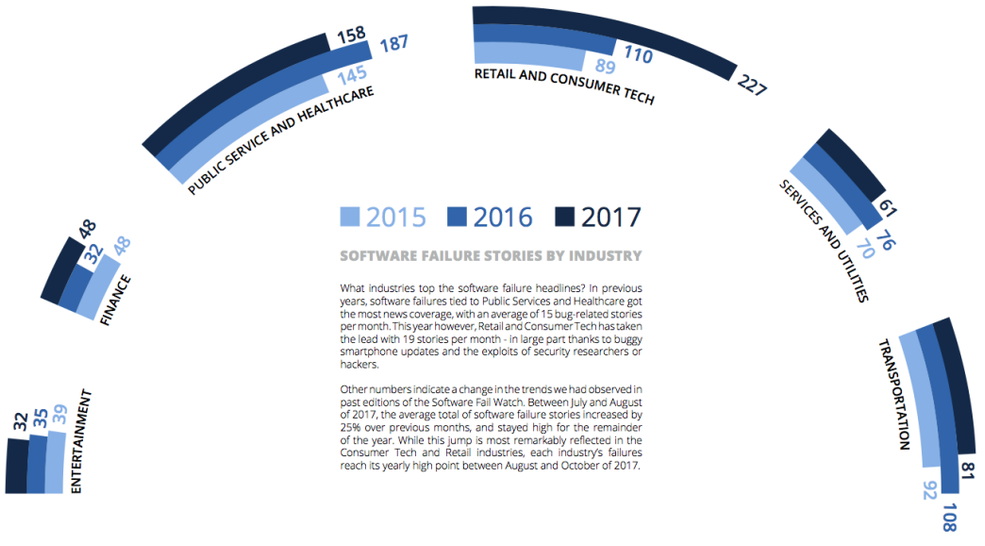
- 사회 각 영역에서 더욱 심각해지고 있는 소프트웨어 오류 문제

금융 소프트웨어 결함 (2012)

인공지능 소프트웨어 결함 (2017)



블록체인 소프트웨어 결함 (2018)



Software fail watch (5th ed) 2017

연구 방향

- Q) 어떻게 안전한 소프트웨어를 손쉽게 만들것인가?
- A) 소프트웨어 자동 **분석**, **패치**, **합성** 기술

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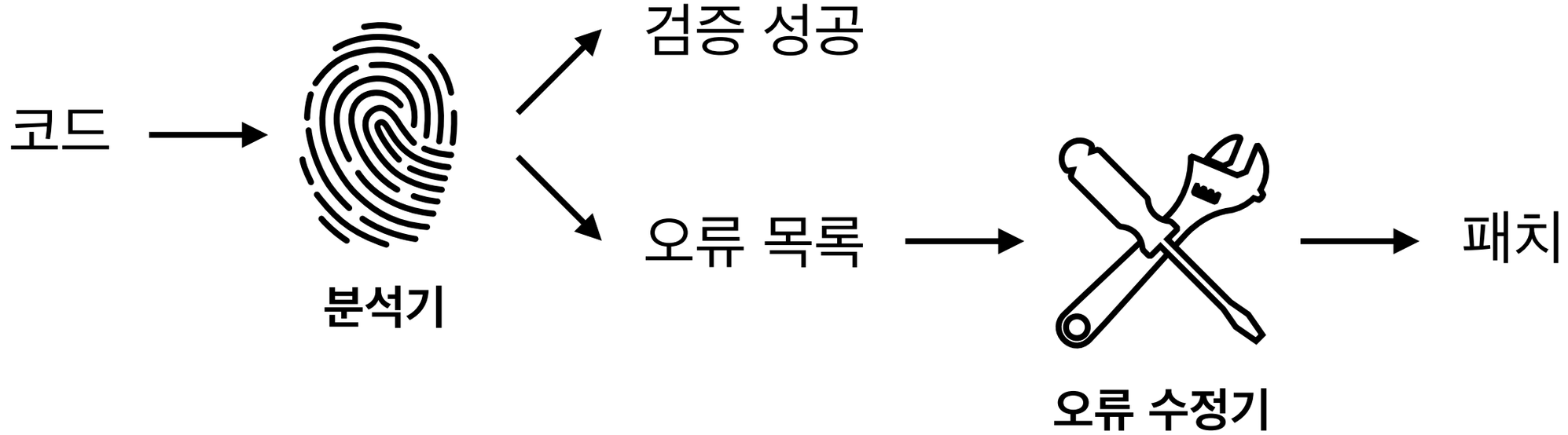

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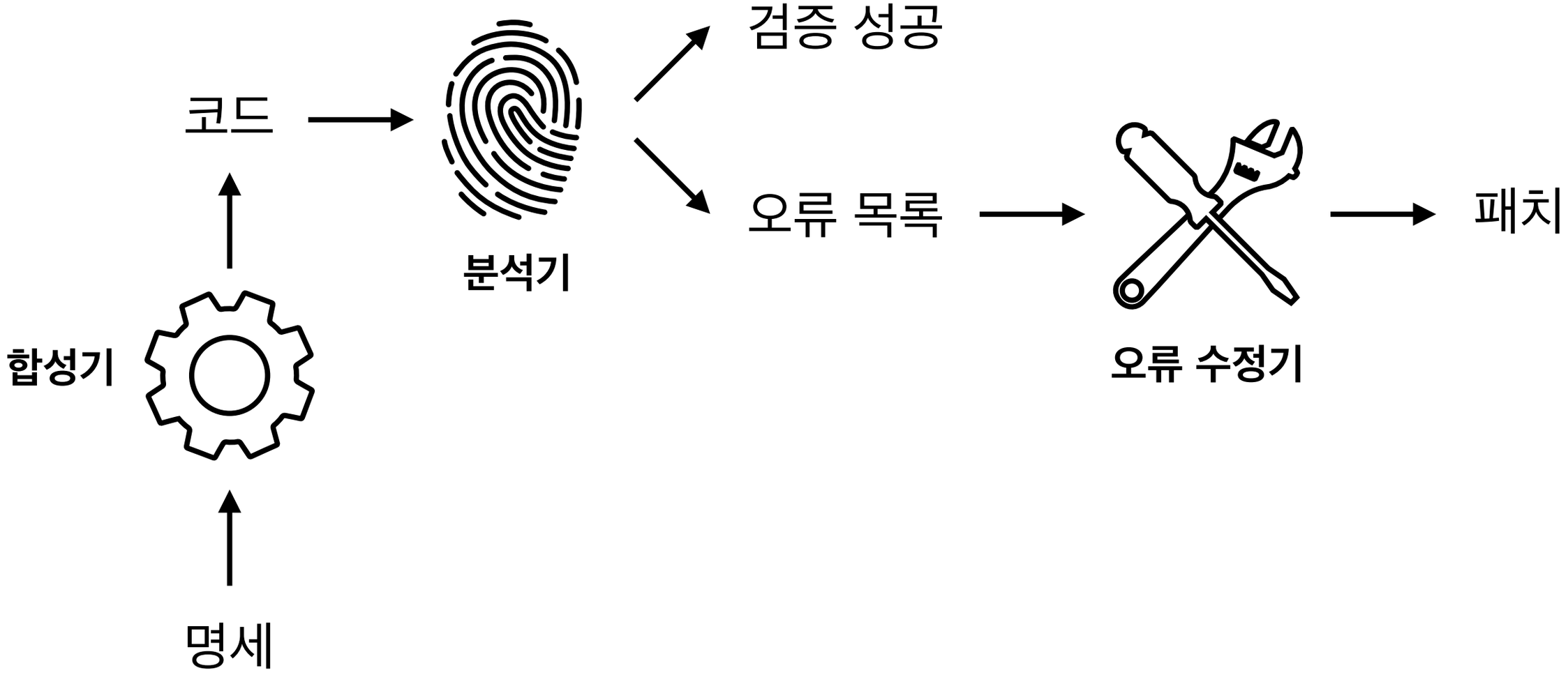

연구 방향

- Q) 어떻게 안전한 소프트웨어를 손쉽게 만들것인가?
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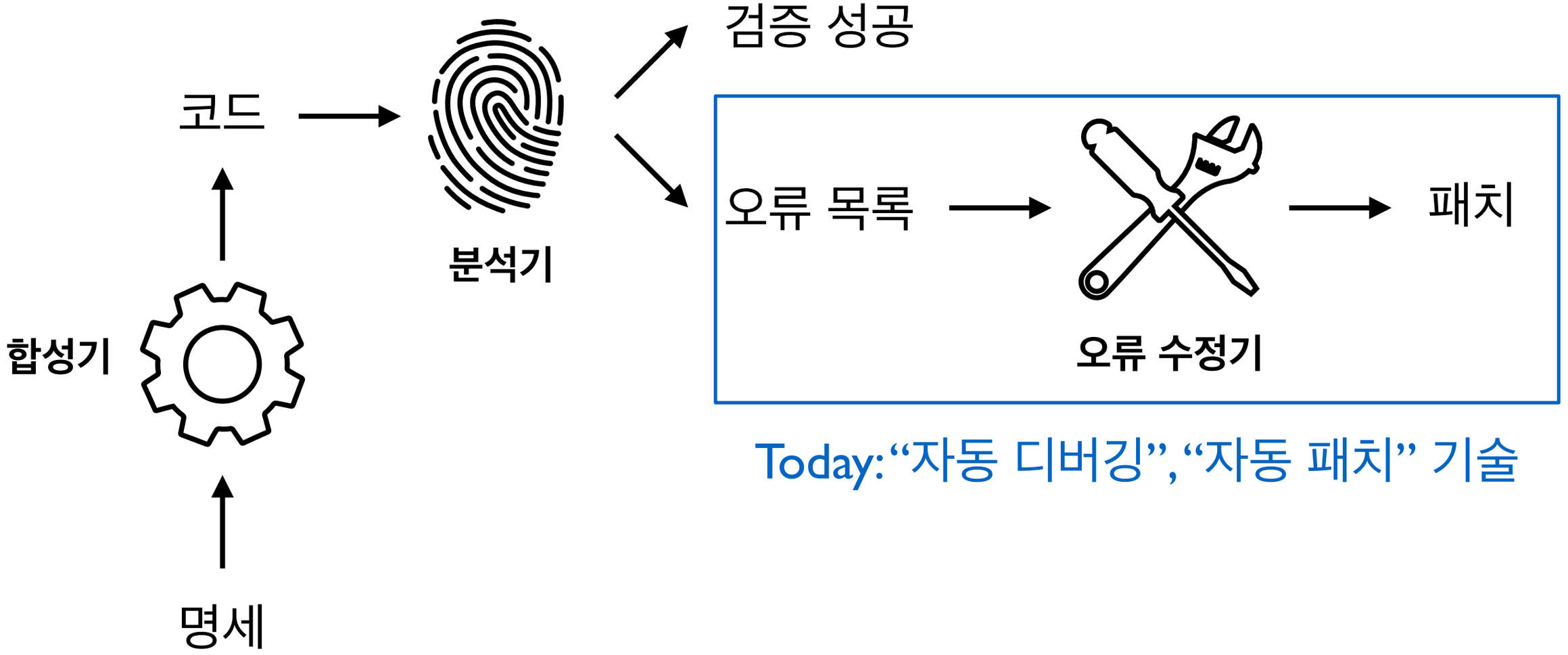


Today: "자동 디버깅", "자동 패치" 기술

왜 필요한가?

