

COSE 215: Theory of Computation

Lecture 16

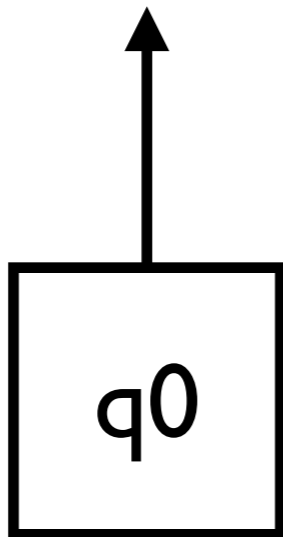
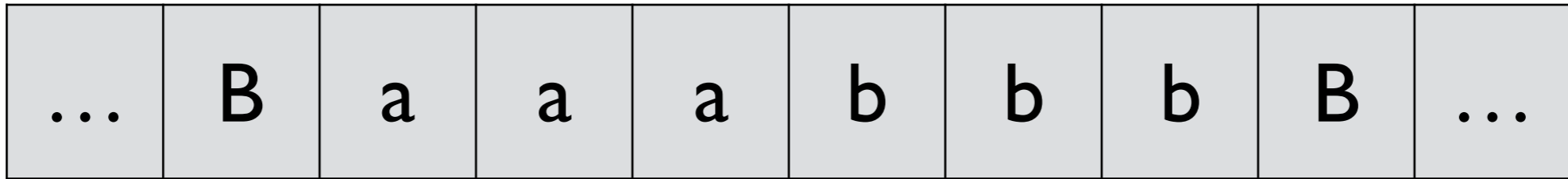
Examples of Turing Machines (I)

Hakjoo Oh
2019 Spring

Example 1. Design a Turing machine that accepts $L = \{a^n b^n \mid n \geq 1\}$.

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$$

$$\begin{array}{lll} \delta(q_0, a) = (q_1, x, R) & \delta(q_2, y) = (q_2, y, L) & \delta(q_0, y) = (q_3, y, R) \\ \delta(q_1, a) = (q_1, a, R) & \delta(q_2, a) = (q_2, a, L) & \delta(q_3, y) = (q_3, y, R) \\ \delta(q_1, y) = (q_1, y, R) & \delta(q_2, x) = (q_0, x, R) & \delta(q_3, B) = (q_4, B, R) \\ \delta(q_1, b) = (q_2, y, L) & & \end{array}$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

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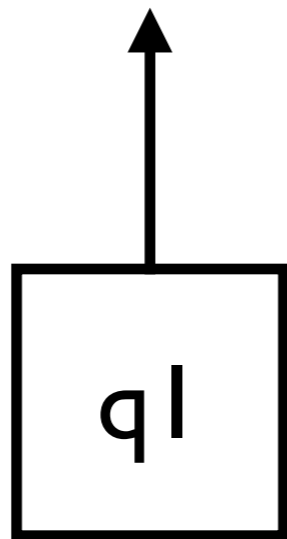
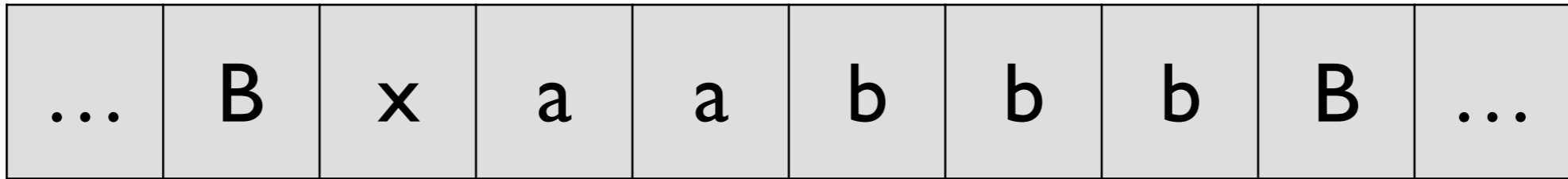
$$\delta(q_2, a) = (q_2, a, L)$$

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$$\delta(q_0, y) = (q_3, y, R)$$

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$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

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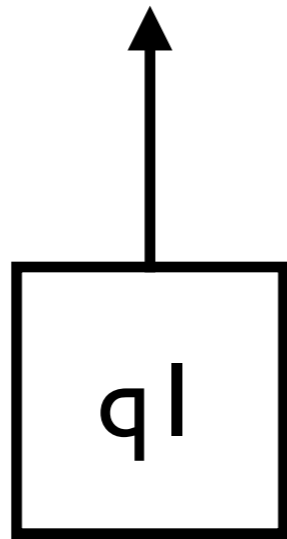
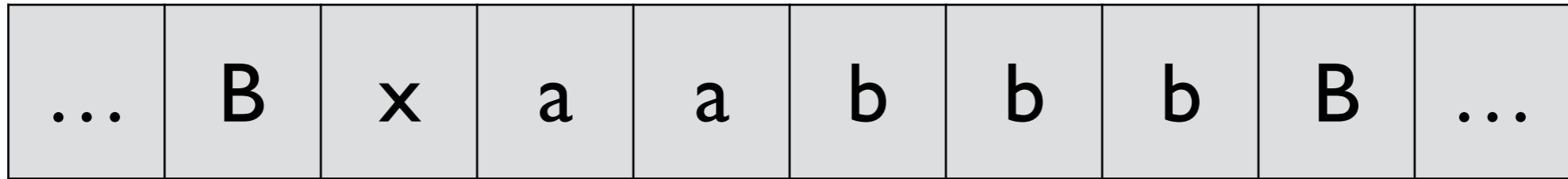
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(In q_1 , move right to search for the leftmost 'b')

