COSE312: Compilers

Lecture 8 — Bottom-Up Parsing

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2015 Fall

Expression Grammar

Expression grammar:

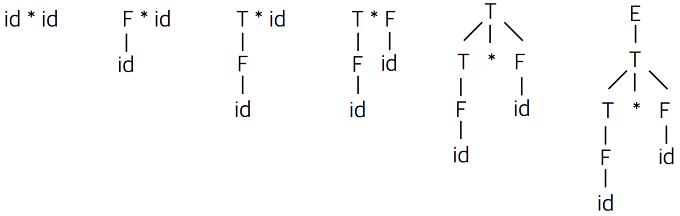
$$\boldsymbol{E \to E + E \mid E * E \mid (E) \mid \mathbf{id}}$$

Unambiguous version:

$$\begin{array}{llll}
(1) & E & \to & E + T \\
(2) & E & \to & T \\
(3) & T & \to & T * F \\
(4) & T & \to & F \\
(5) & F & \to & (E) \\
(6) & F & \to & \mathbf{id}
\end{array}$$

Bottom-Up Parsing

- Construct a parse tree beginning at the leaves and working up towards the root.
- Ex) for input $\mathbf{id} * \mathbf{id}$:



- A process of "reducing" a string w to the start symbol.
- Construct the rightmost-derivation in reverse:

$$E \Rightarrow T \Rightarrow T * F \Rightarrow T * \mathbf{id} \Rightarrow F * \mathbf{id} \Rightarrow \mathbf{id} * \mathbf{id}$$

Handle

- In bottom-up parsing, we have to make decisions about when to reduce and what production to apply.
- For instance, for $T * \mathbf{id}$, we reduce $\mathbf{id}$ to F because reducing T does not lead to a right-sentential form.
- Handle: a substring that matches the body of a production and whose reduction leads to a right-sentential form.
- A bottom-up parsing is a process of finding a handle and reducing it.

Right Sentential Form	Handle	Reducing Production
$\mathbf{id}_1 * \mathbf{id}_2$	$\mathbf{id}_1$	$F \to \mathbf{id}$
$F * \mathbf{id}_2$	F	$T \to F$
$T * \mathbf{id}_2$	$\mathbf{id}_2$	$F \to \mathbf{id}$
$T * F$	$T * F$	$T \to T * F$
T	T	$E \to T$

LR Parsing

- The most prevalent type of bottom-up parsing.
- Handles are recognized by a deterministic finite automaton.
- LR(k)
  - "L": Left-to-right scanning of the input
  - "R": Rightmost-derivation in reverse
  - "k": k-tokens lookahead
- We consider LR(0), SLR, LR(1), LALR(1) parsing algorithms.

Why LR parsing?

- Widely used:
  - Most automatic parser generators are based on LR parsing
- General and powerful:
  - LL(k) $\subseteq$ LR(k)
  - Most programming languages can be described by LR grammars

LR Parsing Overview

An LR parser has a *stack* and *an input*. Based on the lookahead and stack contents, perform two kinds of actions:

- Shift
  - performed when the top of the stack is not a handle
  - move the first input token to the stack
- Reduce
  - performed when the top of the stack is a handle
  - choose a rule $X \rightarrow A\ B\ C$; pop C, B, A; push X

Example: $\mathbf{id} * \mathbf{id}$

$$
\begin{array}{llll}
(1) & E & \rightarrow & E + T \\
(2) & E & \rightarrow & T \\
(3) & T & \rightarrow & T * F \\
(4) & T & \rightarrow & F \\
(5) & F & \rightarrow & (E) \\
(6) & F & \rightarrow & \mathbf{id}
\end{array}
$$

Stack	Input	Action
	$\mathbf{id} * \mathbf{id}\$$	shift
id	$*\mathbf{id}\$$	reduce by $F \rightarrow \mathbf{id}$
F	$*\mathbf{id}\$$	reduce by $T \rightarrow F$
T	$*\mathbf{id}\$$	shift
$T*$	$\mathbf{id}\$$	shift
$T * \mathbf{id}$	$\$$	reduce by $F \rightarrow \mathbf{id}$
$T * F$	$\$$	reduce by $T \rightarrow T * F$
T	$\$$	reduce by $E \rightarrow T$
E	$\$$	shift (accept)

Recognizing Handles

By using a deterministic finite automaton. The transition table (parsing table) for the expression grammar:

State	**id**	$+$	$*$	(	)	\$	E	T	F
0	$s5$			$s4$			$g1$	$g2$	$g3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$g8$	$g2$	$g3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$g9$	$g3$
7	$s5$			$s4$					$g10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Recognizing Handles

- Given a parse state

Stack	Input
$T*$	$\mathbf{id}\$$

1. Run the DFA on stack, treating shift/goto actions as edges of the DFA: $\mathbf{0} \to \mathbf{2} \to \mathbf{7}$.
2. Look up the entry $(\mathbf{7}, \mathbf{id})$ of the transition table: shift 5. (not a handle)
3. Push $\mathbf{id}$ onto the stack.