- 소프트웨어 개발에서 디버깅은 전체 시간의 절반을 차지
  - 상용 소프트웨어 오류 수정에 평균 200일 소요[1)]
  - 오류/취약점은 해마다 증가 개수: e.g., CVE 등록수 4,000('10년), 6,000('15년)
- 다른 개발 단계에 비해 자동화된 도구 지원이 가장 적음
  - cf) 소프트웨어 오류 탐지 분야는 지난 30여년간 눈부신 발전
  - 개발자에 전적으로 의존할수 밖에 없지만 가장 어렵고 부담스러운 단계

1) Kim and Whitehead. How long did it take to fix bugs? MSR 2006

실제 사례
(Linux Kernel)

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {

  goto err;
}
```

```
out = malloc(2);
if (out == NULL) {
  free(in);

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

실제 사례
(Linux Kernel)

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {

  goto err;
}

out = malloc(2);
if (out == NULL) {
  free(in);

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

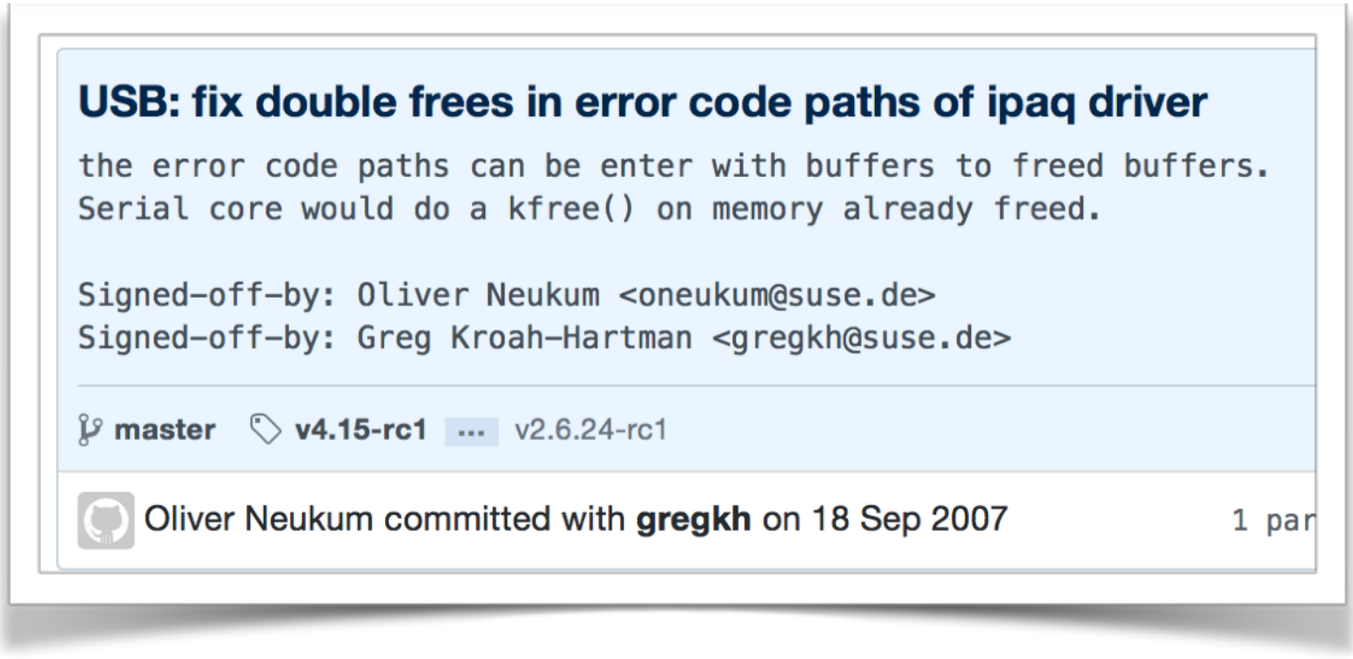
double-free

실제 사례
(Linux Kernel)

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {

  goto err;
}

out = malloc(2);
if (out == NULL) {
  free(in);

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

double-free

실제 사례
(Linux Kernel)




```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {
  out = NULL;
  goto err;
}
```

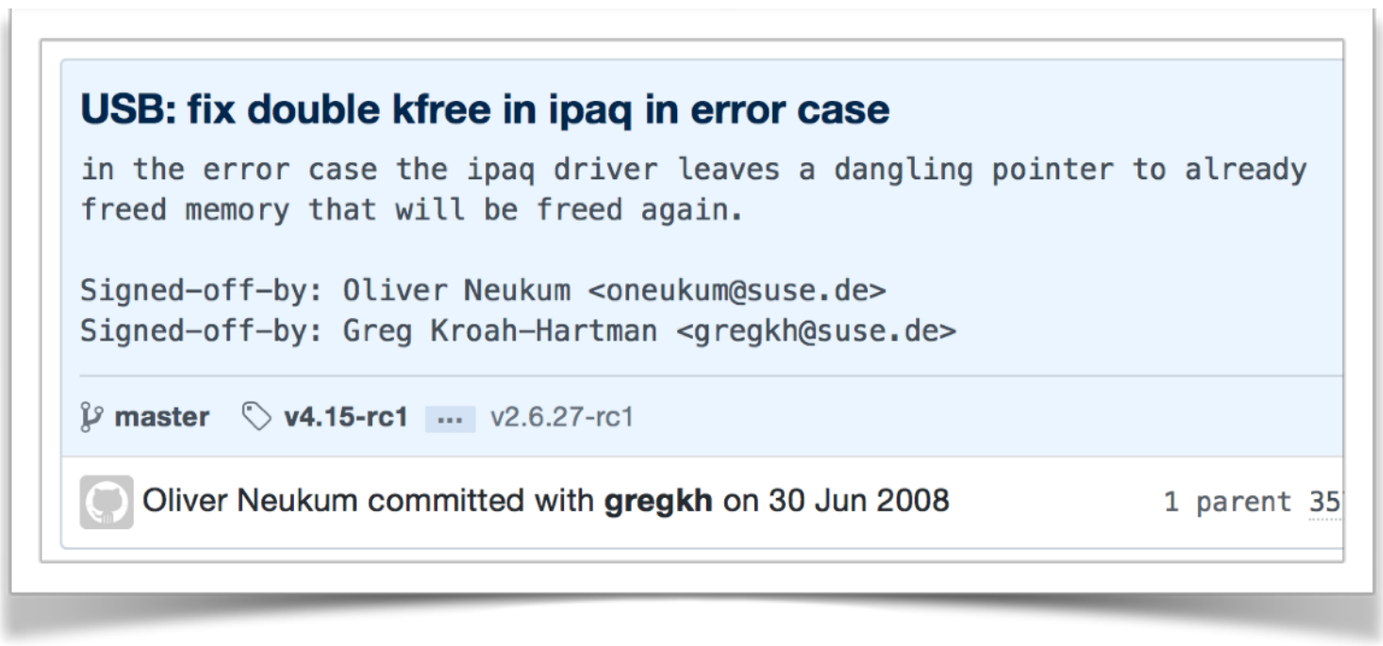
```
out = malloc(2);
if (out == NULL) {
  free(in);
  in = NULL;
  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

실제 사례
(Linux Kernel)

수동 디버깅의 문제 1:
오류가 사라졌는지 확신하기 어려움

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {
  out = NULL;
  goto err;
}

out = malloc(2);
if (out == NULL) {
  free(in);
  in = NULL;
  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

실제 사례
(Linux Kernel)




```
in = malloc(1);
out = malloc(1);
... // use in, out
// removed
free(in);

in = malloc(2);
if (in == NULL) {
  out = NULL;
  goto err;
}
free(out);
out = malloc(2);
if (out == NULL) {
  free(in);
  in = NULL;
  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

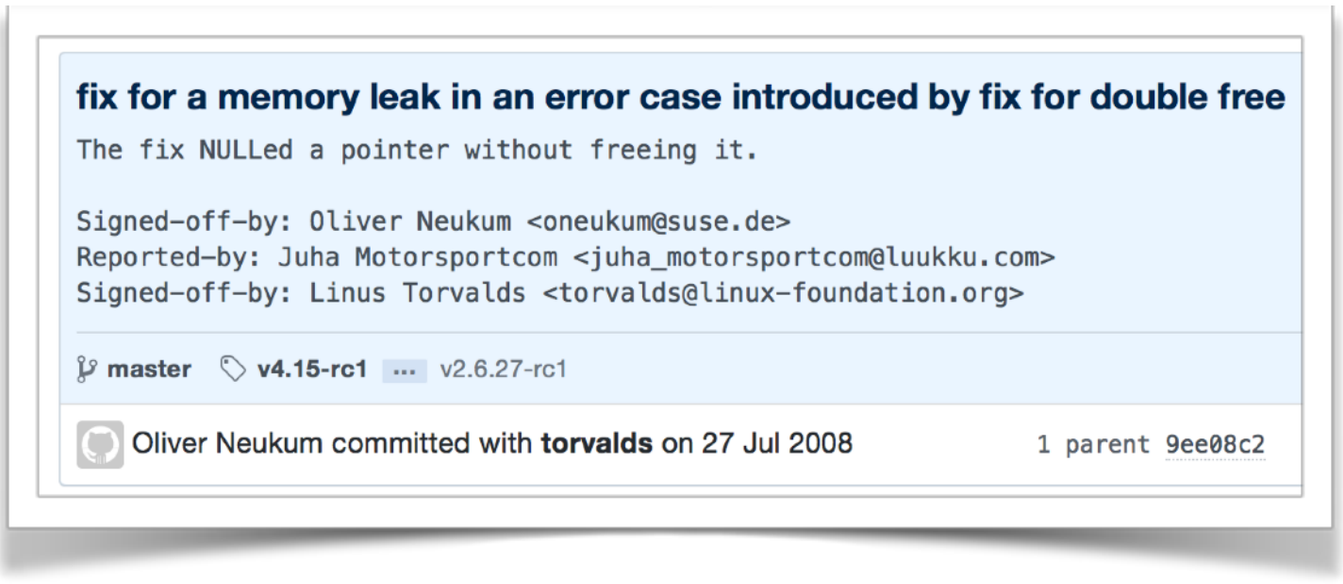
실제 사례
(Linux Kernel)

수동 디버깅의 문제 2:
고치는 과정에서 새로운 오류가 발생

memory leak


```
in = malloc(1);
out = malloc(1);
... // use in, out
// removed
free(in);

in = malloc(2);
if (in == NULL) {
  out = NULL;
  goto err;
}
free(out);
out = malloc(2);
if (out == NULL) {
  free(in);
  in = NULL;
  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

실제 사례
(Linux Kernel)



fix for a memory leak in an error case introduced by fix for double free

The fix NULLed a pointer without freeing it.

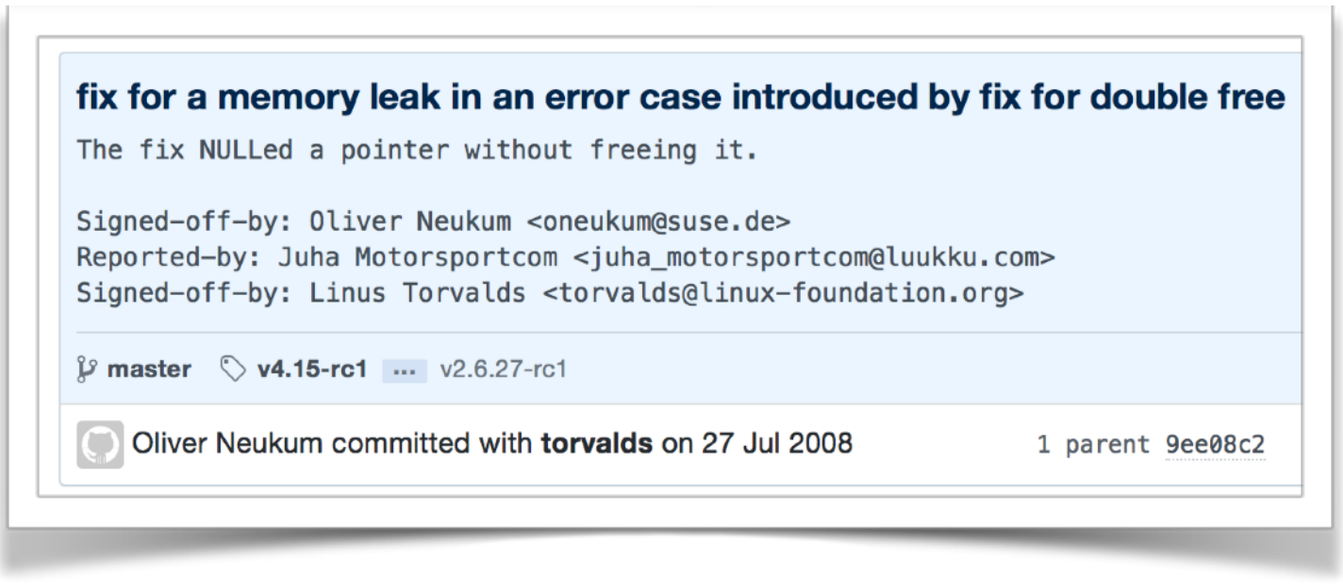
Signed-off-by: Oliver Neukum <oneukum@suse.de>
Reported-by: Juha Motorsportcom <juha_motorsportcom@luukku.com>
Signed-off-by: Linus Torvalds <torvalds@linux-foundation.org>

master v4.15-rc1 ... v2.6.27-rc1

Oliver Neukum committed with **torvalds** on 27 Jul 2008 1 parent 9ee08c2

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);
out = NULL;
in = malloc(2);
if (in == NULL) {
  out = NULL;
  goto err;
}
// removed
out = malloc(2);
if (out == NULL) {
  free(in);
  in = NULL;
  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

실제 사례
(Linux Kernel)



수동 디버깅의 문제 3: 수정된 코드가 복잡

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);
out = NULL;
in = malloc(2);
if (in == NULL) {
  out = NULL;
  goto err;
}
// removed
out = malloc(2);
if (out == NULL) {
  free(in);
  in = NULL;
  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

메모리 오류 자동 수정기

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {
  goto err;
}

out = malloc(2);
if (out == NULL) {
  free(in);

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

패치 자동 생성

```
in = malloc(1);
out = malloc(1);
... // use in, out
// removed
free(in);

in = malloc(2);
if (in == NULL) {
  goto err;
}
free(out);
out = malloc(2);
if (out == NULL) {
  // removed

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

메모리 오류 자동 수정기

```
in = malloc(1);
out = malloc(1);
... // use in, out
free(out);
free(in);

in = malloc(2);
if (in == NULL) {
  goto err;
}

out = malloc(2);
if (out == NULL) {
  free(in);

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

패치 자동 생성

수동 디버깅의 문제 해결:

1. 대상 오류가 반드시 제거됨
2. 새로운 오류가 발생하지 않음
3. 간결한 패치 (최소한의 변경)

=> **수학적 보장**.
추가적인 리뷰 불필요.