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

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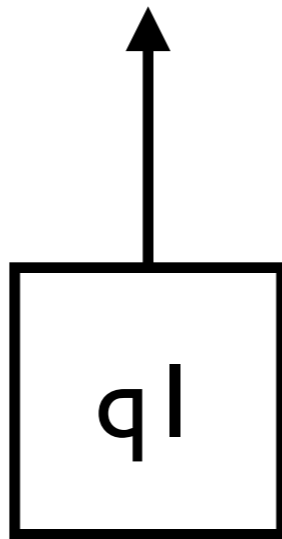
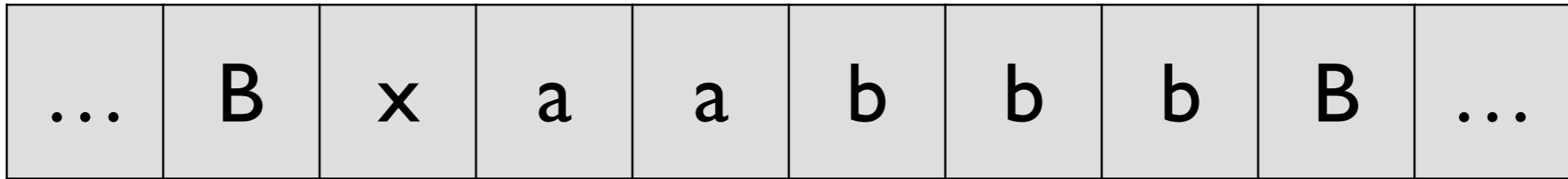
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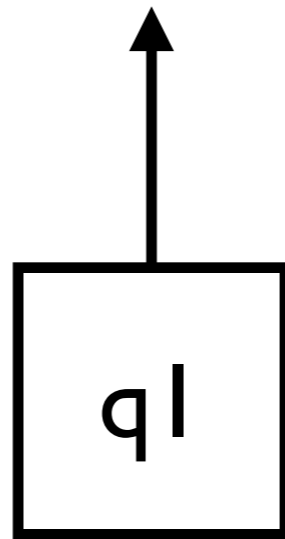
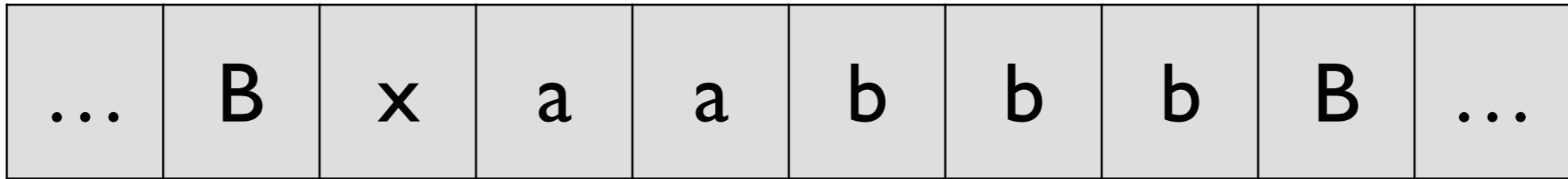
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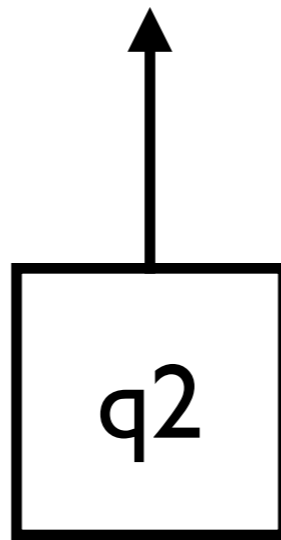
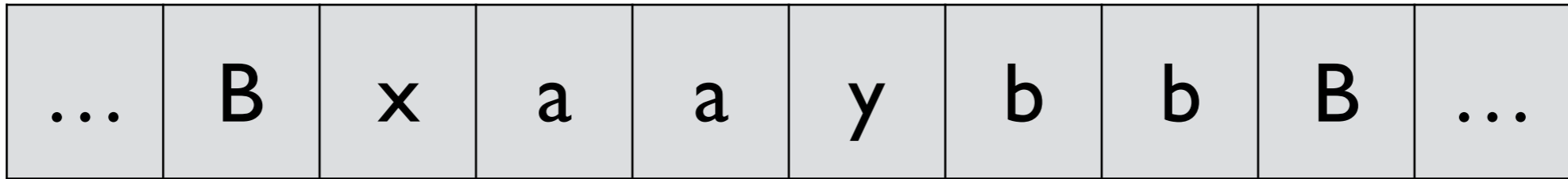
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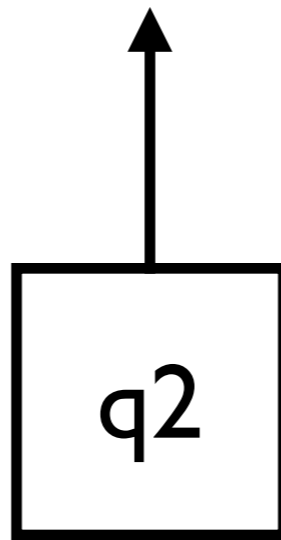
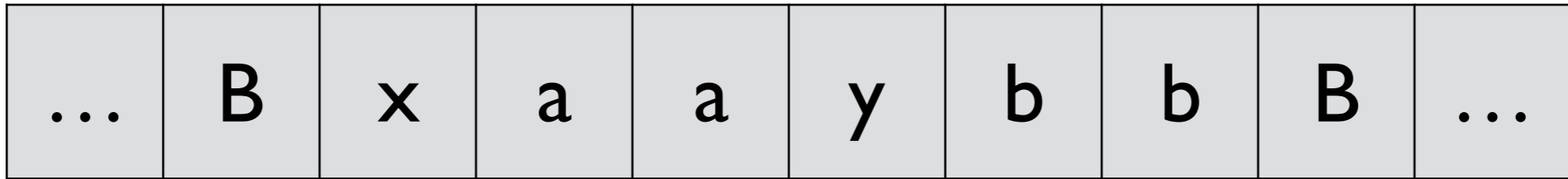
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



(In q_2 , move left to search for the leftmost 'a')