- Given a parse state

Stack	Input
$T * \mathbf{id}$	$\$$

1. Run the DFA on stack: $\mathbf{0} \to \mathbf{2} \to \mathbf{7} \to \mathbf{5}$.
2. Look up the entry $(\mathbf{5}, \$)$ of the transition table: reduce 6. (handle)
3. Reduce by rule 6: $F \to \mathbf{id}$

LR Parsing Process

To avoid rescanning the stack for each token, the stack maintains DFA states:

Stack	Symbols	Input	Action
0		$\mathbf{id} * \mathbf{id}\$$	shift to 5
$\mathbf{0\ 5}$	**id**	$*\mathbf{id}\$$	reduce by 6 ($F \to \mathbf{id}$)
$\mathbf{0\ 3}$	F	$*\mathbf{id}\$$	reduce by 4 ($T \to F$)
$\mathbf{0\ 2}$	T	$*\mathbf{id}\$$	shift to 7
$\mathbf{0\ 2\ 7}$	$T*$	$\mathbf{id}\$$	shift to 5
$\mathbf{0\ 2\ 7\ 5}$	$T * \mathbf{id}$	$\$$	reduce by 6 ($F \to \mathbf{id}$)
$\mathbf{0\ 2\ 7\ 10}$	$T * F$	$\$$	reduce by 3 ($T \to T * F$)
$\mathbf{0\ 2}$	T	$\$$	reduce by 2 ($E \to T$)
$\mathbf{0\ 1}$	E	$\$$	accept

LR Parsing Algorithm

Repeat the following:

1. Look up top stack state, and input symbol, to get an action.
2. If the action is
   - Shift(n): Advance input one token; push n on stack
   - Reduce(k):
     1. Pop stack as many times as the number of symbols on the right hand side of rule k
     2. Let X be the left-hand-side symbol of rule k
     3. In the state now on top of stack, look up X to get "goto n"
     4. Push n on top of stack
   - Accept: Stop parsing, report success.
   - Error: Stop parsing, report failure.

LR(0) and SLR Parser Generation

For the augmented grammar

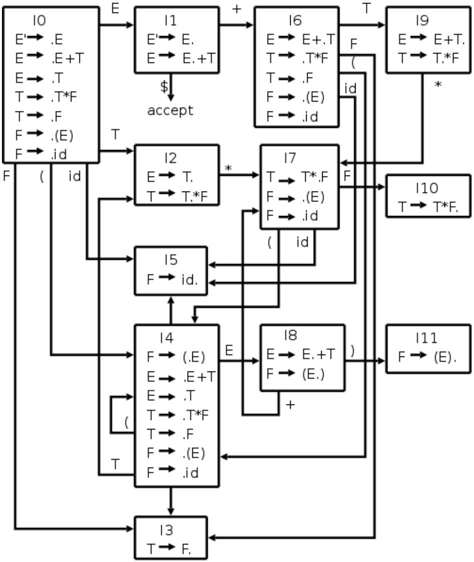
$$
\begin{array}{llcl}
(0) & E' & \rightarrow & E \\
(1) & E & \rightarrow & E + T \\
(2) & E & \rightarrow & T \\
(3) & T & \rightarrow & T * F \\
(4) & T & \rightarrow & F \\
(5) & F & \rightarrow & (E) \\
(6) & F & \rightarrow & \text{id}
\end{array}
$$

construct the parsing table:

State	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$g1$	$g2$	$g3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$g8$	$g2$	$g3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$g9$	$g3$
7	$s5$			$s4$					$g10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

LR(0) Automaton

The parsing table is constructed from the LR(0) automaton:



LR(0) Items

A state is a set of *items*.

- An item is a production with a dot somewhere on the body.
- The items for $A \to XYZ$:

$$\begin{array}{lcl} A & \to & .XYZ \\ A & \to & X.YZ \\ A & \to & XY.Z \\ A & \to & XYZ. \end{array}$$

- $A \to \epsilon$ has only one item $A \to \cdot$.
- An item indicates how much of a production we have seen in parsing.

The Initial Parse State

- Initially, the parser will have an empty stack, and the input will be a complete $\boldsymbol{E}$-sentence, indicated by item

$$\boldsymbol{E'} \to .\boldsymbol{E}$$

where the dot indicates the current position of the parser.