```
in = malloc(1);
out = malloc(1);
... // use in, out
// removed
free(in);

in = malloc(2);
if (in == NULL) {
  goto err;
}
free(out);
out = malloc(2);
if (out == NULL) {
  // removed

  goto err;
}
... // use in, out
err:
  free(in);
  free(out);
  return;
```

메모리 해제 오류

- 메모리 관리를 수동으로 해야하는 언어(e.g., C/C++) 발생
  - Memory-leak (CWE-401): 메모리를 너무 늦게 해제
  - Use-after-free (CWE-416): 메모리를 너무 빨리 해제
  - Double-free (CWE-415): 메모리를 여러번 해제

Memory-Leak

```
p = malloc(1);
...
return;
```

Use-After-Free

```
p = malloc(1);
...
free(p);
...
use(p);
```

Double-Free

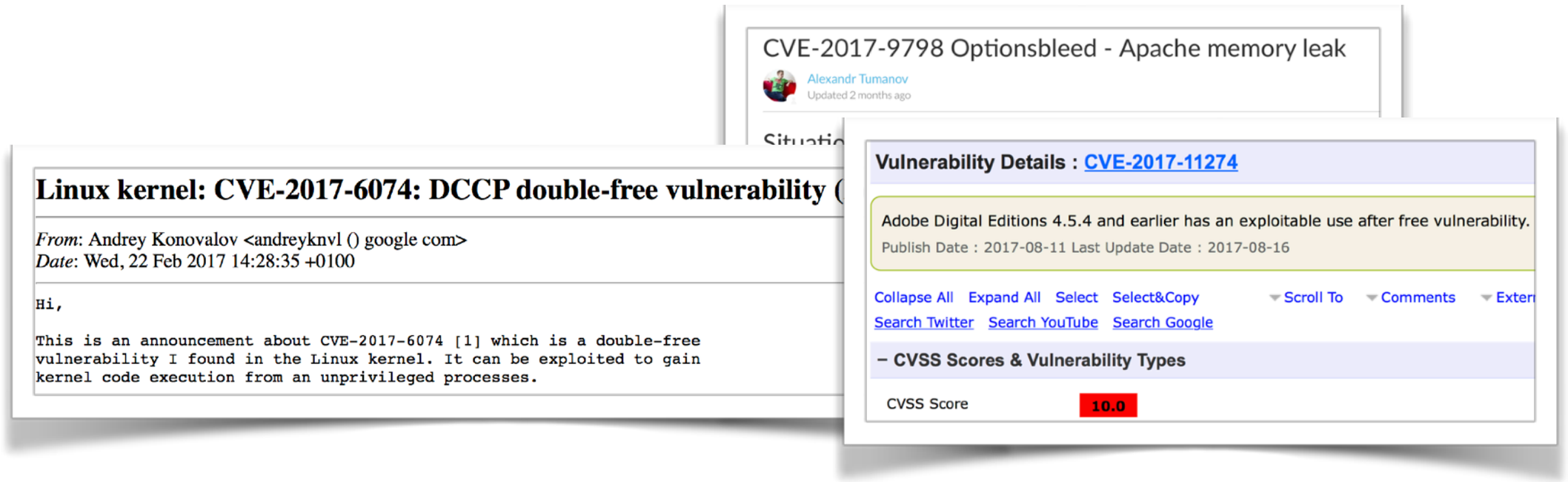
```
p = malloc(1);
...
free(p);
...
free(p);
```

메모리 해제 오류

- C/C++ 프로그램에서 가장 골칫거리중 하나

Repository	#commits	ML	DF	UAF	Total	*-overflow
linux	721,119	3,740	821	1,986	**6,363**	5,092
openssl	21,009	220	36	12	**264**	61
numpy	17,008	58	2	2	**59**	53
php	105,613	1,129	148	197	**1,449**	649
git	49,475	350	19	95	**442**	258

- 소프트웨어 결함의 주요 원인이지만 정확한 수정이 까다로움



FSE'18

MemFix

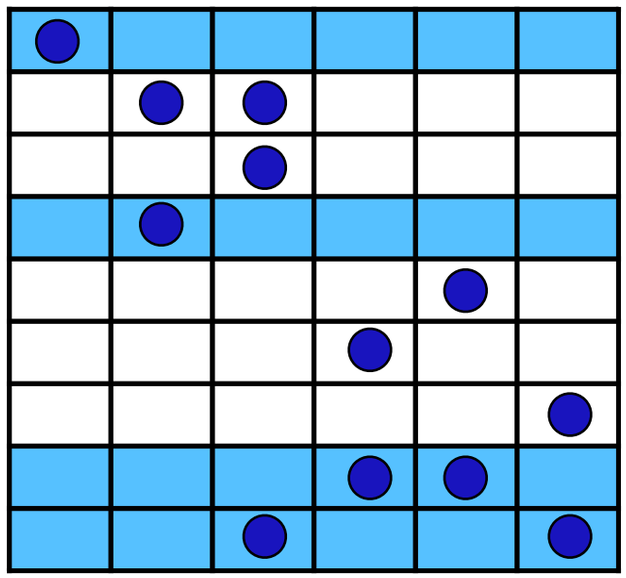
- Automatically repairs deallocation errors
  - **memory-leak**, **double-free** and **use-after-free**
- Key features
  - **sound**: generated patch is guaranteed to be correct
  - **safe**: no new errors are introduced
- Approach: **Static Analysis** + **Exact Cover Problem**

Key Insight

```
1   out = malloc(1);
2   in = malloc(1);
3   … // use in, out
4   free(out);
5   free(in);
6
7   in = malloc(2);
8   if(in == NULL) {
9
10      goto err;
11  }
12
13  out = malloc(2);
14  if(out == NULL) {
15      free(in);
16
17      goto err;
18  }
19  … // use in, out
20 err:
21  free(in);
22  free(out);
```

Find a set of free-statements

|||

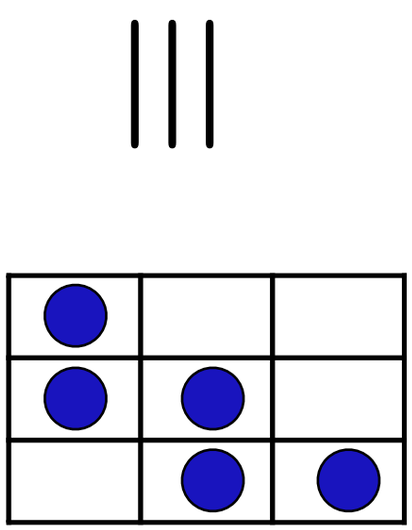


Solve an Exact Cover Problem

```
1   out = malloc(1);
2   in = malloc(1);
3   … // use in, out
4   // -
5   free(in);
6
7   in = malloc(2);
8   if(in == NULL) {
9
10      goto err;
11  }
12  free(out); // +
13  out = malloc(2);
14  if(out == NULL) {
15      // -
16
17      goto err;
18  }
19  … // use in, out
20 err:
21  free(in);
22  free(out);
```

Example: Double Free

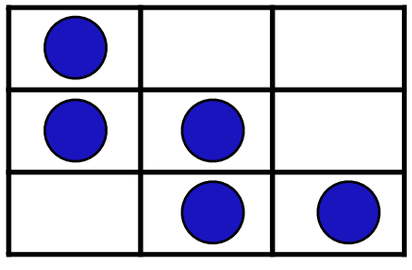
```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```