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

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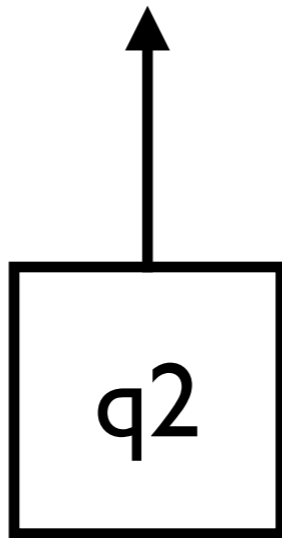
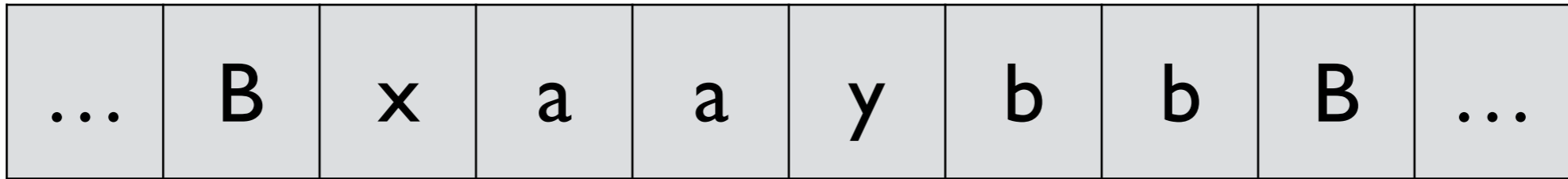
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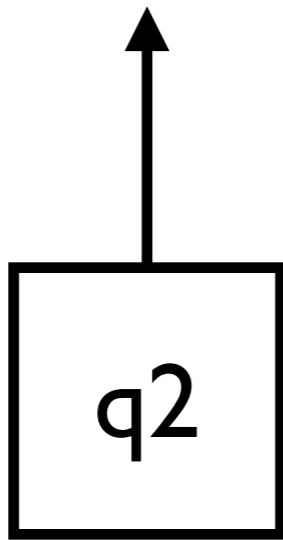
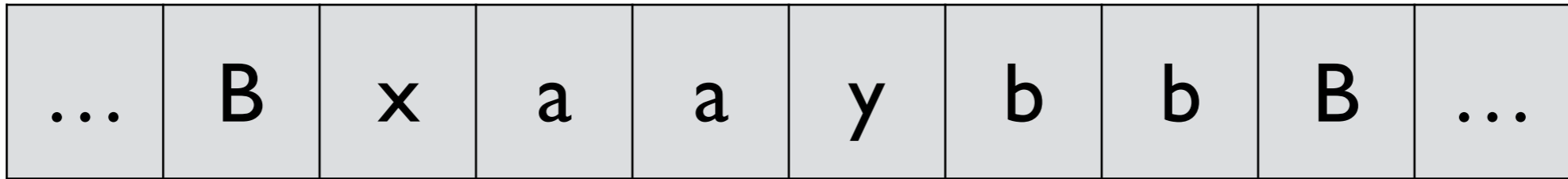
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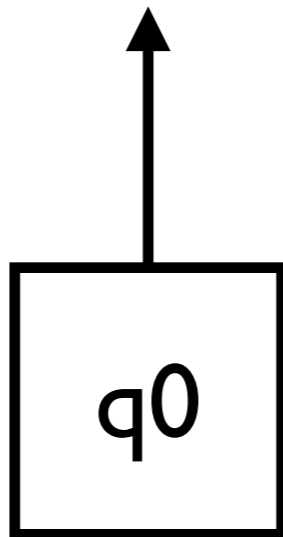
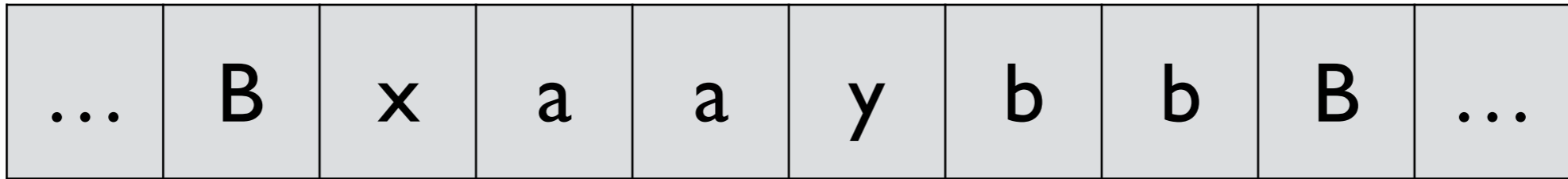
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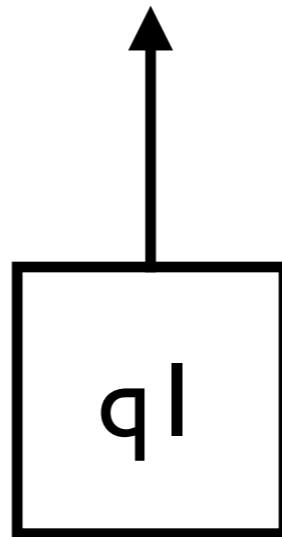
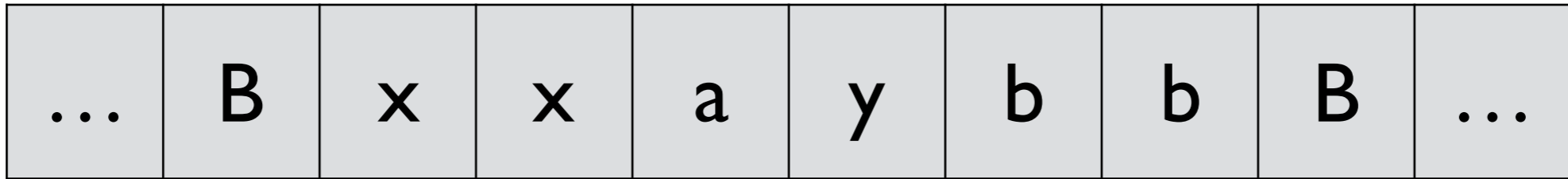
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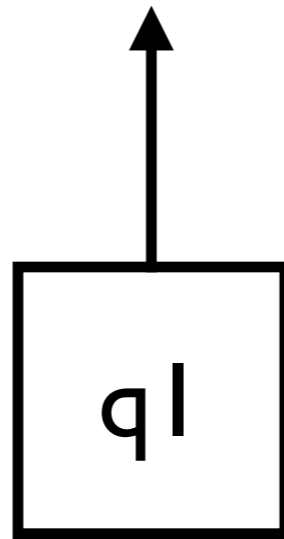
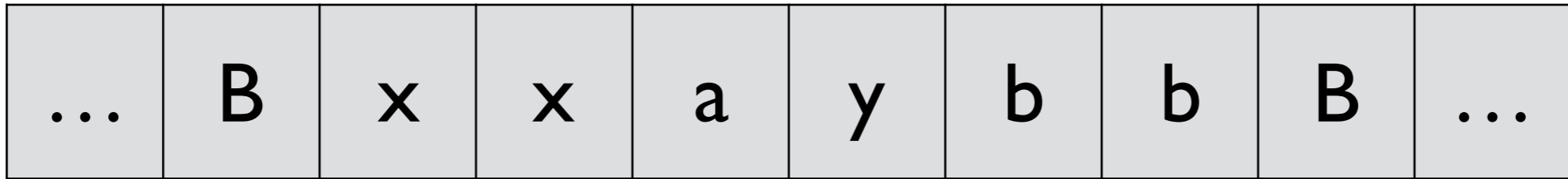
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$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



