COSE215: Theory of Computation

Lecture 9-2 — More Exercises on CFG

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Exercise 1

Design a context-free grammar for the language:

$$L = \{b^n a^m b^{2n} \mid n, m \geq 0\}$$

Exercise 2

Design a CFG for well-formed nested parentheses and square brackets: e.g.,

```
([ [ [ ()() [ ][ ] ] ]([ ]) ])
```

Exercise 3

Describe the language defined by the context-free grammar:

$$S \rightarrow aSbS \mid bSaS \mid \epsilon$$

Exercise 4

Consider the set of all regular expressions:

$$L = \{\epsilon, \emptyset, a, b, a + b, a \cdot b, a^*, b^*, \epsilon^*, \emptyset^*, (a + b)^*, (a \cdot b)^*, \ldots\}$$

Is L regular or context-free?

Exercise 5

Consider the following fragment of the English language:

$$
\begin{array}{rcl}
\langle \text{SENTENCE} \rangle & \rightarrow & \langle \text{NOUN-PHRASE} \rangle \langle \text{VERB-PHRASE} \rangle \\
\langle \text{NOUN-PHRASE} \rangle & \rightarrow & \langle \text{CMPLX-NOUN} \rangle \mid \langle \text{CMPLX-NOUN} \rangle \langle \text{PREP-PHRASE} \rangle \\
\langle \text{VERB-PHRASE} \rangle & \rightarrow & \langle \text{CMPLX-VERB} \rangle \mid \langle \text{CMPLX-VERB} \rangle \langle \text{PREP-PHRASE} \rangle \\
\langle \text{PREP-PHRASE} \rangle & \rightarrow & \langle \text{PREP} \rangle \langle \text{CMPLX-NOUN} \rangle \\
\langle \text{CMPLX-NOUN} \rangle & \rightarrow & \langle \text{ARTICLE} \rangle \langle \text{NOUN} \rangle \\
\langle \text{CMPLX-VERB} \rangle & \rightarrow & \langle \text{VERB} \rangle \mid \langle \text{VERB} \rangle \langle \text{NOUN-PHRASE} \rangle \\
\langle \text{ARTICLE} \rangle & \rightarrow & \texttt{a} \mid \texttt{the} \\
\langle \text{NOUN} \rangle & \rightarrow & \texttt{boy} \mid \texttt{girl} \mid \texttt{flower} \\
\langle \text{VERB} \rangle & \rightarrow & \texttt{touches} \mid \texttt{likes} \mid \texttt{sees} \\
\langle \text{PREP} \rangle & \rightarrow & \texttt{with}
\end{array}
$$

Is "`a girl with a flower likes the boy`" English? If so, show a derivation.

cf) Grammar for the C programming language

- ANSI C grammar:
  `http://www.lysator.liu.se/c/ANSI-C-grammar-y.html`
- C18 standard:
  `https://web.archive.org/web/20181230041359if_/http://www.open-std.org/jtc1/sc22/wg14/www/abq/c17_updated_proposed_fdis.pdf`

Exercise 6

Consider the context-free grammar:

$$E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$$

and consider the string "$+ * -xyxy$".

1. Find the leftmost derivation of the string.
2. Find the rightmost derivation of the string.