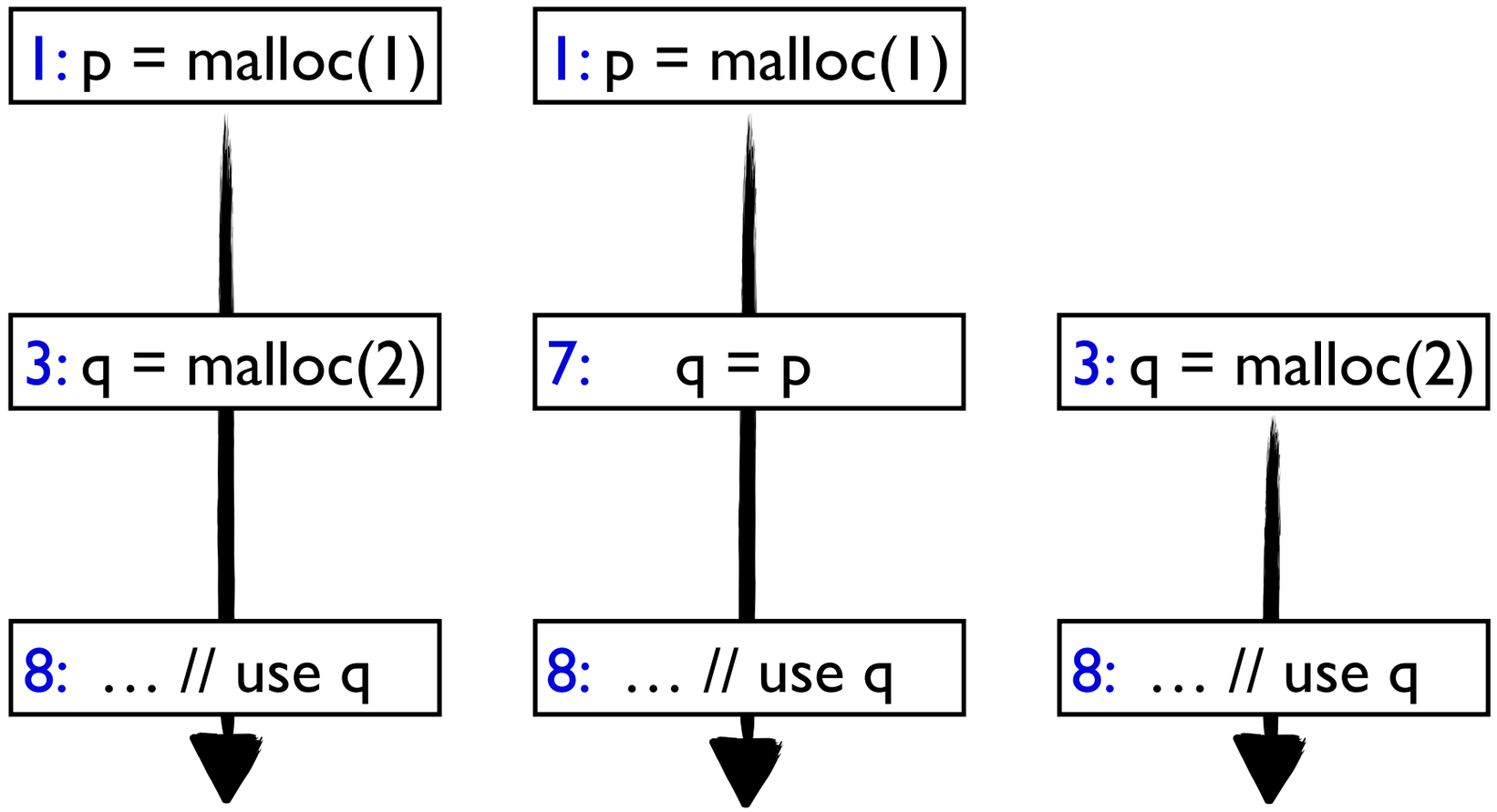


Enumerate All Object Traces

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```

|||

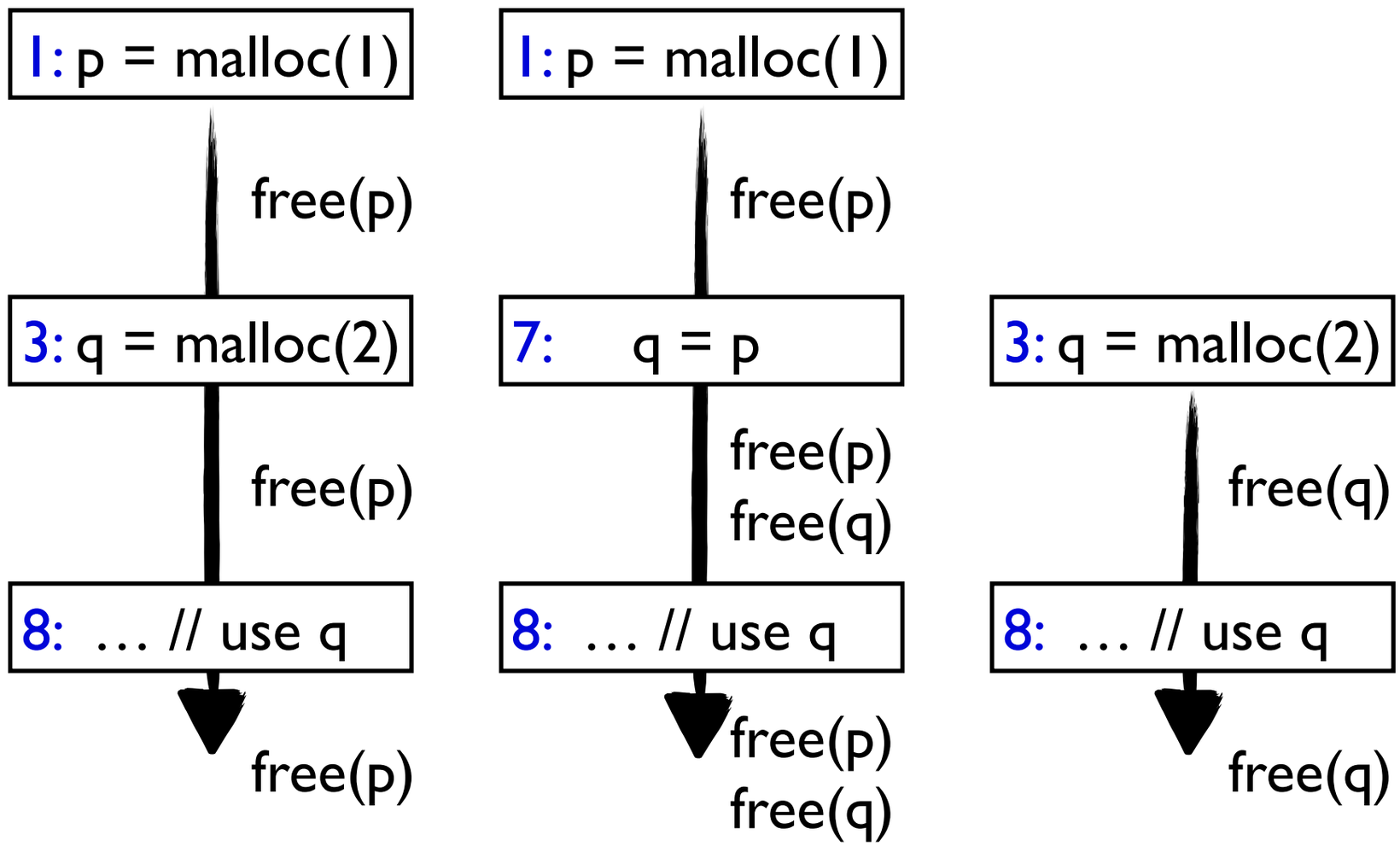




Object traces

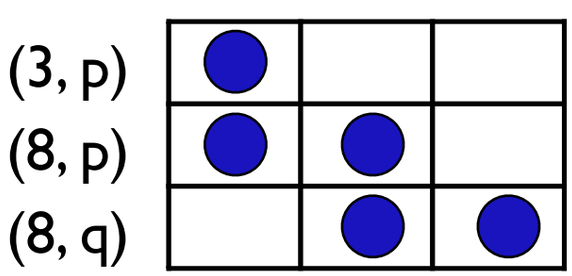
Find Safe Patches for Each Trace

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```



Object traces

|||



Find Safe Patches for Each Trace

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
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5   }
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8   … // use q
9   free(p);
10  free(q);
```
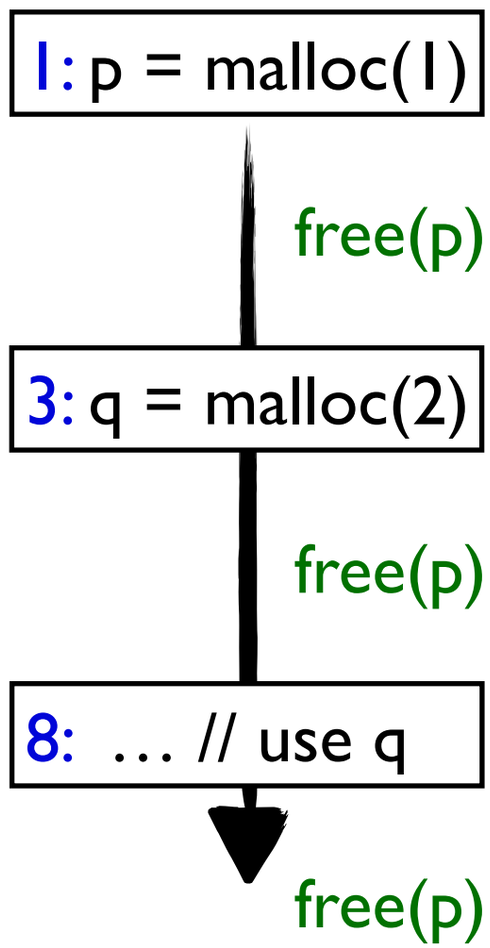
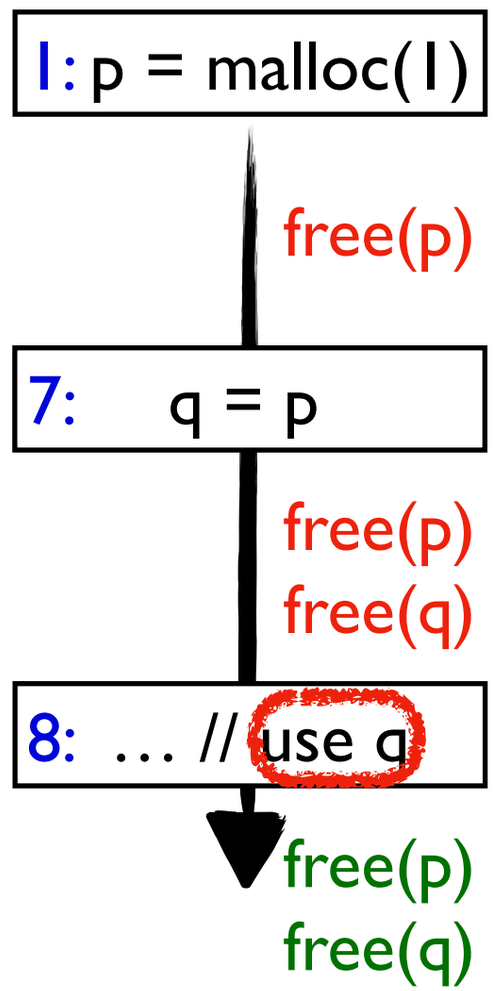
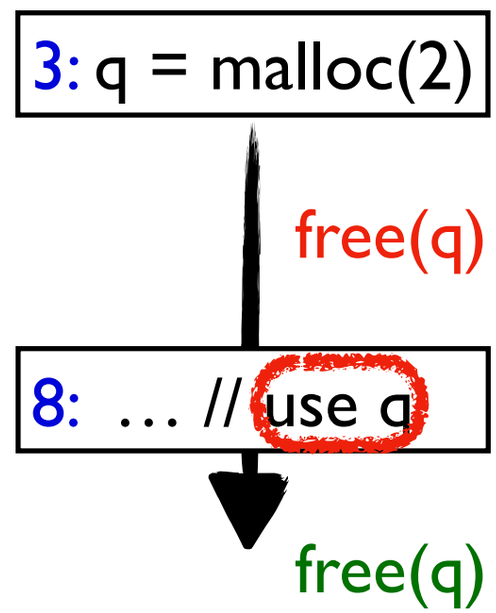
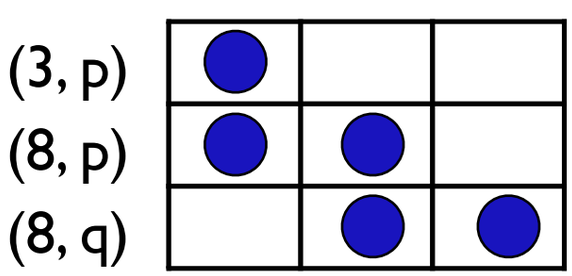







Object traces

|||



Find Safe Patches for Each Trace

```
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```
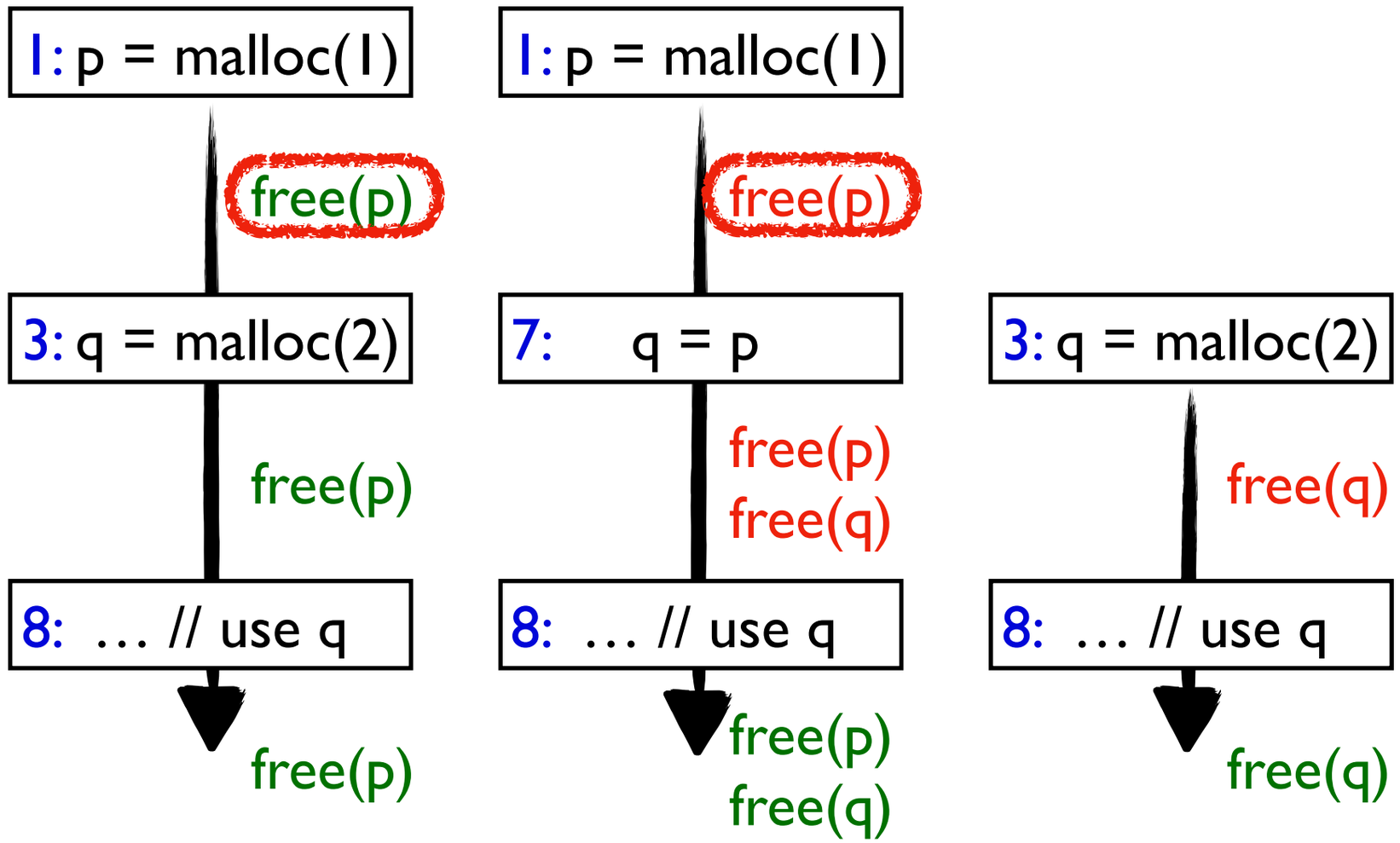
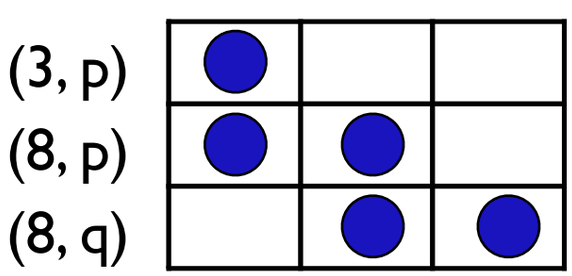