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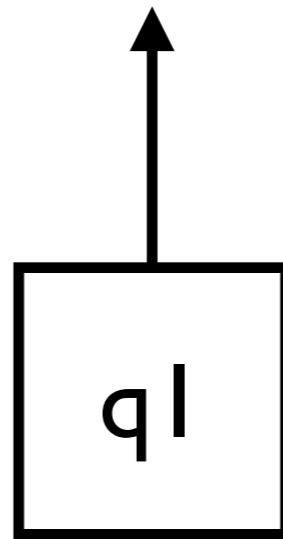
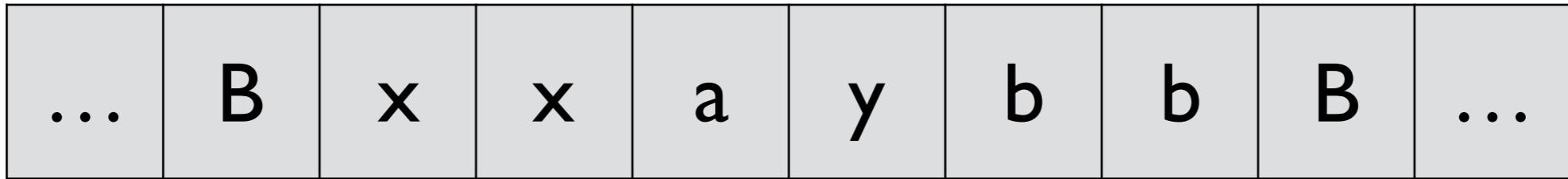
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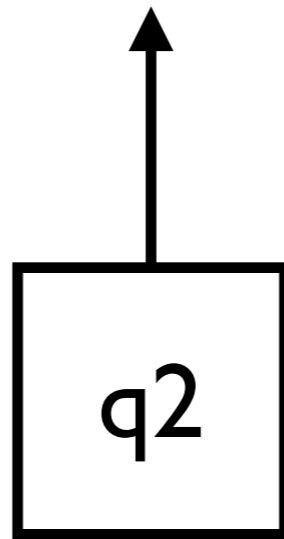
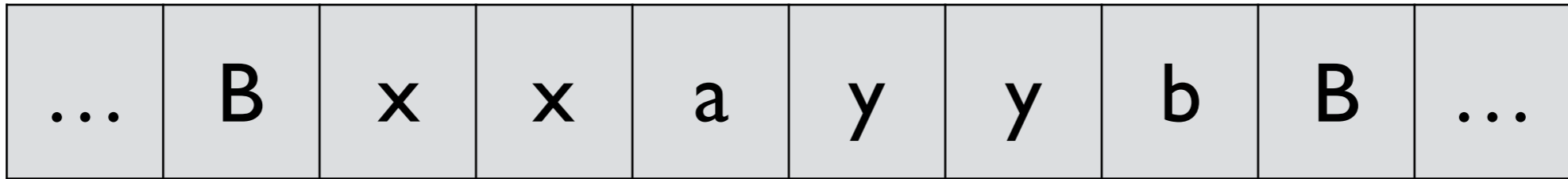
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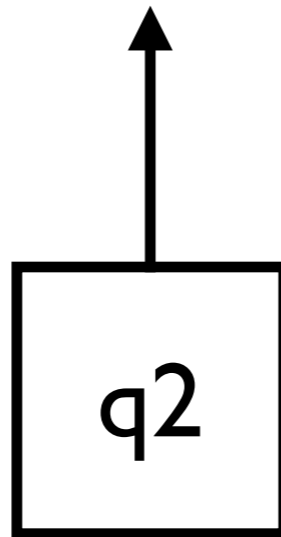
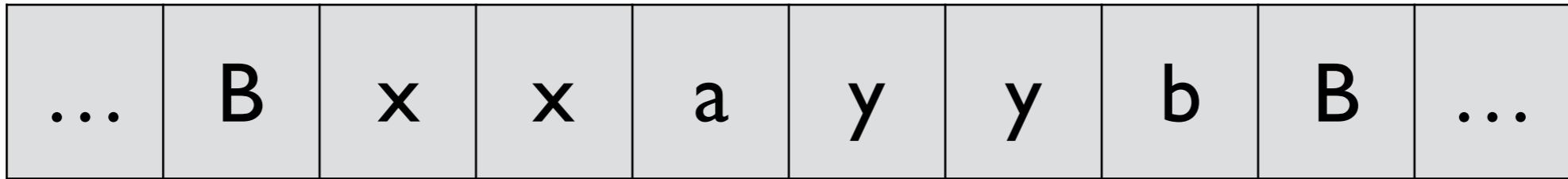
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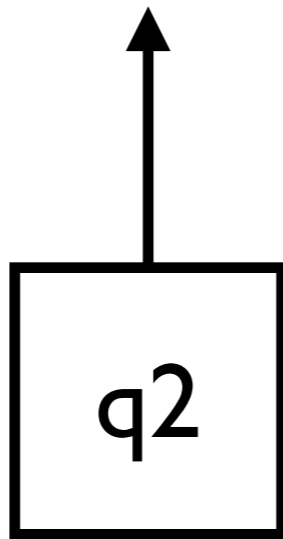
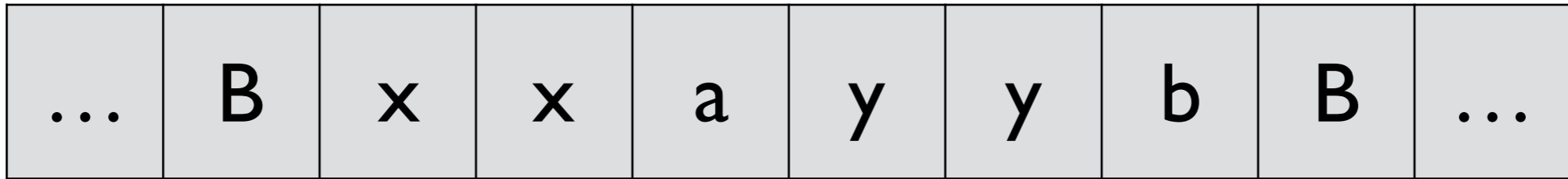
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