- Collect all of the items reachable from the initial item without consuming any input tokens:

$$\boldsymbol{I_0} = \boxed{\begin{array}{lcl} \boldsymbol{E'} & \to & .\boldsymbol{E} \\ \boldsymbol{E} & \to & .\boldsymbol{E} + \boldsymbol{T} \\ \boldsymbol{E} & \to & .\boldsymbol{T} \\ \boldsymbol{T} & \to & .\boldsymbol{T} * \boldsymbol{F} \\ \boldsymbol{T} & \to & .\boldsymbol{F} \\ \boldsymbol{F} & \to & .(\boldsymbol{E}) \\ \boldsymbol{F} & \to & .\mathbf{id} \end{array}}$$

Closure of Item Sets

IF I is a set of items for a grammar G, then $CLOSURE(I)$ is the set of items constructed from I by the two rules:

1. Initially, add every item in I to $CLOSURE(I)$.
2. If $A \to \alpha.B\beta$ is in $CLOSURE(I)$ and $B \to \gamma$ is a production, then add the item $B \to .\gamma$ to $CLOSURE(I)$, if it is not already there. Apply this rule until no more new items can be added to $CLOSURE(I)$.

In algorithm:

$CLOSURE(I) =$

repeat

  for any item $A \to \alpha.B\beta$ in I

    for any production $B \to \gamma$

      $I = I \cup \{X \to .\gamma\}$

until I does not change

return I

Construction of LR(0) Automaton

For the initial state

$$I_0 = \boxed{\begin{array}{lcl} E' & \rightarrow & .E \\ E & \rightarrow & .E + T \\ E & \rightarrow & .T \\ T & \rightarrow & .T * F \\ T & \rightarrow & .F \\ F & \rightarrow & .(E) \\ F & \rightarrow & .\mathrm{id} \end{array}}$$

construct the next states for each grammar symbol.

Consider E:

1. Find all items of form $A \rightarrow \alpha.E\beta$: $\{E' \rightarrow .E, E \rightarrow .E + T\}$
2. Move the dot over E: $\{E' \rightarrow E., E \rightarrow E. + T\}$
3. Closure it:

$$I_1 = \boxed{\begin{array}{lcl} E' & \rightarrow & E. \\ E & \rightarrow & E. + T \end{array}}$$

Construction of LR(0) Automaton

$$I_0 = \boxed{\begin{array}{lcl} E' & \rightarrow & .E \\ E & \rightarrow & .E+T \\ E & \rightarrow & .T \\ T & \rightarrow & .T*F \\ T & \rightarrow & .F \\ F & \rightarrow & .(E) \\ F & \rightarrow & .\mathrm{id} \end{array}}$$

Consider (:

1. Find all items of form $A \rightarrow \alpha.(\beta$: $\{F \rightarrow .(E)\}$
2. Move the dot over E: $\{F \rightarrow (.E)\}$
3. Closure it:

$$I_4 = \boxed{\begin{array}{lcl} F & \rightarrow & (.E) \\ E & \rightarrow & .E+T \\ E & \rightarrow & .T \\ T & \rightarrow & .T*F \\ T & \rightarrow & .F \\ F & \rightarrow & .(E) \\ F & \rightarrow & .\mathrm{id} \end{array}}$$

Goto

When I is a set of items and X is a grammar symbol (terminals and nonterminals, $GOTO(I, X)$ is defined to be the closure of the set of all items $A \rightarrow \alpha X.\beta$ such that $A \rightarrow \alpha.X\beta$ is in I.
In algorithm:

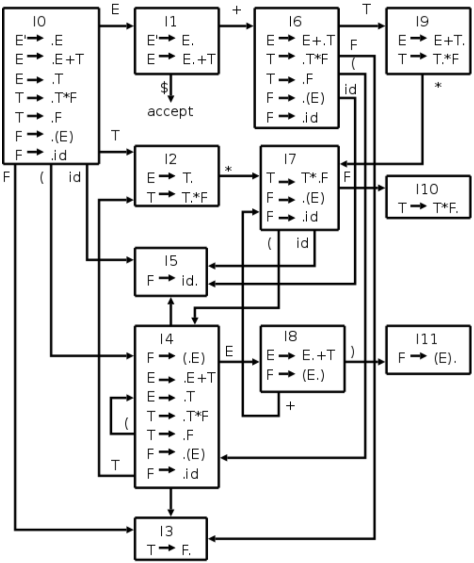
$GOTO(I, X) =$
  set J to the empty set
    for any item $A \rightarrow \alpha.X\beta$ in I
      add $A \rightarrow \alpha X.\beta$ to J
  return $CLOSURE(J)$