Object traces

|||



Find Safe Patches for Each Trace

```
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5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```
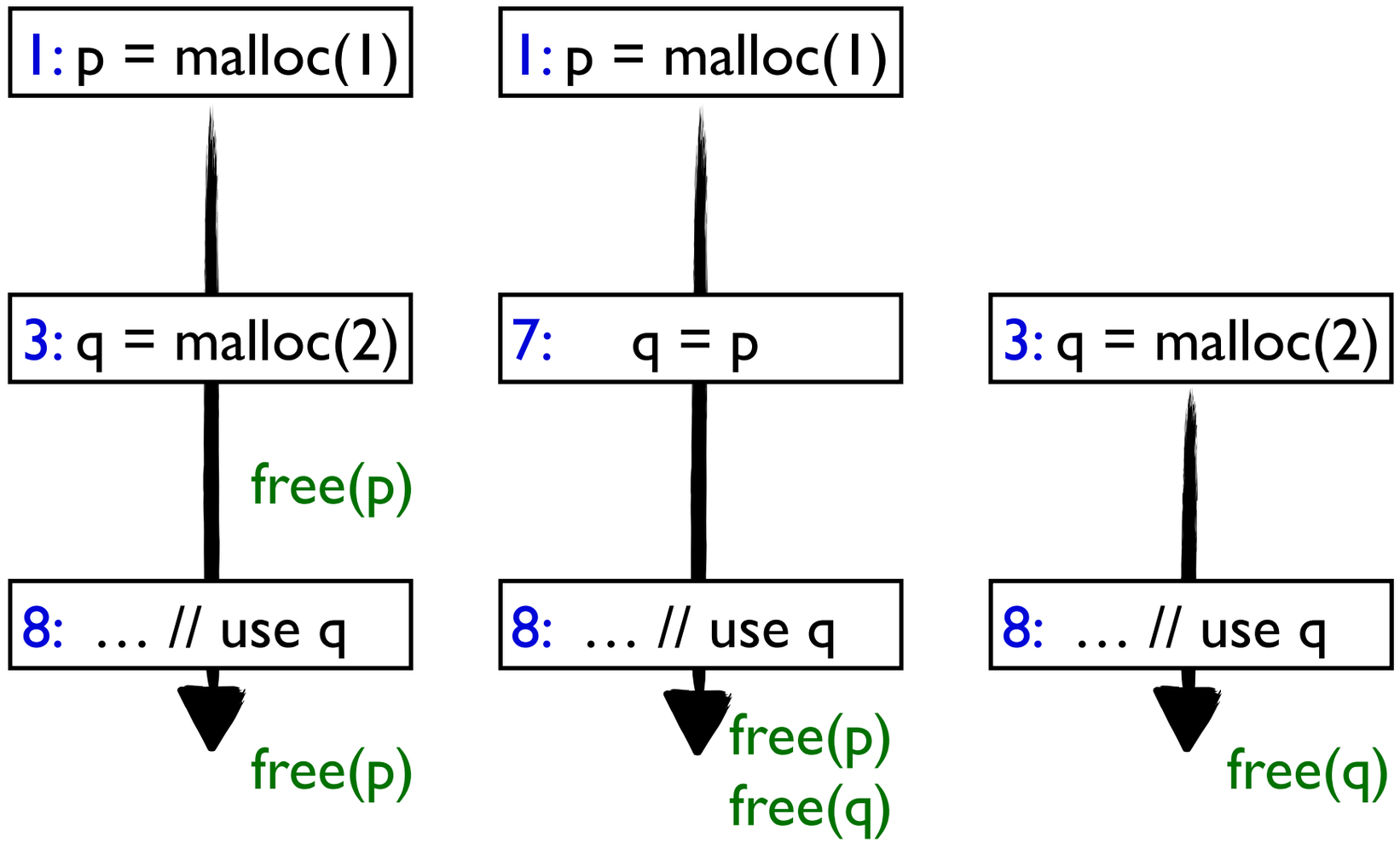
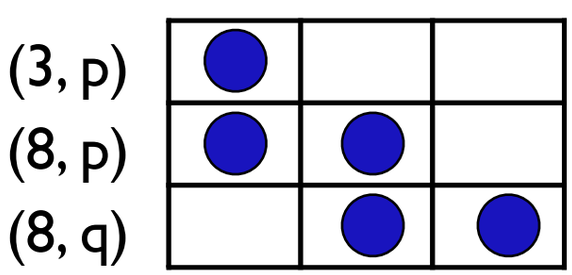



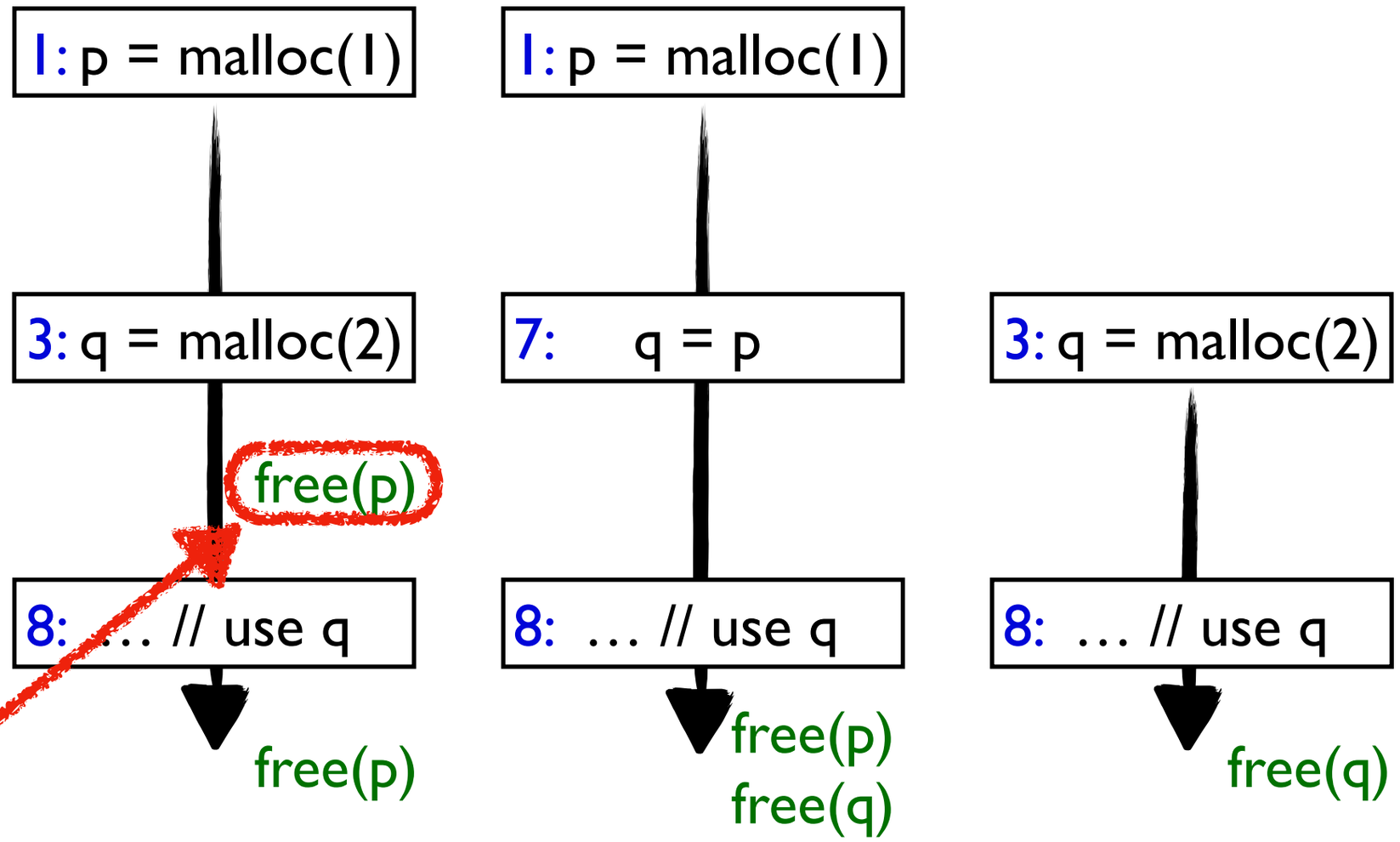
Object traces

|||



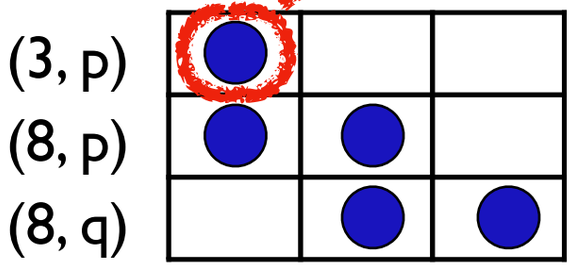
Find Safe Patches for Each Trace

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```



Object traces

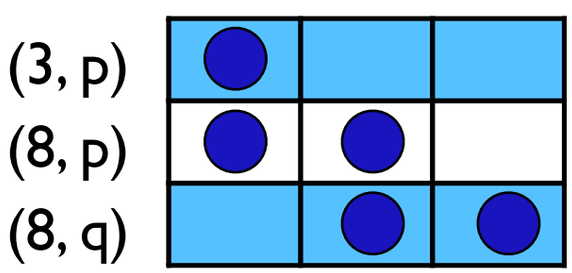
|||

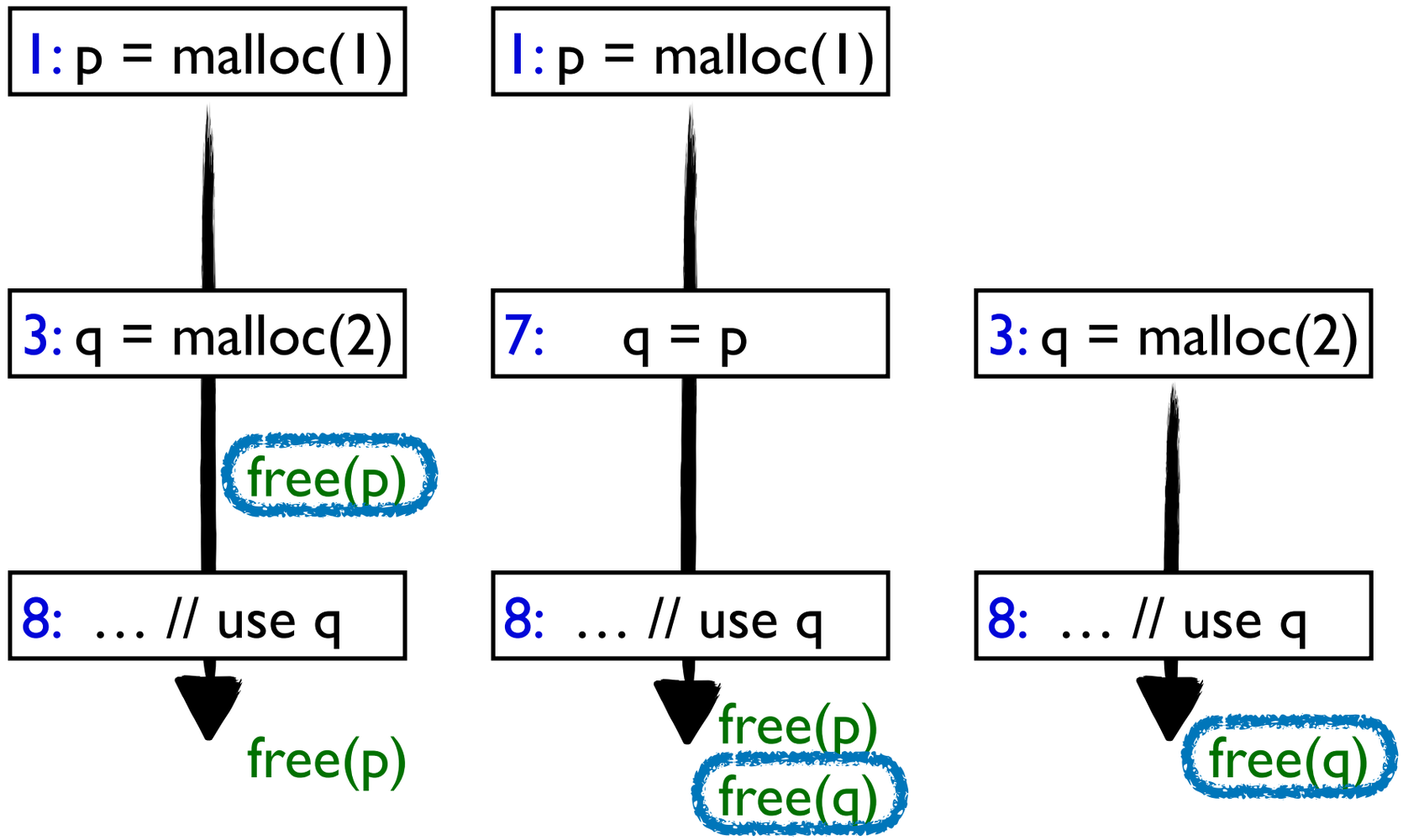


Find Safe Patches for Each Trace

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```

|||

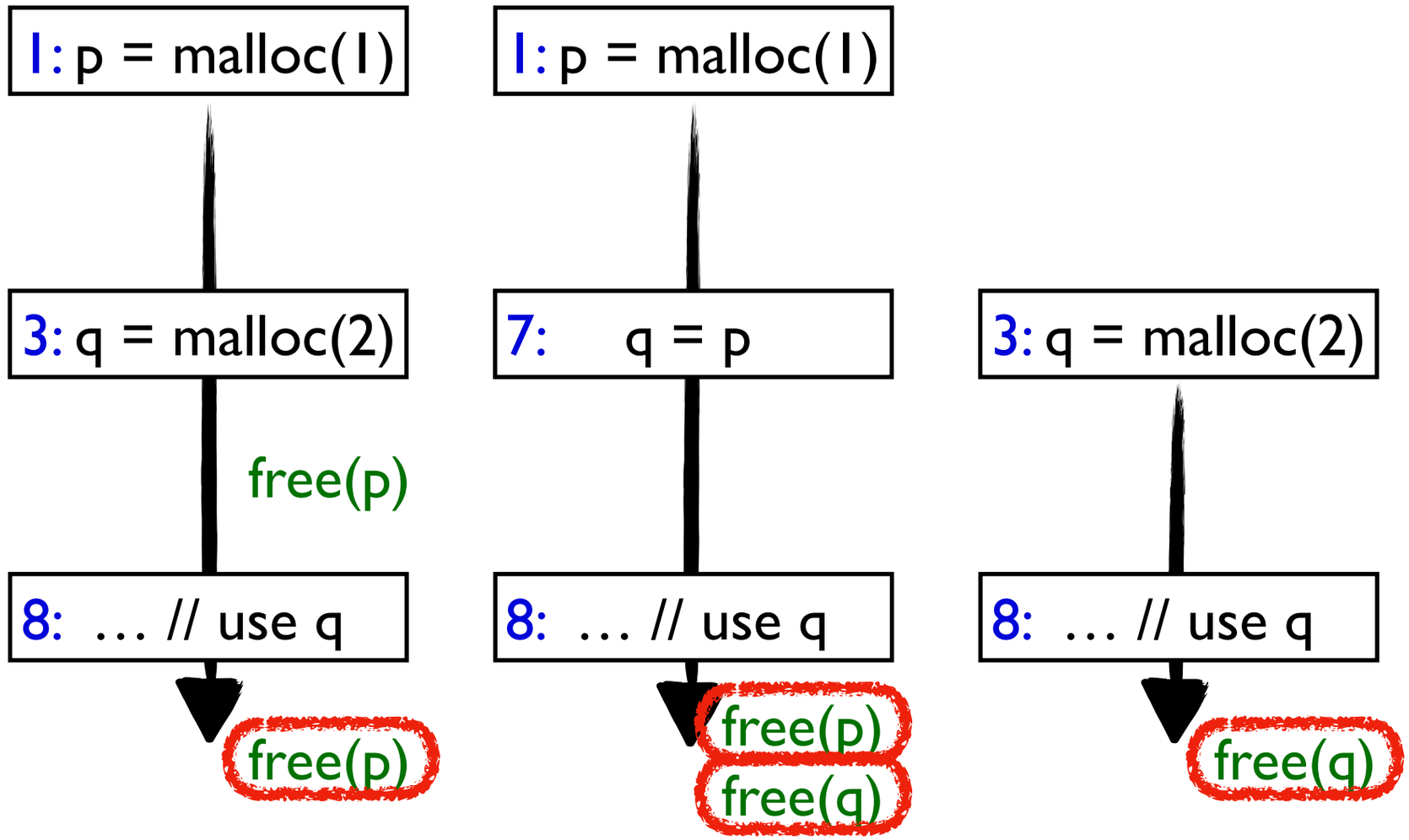




Exact Cover!

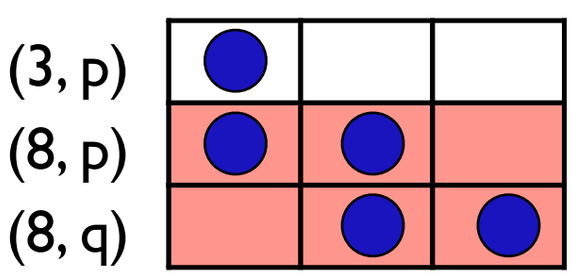
Non-Exact Cover

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```



Double free!

|||



Applying Generated Patches

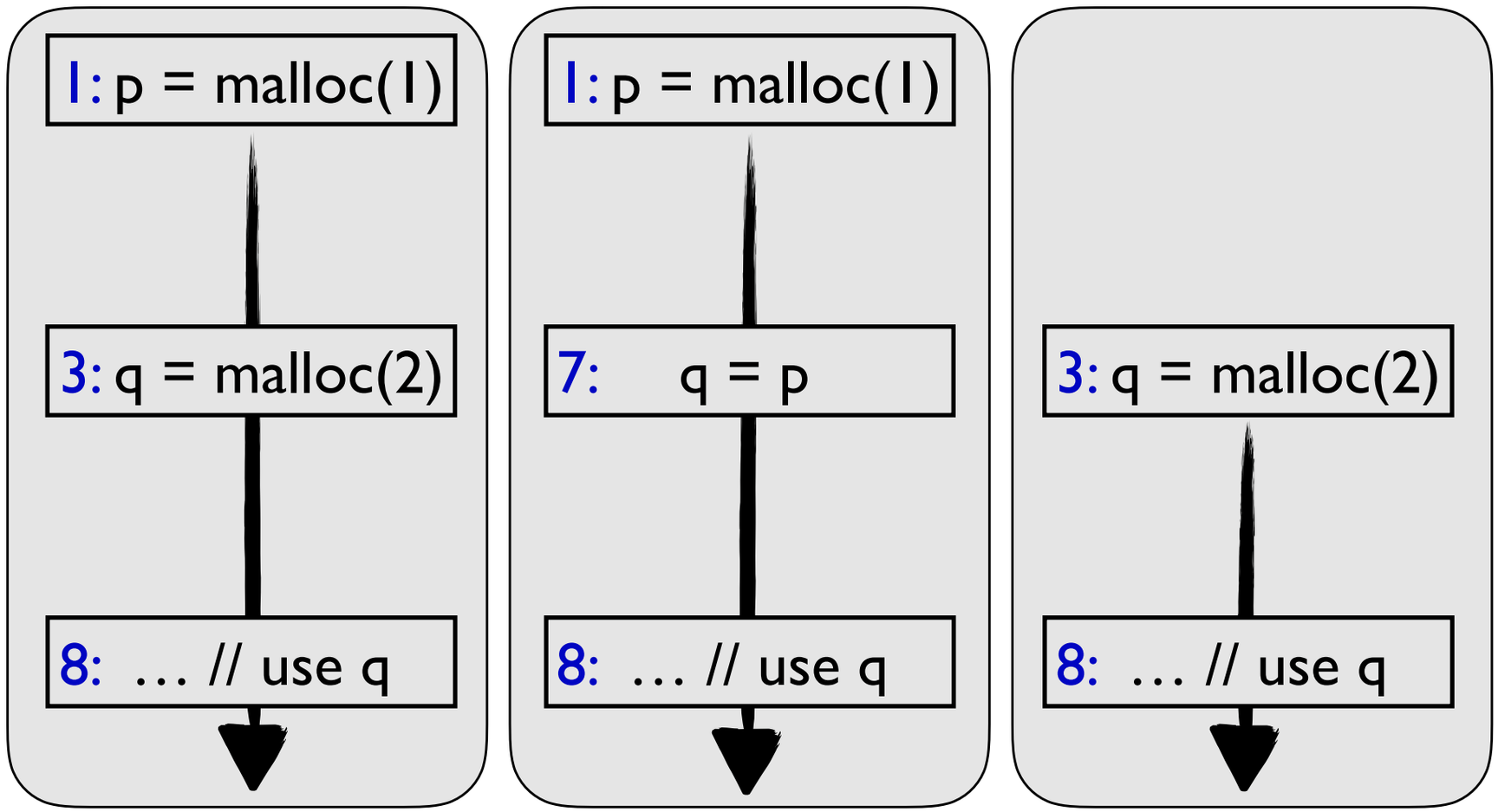
```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```