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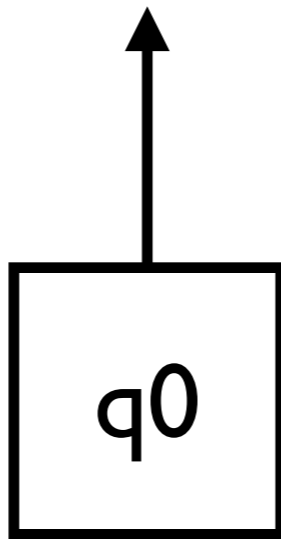
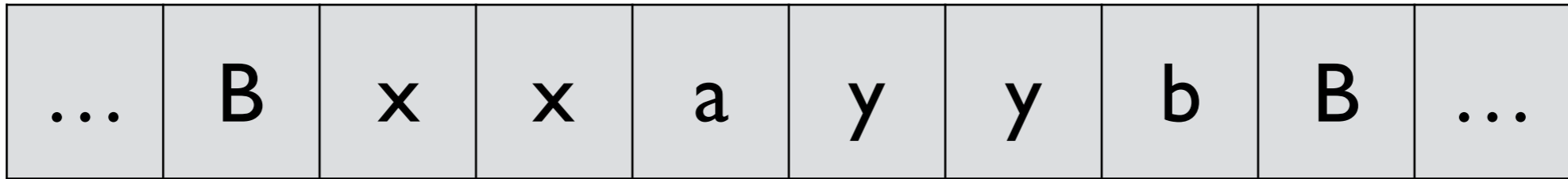
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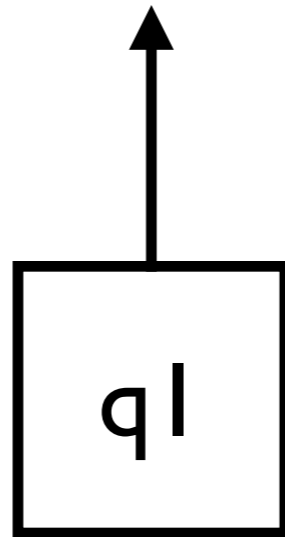
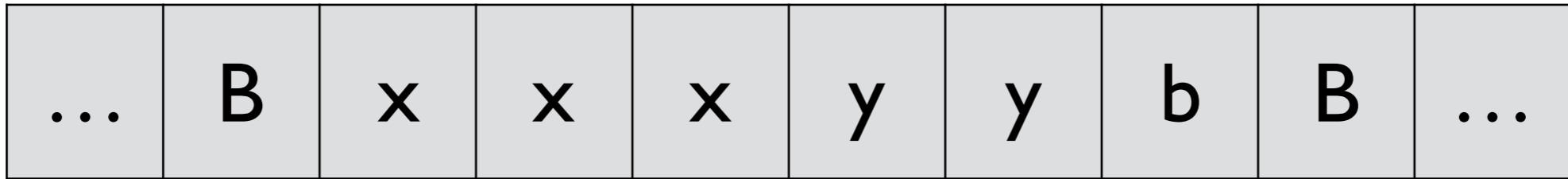
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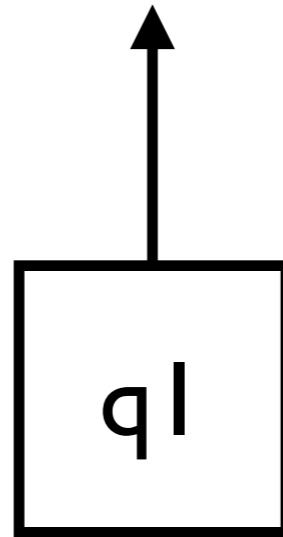
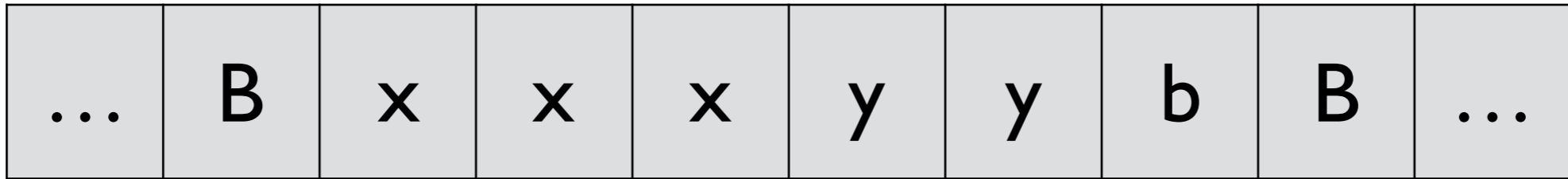
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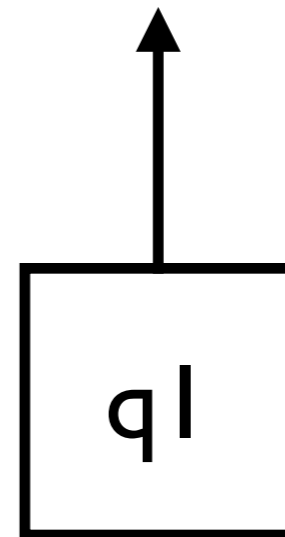
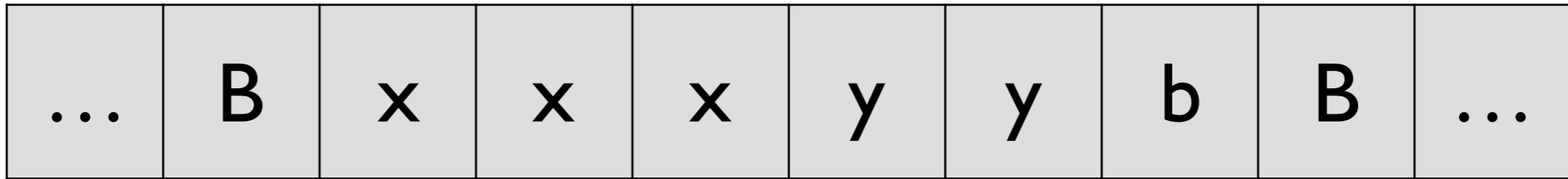
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

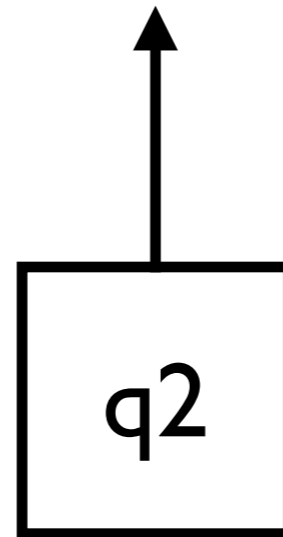
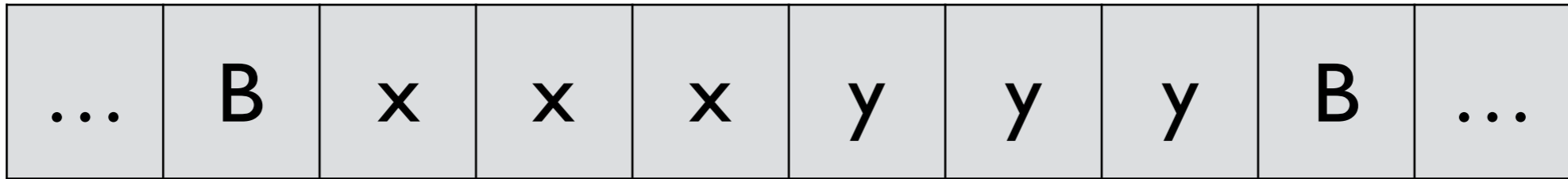
$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