Construction of LR(0) Automaton

- T: the set of states
- E: the set of edges

Initialize T to $\{CLOSURE(\{S' \to S\})\}$
Initialize E to empty
repeat
  for each state I in T
    for each item $A \to \alpha.X\beta$ in I
      let J be $GOTO(I, X)$
      $T = T \cup \{J\}$
      $E = E \cup \{I \xrightarrow{X} J\}$
until E and T do not change

LR(0) Automaton



Construction of LR(0) Parsing Table

- For each edge $I \xrightarrow{X} J$ where X is a terminal, we put the action *shift* J at position (I, X) of the table.
- If X is a nonterminal, we put an *goto* J at position (I, X).
- For each state I containing an item $S' \to S.$, we put an *accept* action at $(I, \$)$.
- Finally, for a state containing an item $A \to \gamma.$ (production n with the dot at the end), we put a *reduce* n action at (I, Y) for every token Y.

LR(0) Parsing Table

State	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$g1$	$g2$	$g3$
1		$s6$				acc			
2	$r2$	$r2$	$r2, s7$	$r2$	$r2$	$r2$			
3	$r4$	$r4$	$r4$	$r4$	$r4$	$r4$			
4	$s5$			$s4$			$g8$	$g2$	$g3$
5	$r6$	$r6$	$r6$	$r6$	$r6$	$r6$			
6	$s5$			$s4$				$g9$	$g3$
7	$s5$			$s4$					$g10$
8		$s6$			$s11$				
9	$r1$	$r1$	$r1, s7$	$r1$	$r1$	$r1$			
10	$r3$	$r3$	$r3$	$r3$	$r3$	$r3$			
11	$r5$	$r5$	$r5$	$r5$	$r5$	$r5$			

Conflicts

The parsing table may contain conflicts (duplicated entries). Two kinds of conflicts:

- Shift/reduce conflicts: the parser cannot tell whether to shift or reduce.
- Reduce/reduce conflicts: the parser knows to reduce, but cannot tell which reduction to perform.

If the LR(0) parsing table for a grammar contains no conflicts, the grammar is in LR(0) grammar.

Construction of SLR Parsing Table

- For each edge $I \xrightarrow{X} J$ where X is a terminal, we put the action *shift* J at position (I, X) of the table.
- If X is a nonterminal, we put an *goto* J at position (I, X).
- For each state I containing an item $S' \to S.$, we put an *accept* action at $(I, \$)$.
- Finally, for a state containing an item $A \to \gamma.$ (production n with the dot at the end), we put a *reduce* n action at (I, Y) for every token $Y \in FOLLOW(A)$.

SLR Parsing Table

State	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$g1$	$g2$	$g3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$g8$	$g2$	$g3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$g9$	$g3$
7	$s5$			$s4$					$g10$
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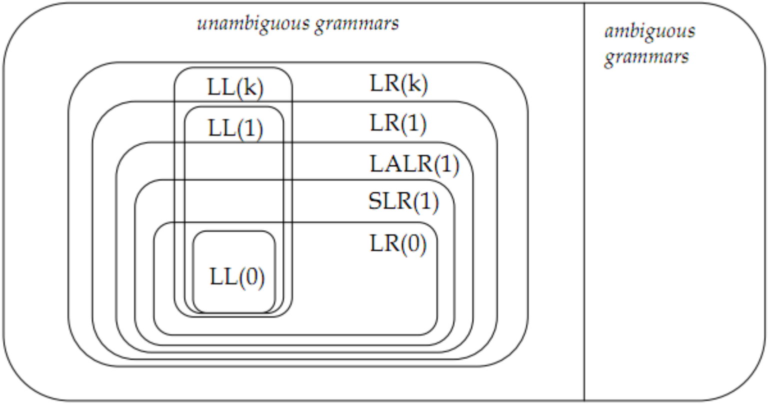
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

More Powerful LR Parsers

We can extend LR(0) parsing to use one symbol of lookahead on the input:

- LR(1) parsing:
  - The parsing table is based on LR(1) items, $(A \to \alpha.B\beta, a)$
  - Make full use of the lookahead symbol.
  - Generate a large set of states.
- LALR(1) parsing.
  - Based on the LR(0) items.
  - Introducting lookaheads into the LR(0) items.
  - Parsing tables have many fewer states than LR(1), no bigger than that of SLR.

Summary


unambiguous grammars
ambiguous grammars
LL(k)
LR(k)
LL(1)
LR(1)
LALR(1)
SLR(1)
LR(0)
LL(0)