Apply the patch (3, p), (8, q)

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4       free(p); // +
5   }
6   else
7       q = p;
8   … // use q
9
10  free(q); // -
```

Hurdle 1: Unbounded Traces

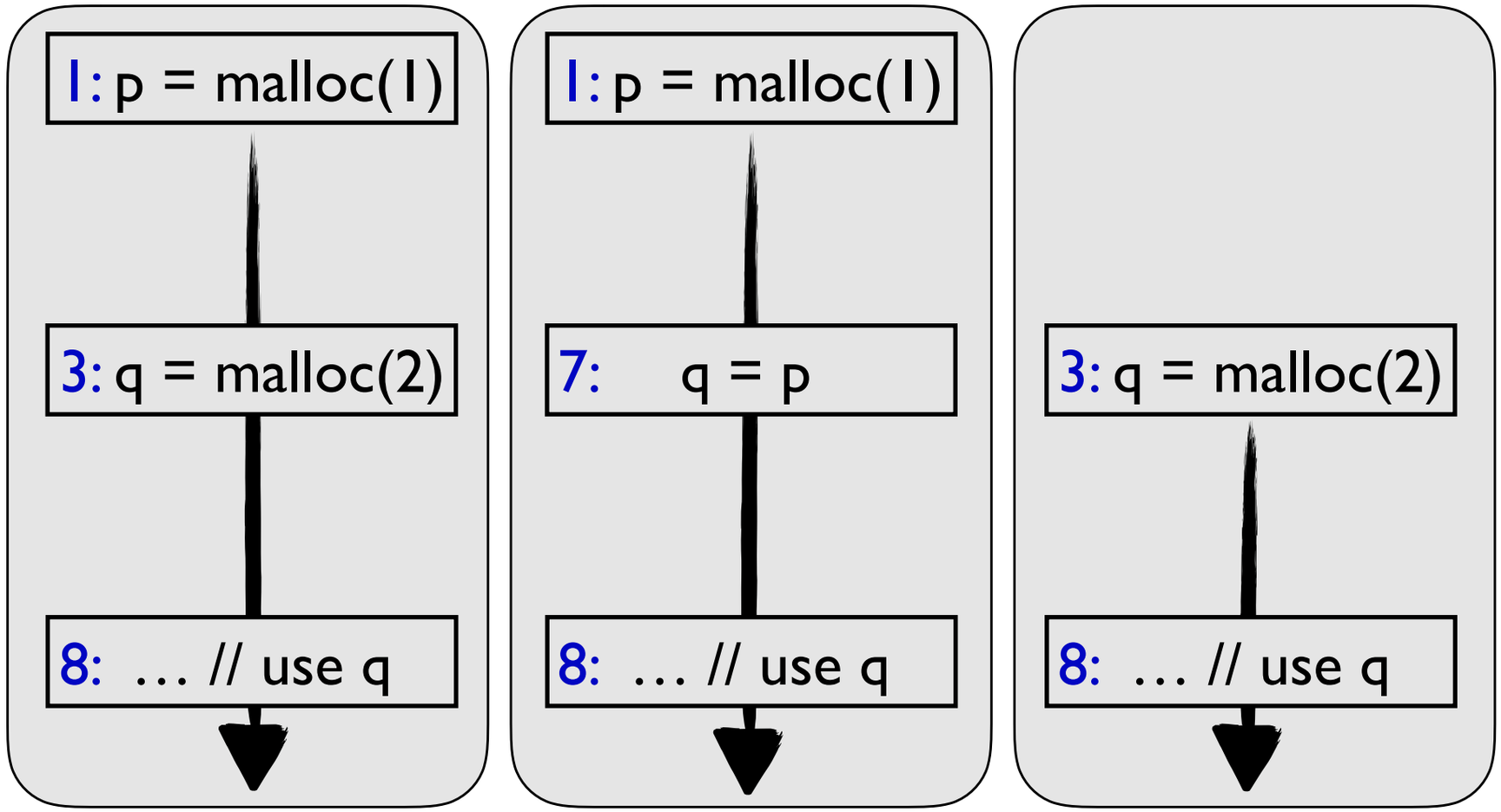
```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```