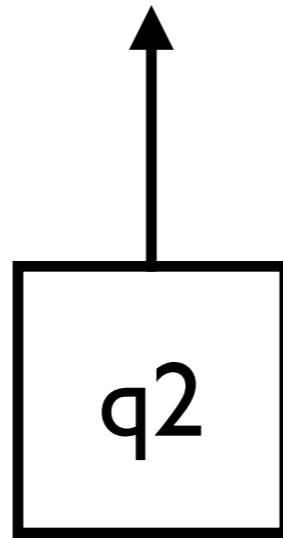
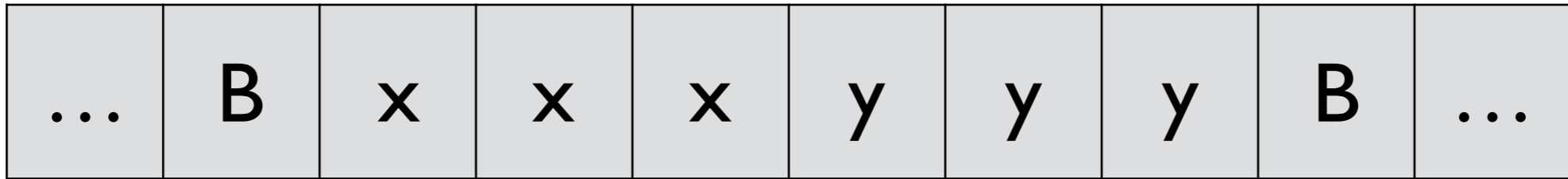
$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

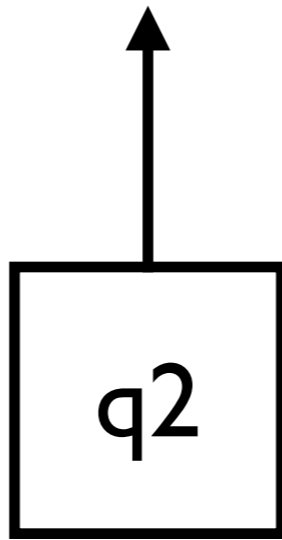
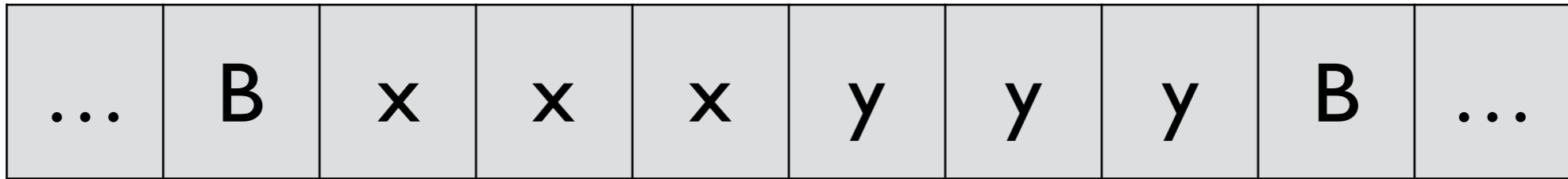
$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

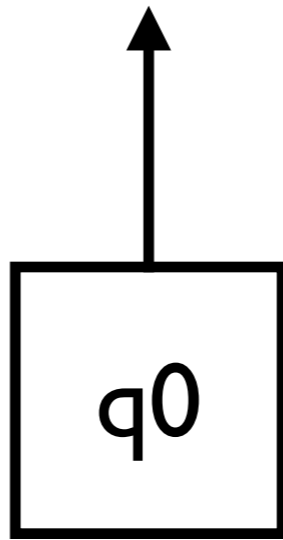
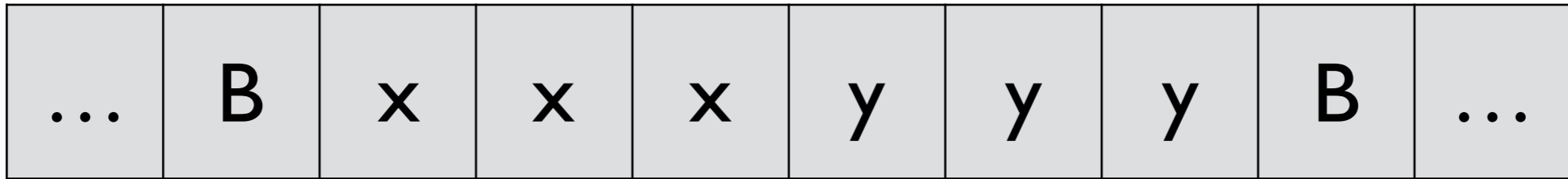
$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

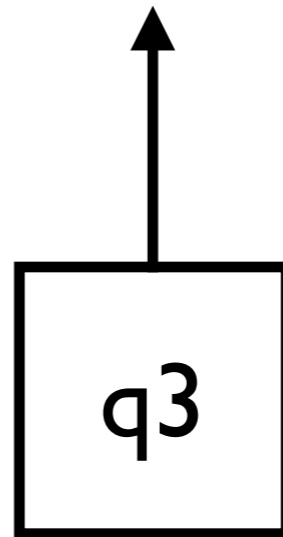
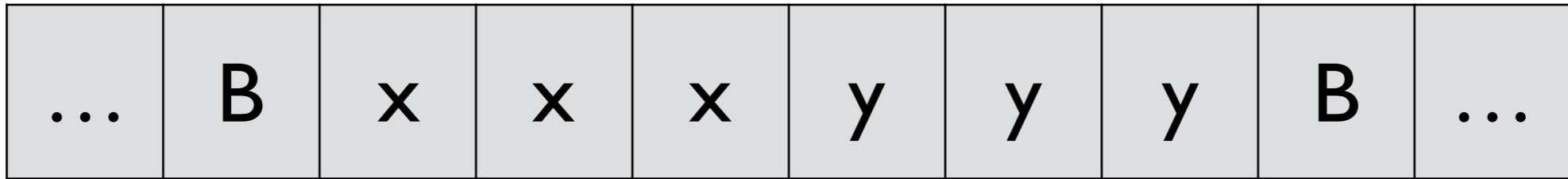
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

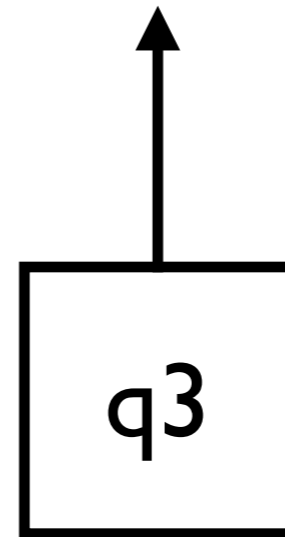
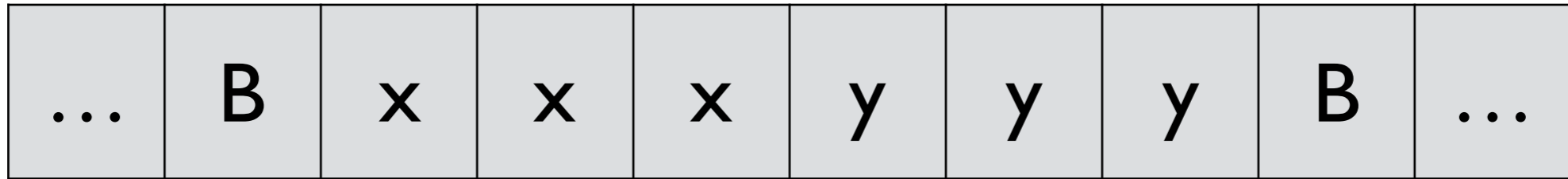
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



(In q_3 , move right to check that there are no more b 's)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

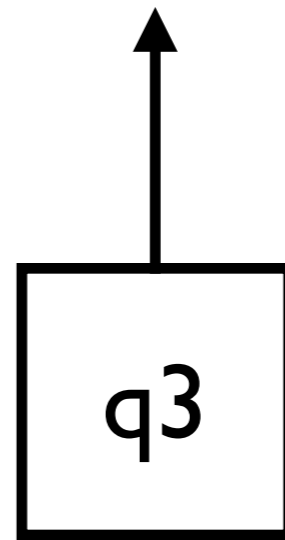
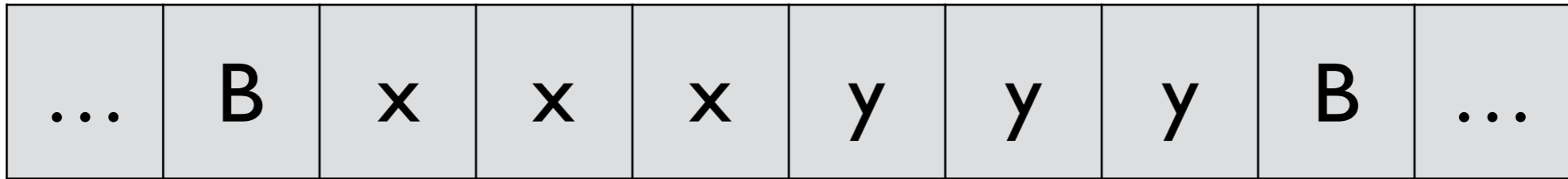
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

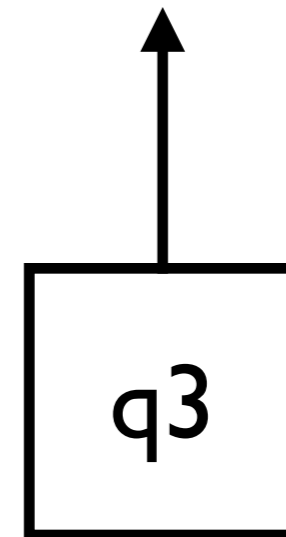
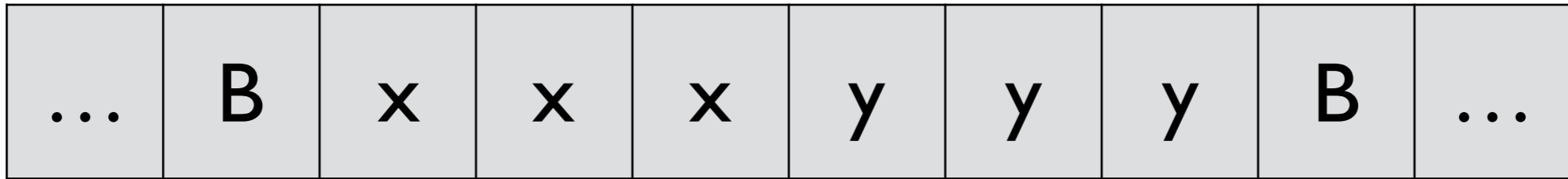
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



(no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

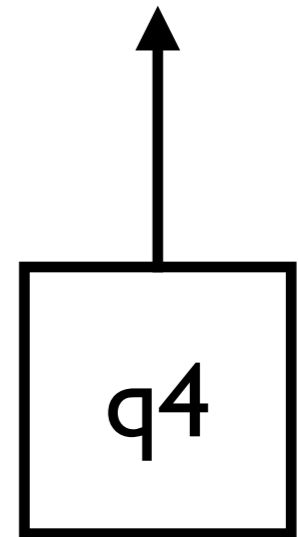
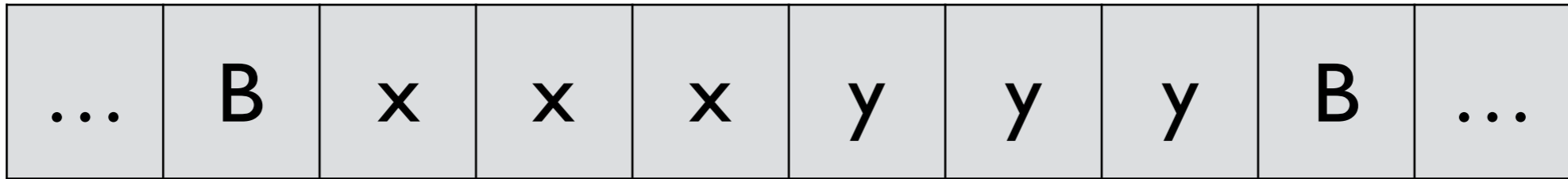
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



“final state”

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

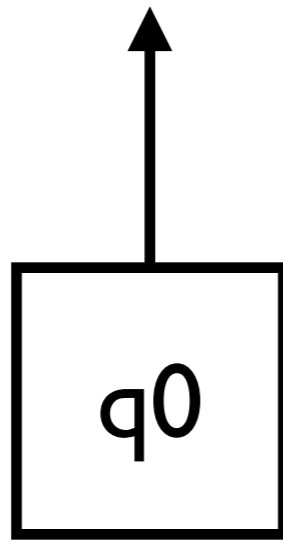
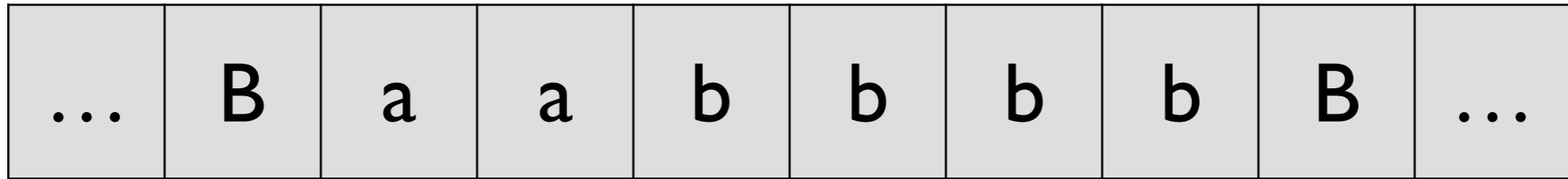
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