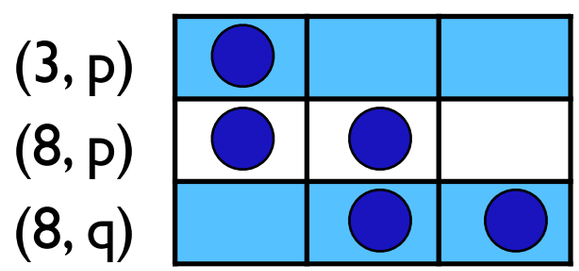
Unbounded number of object traces

Hurdle 2: Finding Exact Cover

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```



Unbounded number of object traces



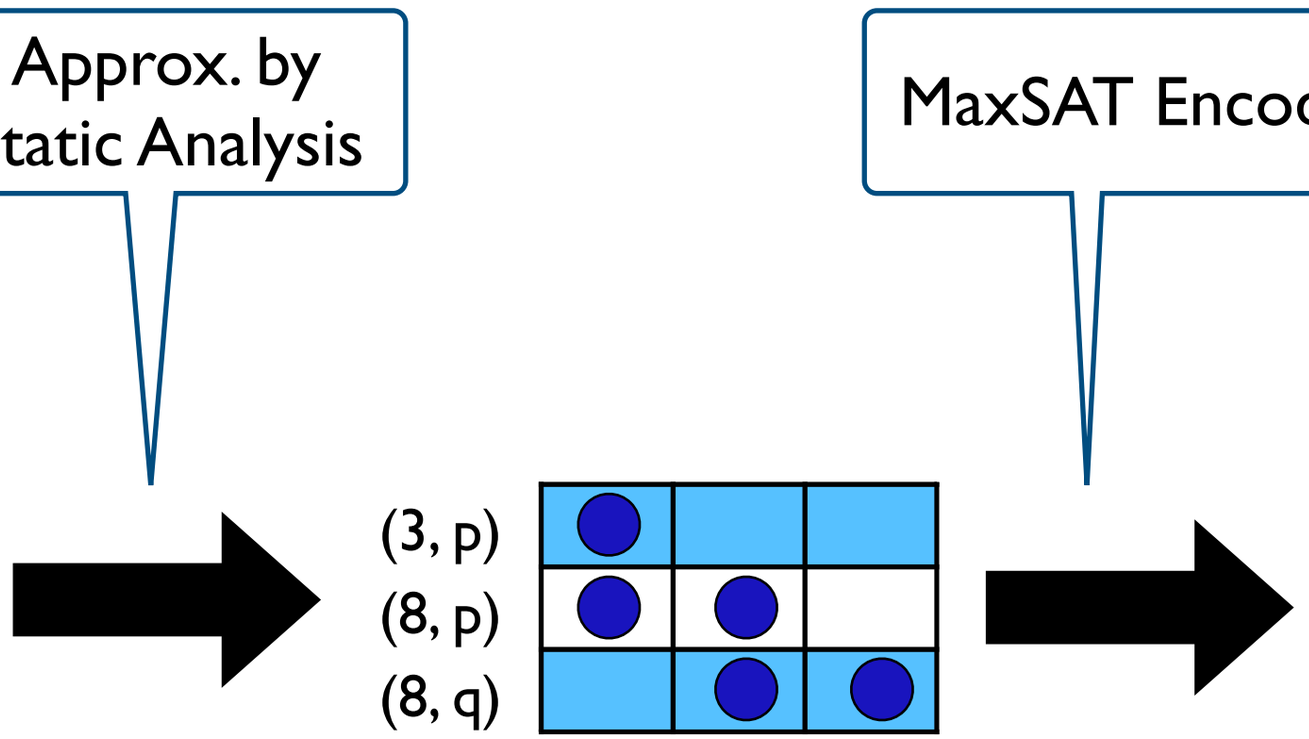
Well-known NP-complete problem

MemFix Algorithm

Approx. by Static Analysis

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4
5   }
6   else
7       q = p;
8   … // use q
9   free(p);
10  free(q);
```

Buggy Program



"Approximated" Exact Cover Problem

MaxSAT Encoding

```
1   p = malloc(1);
2   if(…) {
3       q = malloc(2);
4       free(p); // +
5   }
6   else
7       q = p;
8   … // use q
9
10  free(q); // -
```

Patched Program

Static Analysis by Abstract Interpretation

Abstract Domain $\mathbb{D} : \mathbb{C} \rightarrow \mathbb{S}$

<AllocSite, Must, MustNot, Patch, PatchNot> $\in \mathbb{S}$

```
1  while(…) {
2      p = malloc(1);
3      … // use p
4  }
5  … // use p
```

- Use-after-free
- Double-free
- **Uncertain**

Infinite number of object traces

- 2→3→1→5
- 2→3→1→2→3→1→5
- 2→3→1→2→3→1→2→3→1→5
- …

Static Analysis by Abstract Interpretation

Abstract Domain $\mathbb{D} : \mathbb{C} \rightarrow \mathbb{S}$

<AllocSite, Must, MustNot, Patch, PatchNot> $\in \mathbb{S}$

```
1  while(…) {
2      p = malloc(1);
3      … // use p
4  }
5  … // use p
```

- Use-after-free
- Double-free
- **Uncertain**

AllocSite	Must	MustNot	Patch	PatchNot
2	p	{}	(5, p)	…
2	{}	p	(1, p)	…

Experiments

ICSE'15

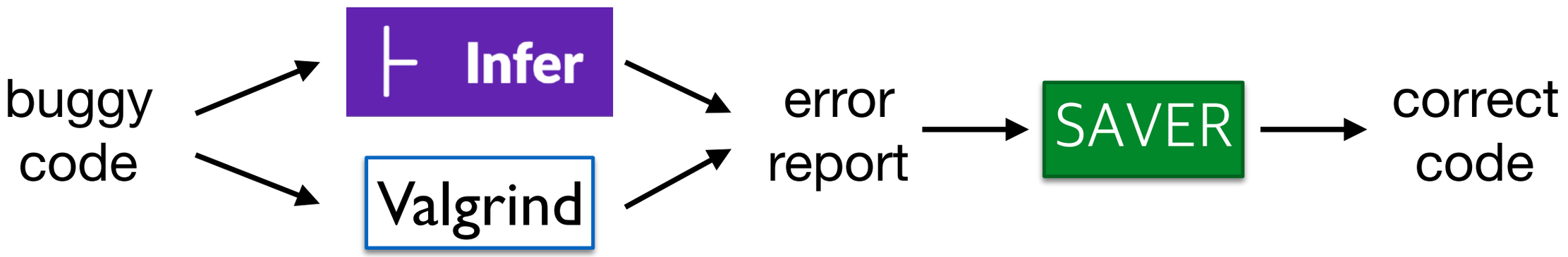
- 실험 1) Core utils
- 실험 2) Open-sources

Repo.	ML	DF	UAF	Total
	Fix/#Pgm.	Fix/#Pgm.	Fix/#Pgm.	Fix/#Pgm.
Binutils	4/10	1/5	2/5	7/20 (35%)
Git	1/10	1/4	2/6	4/20 (20%)
OpenSSH	6/10	5/7	1/3	12/20 (60%)
OpenSSL	5/10	3/5	1/5	9/20 (45%)
Tmux	5/10	0/3	0/7	5/20 (25%)
Total	21/50 (42%)	10/24 (42%)	6/26 (23%)	37/100 (37%)

Programs	LoC	#Al.	MemFIx		LeakFix	
			#Ins.	sec	#Ins.	sec
yes	553	1	1	< 1.0	✗	< 1.0
users	577	1	1	< 1.0	✗	< 1.0
unexpand	707	1	1	< 1.0	✗	< 1.0
tee	779	1	1	< 1.0	1	< 1.0
mktemp	794	4	✗	1.3	✗	< 1.0
tsort	920	3	✗	1.4	✗	< 1.0
paste	982	3	3	2.4	△/3	< 1.0
date	1,054	1	1	3.5	✗	< 1.0
cut	1,056	1	✗	2.0	✗	< 1.0
nl	1,063	4	4	4.0	✗	< 1.0
pinky	1,120	3	4	5.2	✗	< 1.0
cat	1,209	3	✗	9.3	✗	< 1.0
ln	1,258	2	✗	5.2	✗	< 1.0
printf	1,288	1	1	3.0	✗	< 1.0
stdbuf	1,605	3	3	1.3	✗	< 1.0
wc	1,669	1	1	7.3	△/2	< 1.0
shred	1,822	5	✗	31.1	✗	< 1.0
cp	1,926	8	✗	430.7	✗	< 1.0
install	2,076	1	✗	13.4	✗	< 1.0
who	2,156	8	✗	36.8	✗	< 1.0
tr	2,304	10	✗	20.0	✗	< 1.0
expr	2,378	9	✗	13.0	✗	< 1.0
stat	2,439	10	6	130.3	✗	< 1.0
dd	3,475	2	✗	52.2	✗	< 1.0

후속 연구

- SAVER (**S**calable, **A**utomatic, and **V**erified **E**rror **R**epair)
  - Scalability: 수십만 라인 코드에 적용 가능
  - Verifiability: 패치의 안전성 보장
  - Repairability: 평균 60% 이상 패치 성공
- 정적/동적 오류 탐지 도구(e.g., Infer, Valgrind)와 연동



Application to Intelligent Tutoring System

- 오류 수정 기술을 프로그래밍 교육에 적용 가능
- 현재 코딩 교육 자동 도구들의 한계: 개인화된 피드백 제공 못함

CodeOnWeb

oncoder

```
let rec diff : aexp * string -> aexp
= fun (e, x) ->
   match e with
   | Const n -> Const 0
   | Var a -> if (a <> x) then Const 0 else Const 1
   | Power (a, n) -> if (a <> x) then Const 0 else Times [Const n; Power (a, n-1)]
   | Times l ->
      begin
      match l with
      | [] -> Const 0
      | hd::tl -> Sum [Times ((diff (hd, x))::tl); Times [hd; diff (Times tl, x)]]
      end
   | Sum l -> Sum (List.map (fun e -> diff (e,x)) l)
```

제공된 솔루션

```
type aexp =
   |CONST of int
   | VAR of string
   | POWER of string * int
   | TIMES of aexp list
   | SUM of aexp list

type env = (string * int * int) list

let diff : aexp * string -> aexp
= fun (aexp, x) ->

   let rec deployEnv : env -> int -> aexp list
   = fun env flag ->
   match env with
   | hd::tl ->
   (
    match hd with
    |(x, c, p) ->
      if (flag = 0 && c = 0) then deployEnv tl flag
      else if (x = "const" && flag = 1 && c = 1) then deployEnv tl flag
      else if (p = 0) then (CONST c)::(deployEnv tl flag)
      else if (c = 1 && p = 1) then (VAR x)::(deployEnv tl flag)
      else if (p = 1) then TIMES[CONST c; VAR x]::(deployEnv tl flag)
      else if (c = 1) then POWER(x, p)::(deployEnv tl flag)
      else TIMES [CONST c; POWER(x, p)]::(deployEnv tl flag)
   )
   | [] -> []
   in

   let rec updateEnv : (string * int * int) -> env -> int -> env
   = fun elem env flag ->
   match env with
   | (hd::tl) ->
      (
      match hd with
      | (x, c, p) ->
         (
         match elem with
         |(x2, c2, p2) ->
            if (flag = 0) then
               if (x = x2 && p = p2) then (x, (c + c2), p)::tl
               else hd::(updateEnv elem tl flag)
            else
               if (x = x2) then (x, (c*c2), (p + p2))::tl
               else hd::(updateEnv elem tl flag)
         )
      )
   | [] -> elem::[]
   in

   let rec doDiff : aexp * string -> aexp
   = fun (aexp, x) ->
   match aexp with
   | CONST _ -> CONST 0
   | VAR v ->
      if (x = v) then CONST 1
      else CONST 0
   | POWER (v, p) ->
      if (p = 0) then CONST 0
      else if (x = v) then TIMES ((CONST p)::POWER (v, p-1)::[])
      else CONST 0
   | TIMES lst ->
      (
         match lst with
         (
            match (hd, diff_hd, tl, diff_tl) with
            | (CONST p, CONST s, [CONST r], CONST q) -> CONST (p*q + r*s)
            | (CONST p, _, _, CONST q) ->
               if (diff_hd = CONST 0 || tl = [CONST 0]) then CONST (p*q)
               else SUM [CONST(p*q); TIMES(diff_hd::tl)]
            | (_, CONST s, [CONST r], _) ->
               if (hd = CONST 0 || diff_tl = CONST 0) then CONST (r*s)
               else  SUM [TIMES [hd; diff_tl]; CONST(r*s)]
            | _ ->
               if (hd = CONST 0 || diff_tl = CONST 0) then TIMES(diff_hd::tl)
               else if (tl = [CONST 0] || diff_hd = CONST 0) then TIMES [hd; diff_tl]
               else SUM [TIMES [hd; diff_tl]; TIMES (diff_hd::tl)]
            )
         | [] -> CONST 0
      )
   | SUM lst -> SUM(List.map (fun aexp -> doDiff(aexp, x)) lst)
   in

   let rec simplify : aexp -> env -> int -> aexp list
   = fun aexp env flag ->
   match aexp with
   | SUM lst ->
      (
         match lst with
         | (CONST c)::tl -> simplify (SUM tl) (updateEnv ("const", c, 0) env 0) 0
         | (VAR x)::tl -> simplify (SUM tl) (updateEnv (x, 1, 1) env 0) 0
         | (POWER (x, p))::tl -> simplify (SUM tl) (updateEnv (x, 1, p) env 0) 0
         | (SUM lst)::tl -> simplify (SUM (List.append lst tl)) env 0
         | (TIMES lst)::tl ->
            (
               let l = simplify (TIMES lst) [] 1 in
               match l with
               | h::t ->
                  if (t = []) then List.append l (simplify (SUM tl) env 0)
                  else List.append (TIMES l::[]) (simplify (SUM tl) env 0)
               | [] -> []
            )
         | [] -> deployEnv env 0
      )
   | TIMES lst ->
      (
         match lst with
         | (CONST c)::tl -> simplify (TIMES tl) (updateEnv ("const", c, 0) env 1) 1
         | (VAR x)::tl -> simplify (TIMES tl) (updateEnv (x, 1, 1) env 1) 1
         | (POWER (x, p))::tl -> simplify (TIMES tl) (updateEnv (x, 1, p) env 1) 1
         | (SUM lst)::tl ->
            (
               let l = simplify (SUM lst) [] 0 in
               match l with
               | h::t ->
                  if (t = []) then List.append l (simplify (TIMES tl) env 1)
                  else List.append (SUM l::[]) (simplify (TIMES tl) env 1)
               | [] -> []  (* Feedback : Replace [] by ((Sum lst) :: tl) *)
            )
         | (TIMES lst)::tl -> simplify (TIMES (List.append lst tl)) env 1
         | [] -> deployEnv env 1
      )
   in

   let result = doDiff (aexp, x) in
   match result with
   | SUM _ -> SUM (simplify result [] 0)
   | TIMES _ -> TIMES (simplify result [] 1)
   | _ -> result
```

학생 제출 답안

Application to Intelligent Tutoring System

- 오류 수정 기술을 프로그래밍 교육에 적용 가능
- 현재 코딩 교육 자동 도구들의 한계: 개인화된 피드백 제공 못함

CodeOnWeb

oncoder

OOPSLA'18

FixML-generated feedback: ((Sum lst)::tl)

```
let rec diff : aexp * string -> aexp
= fun (e, x) ->
   match e with
   | Const n -> Const 0
   | Var a -> if (a <> x) then Const 0 else Const 1
   | Power (a, n) -> if (a <> x) then Const 0 else Times [Const n; Power (a, n-1)]
   | Times l ->
      begin
      match l with
      | [] -> Const 0
      | hd::tl -> Sum [Times ((diff (hd, x))::tl); Times [hd; diff (Times tl, x)]]
      end
   | Sum l -> Sum (List.map (fun e -> diff (e,x)) l)
```

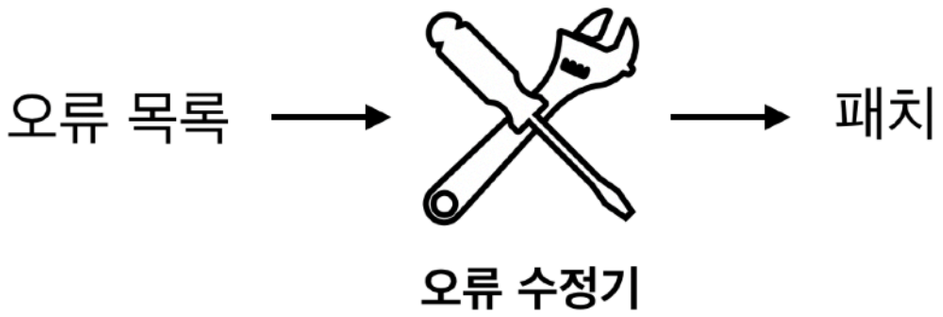
제공된 솔루션

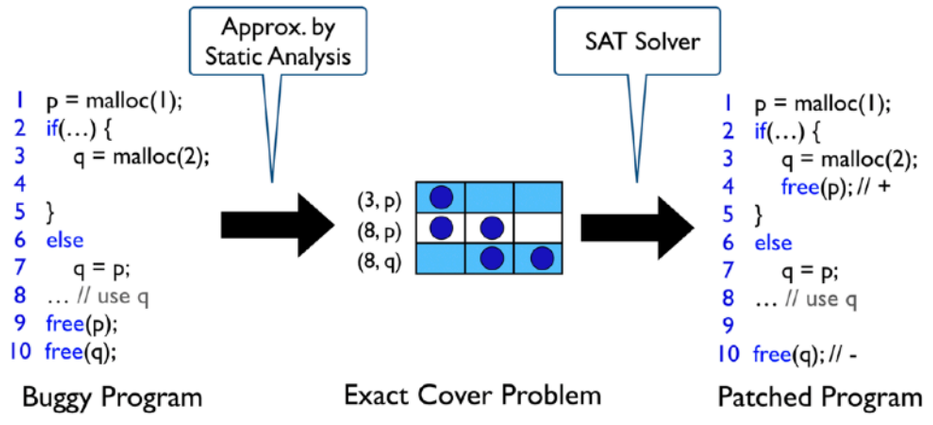
```
type aexp =
   |CONST of int
   | VAR of string
   | POWER of string * int
   | TIMES of aexp list
   | SUM of aexp list

type env = (string * int * int) list

let diff : aexp * string -> aexp
= fun (aexp, x) ->

   let rec deployEnv : env -> int -> aexp list
   = fun env flag ->
   match env with
   | hd::tl ->
   (
    match hd with
    |(x, c, p) ->
      if (flag = 0 && c = 0) then deployEnv tl flag
      else if (x = "const" && flag = 1 && c = 1) then deployEnv tl flag
      else if (p = 0) then (CONST c)::(deployEnv tl flag)
      else if (c = 1 && p = 1) then (VAR x)::(deployEnv tl flag)
      else if (p = 1) then TIMES[CONST c; VAR x]::(deployEnv tl flag)
      else if (c = 1) then POWER(x, p)::(deployEnv tl flag)
      else TIMES [CONST c; POWER(x, p)]::(deployEnv tl flag)
   )
   | [] -> []
   in

   let rec updateEnv : (string * int * int) -> env -> int -> env
   = fun elem env flag ->
   match env with
   | (hd::tl) ->
    (
    match hd with
    | (x, c, p) ->
      (
      match elem with
      |(x2, c2, p2) ->
        if (flag = 0) then
          if (x = x2 && p = p2) then (x, (c + c2), p)::tl
          else hd::(updateEnv elem tl flag)
        else
          if (x = x2) then (x, (c*c2), (p + p2))::tl
          else hd::(updateEnv elem tl flag)
      )
    )
   | [] -> elem::[]
   in

   let rec doDiff : aexp * string -> aexp
   = fun (aexp, x) ->
   match aexp with
   | CONST _ -> CONST 0
   | VAR v ->
     if (x = v) then CONST 1
     else CONST 0
   | POWER (v, p) ->
     if (p = 0) then CONST 0
     else if (x = v) then TIMES ((CONST p)::POWER (v, p-1)::[])
     else CONST 0
   | TIMES lst ->
     (
       match lst with
       (
         match (hd, diff_hd, tl, diff_tl) with
         | (CONST p, CONST s, [CONST r], CONST q) -> CONST (p*q + r*s)
         | (CONST p, _, _, CONST q) ->
           if (diff_hd = CONST 0 || tl = [CONST 0]) then CONST (p*q)
           else SUM [CONST(p*q); TIMES(diff_hd::tl)]
         | (_, CONST s, [CONST r], _) ->
           if (hd = CONST 0 || diff_tl = CONST 0) then CONST (r*s)
           else  SUM [TIMES [hd; diff_tl]; CONST(r*s)]
         | _ ->
           if (hd = CONST 0 || diff_tl = CONST 0) then TIMES(diff_hd::tl)
           else if (tl = [CONST 0] || diff_hd = CONST 0) then TIMES [hd; diff_tl]
           else SUM [TIMES [hd; diff_tl]; TIMES (diff_hd::tl)]
       )
     | [] -> CONST 0
     )
   | SUM lst -> SUM(List.map (fun aexp -> doDiff(aexp, x)) lst)
   in

   let rec simplify : aexp -> env -> int -> aexp list
   = fun aexp env flag ->
   match aexp with
   | SUM lst ->
     (
       match lst with
       | (CONST c)::tl -> simplify (SUM tl) (updateEnv ("const", c, 0) env 0) 0
       | (VAR x)::tl -> simplify (SUM tl) (updateEnv (x, 1, 1) env 0) 0
       | (POWER (x, p))::tl -> simplify (SUM tl) (updateEnv (x, 1, p) env 0) 0
       | (SUM lst)::tl -> simplify (SUM (List.append lst tl)) env 0
       | (TIMES lst)::tl ->
         (
           let l = simplify (TIMES lst) [] 1 in
           match l with
           | h::t ->
             if (t = []) then List.append l (simplify (SUM tl) env 0)
             else List.append (TIMES l::[]) (simplify (SUM tl) env 0)
           | [] -> []
         )
       | [] -> deployEnv env 0
     )
   | TIMES lst ->
     (
       match lst with
       | (CONST c)::tl -> simplify (TIMES tl) (updateEnv ("const", c, 0) env 1) 1
       | (VAR x)::tl -> simplify (TIMES tl) (updateEnv (x, 1, 1) env 1) 1
       | (POWER (x, p))::tl -> simplify (TIMES tl) (updateEnv (x, 1, p) env 1) 1
       | (SUM lst)::tl ->
         (
           let l = simplify (SUM lst) [] 0 in
           match l with
           | h::t ->
             if (t = []) then List.append l (simplify (TIMES tl) env 1)
             else List.append (SUM l::[]) (simplify (TIMES tl) env 1)
           | [] -> []  (* Feedback : Replace [] by ((Sum lst) :: tl) *)
         )
       | (TIMES lst)::tl -> simplify (TIMES (List.append lst tl)) env 1
       | [] -> deployEnv env 1
     )
   in

   let result = doDiff (aexp, x) in
   match result with
   | SUM _ -> SUM (simplify result [] 0)
   | TIMES _ -> TIMES (simplify result [] 1)
   | _ -> result
```

학생 제출 답안

Summary

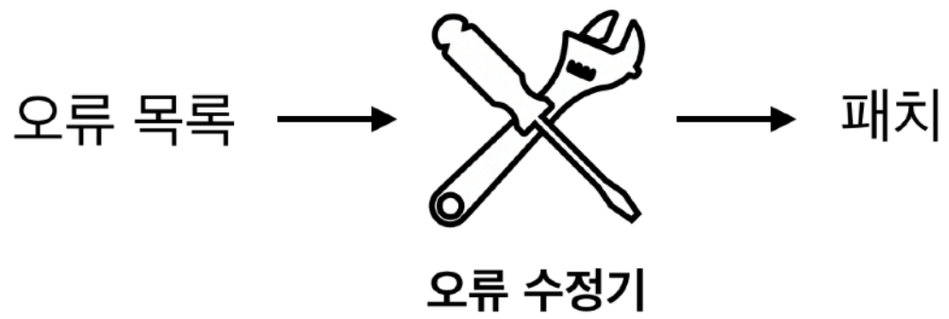
- Technology for automatic software repair
- MemFix focuses on memory deallocation errors
- Very exciting and new research area!

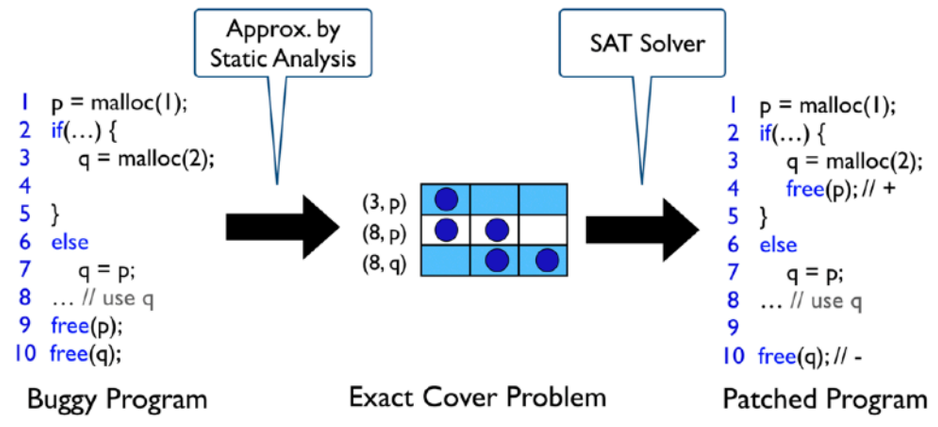




Summary

- Technology for automatic software repair
- MemFix focuses on memory deallocation errors
- Very exciting and new research area!





Thank you