When the input string is not in the language:



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

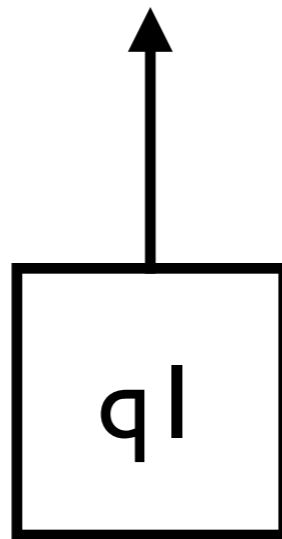
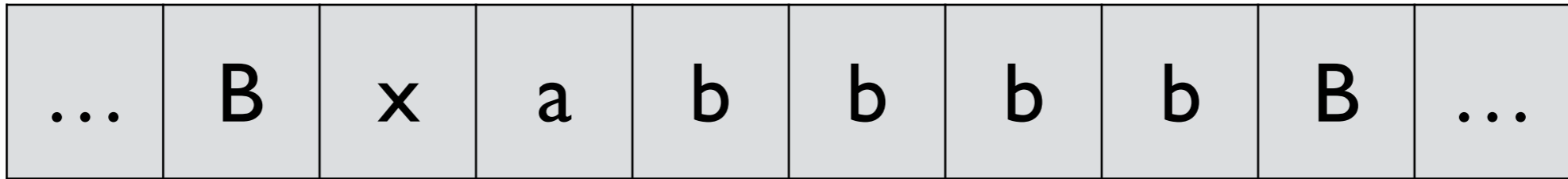
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

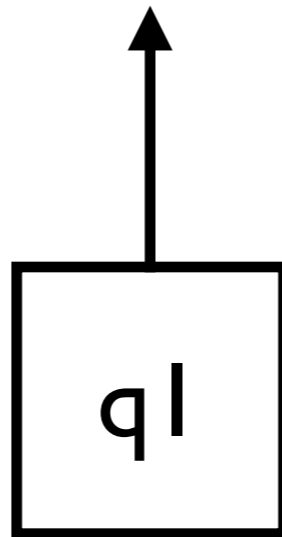
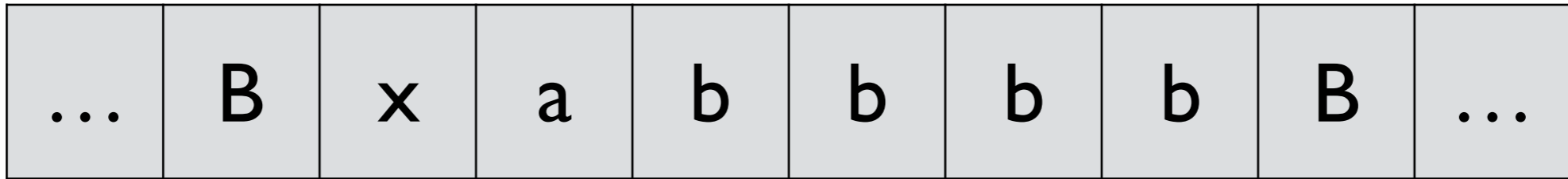
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

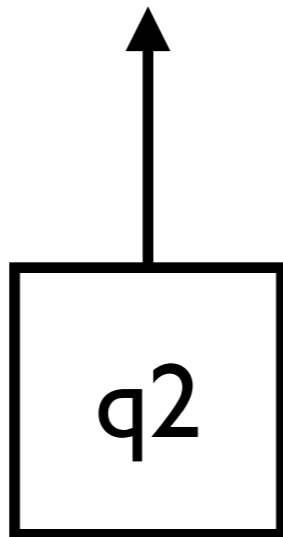
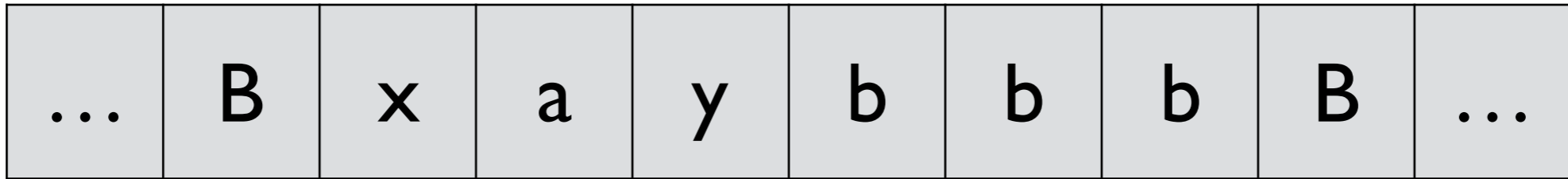
$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

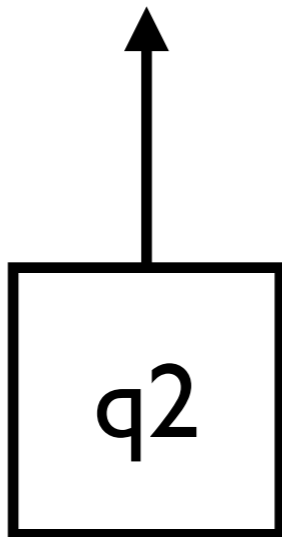
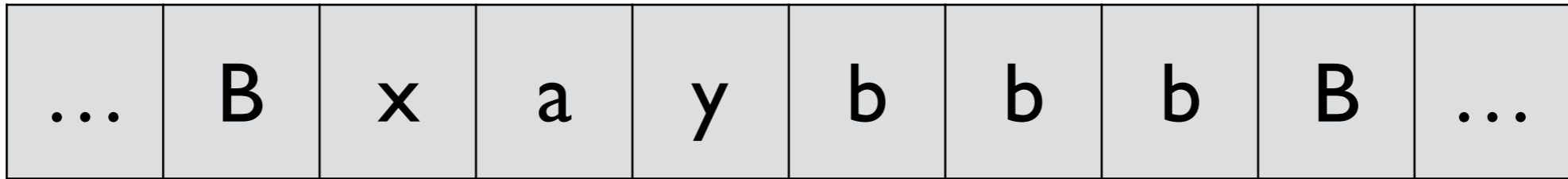
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

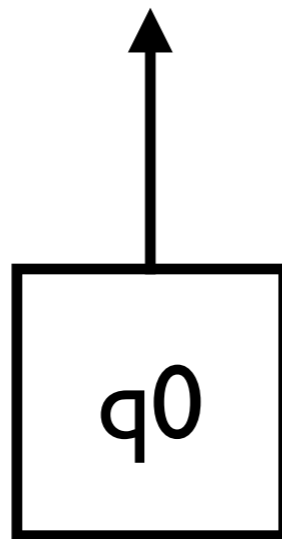
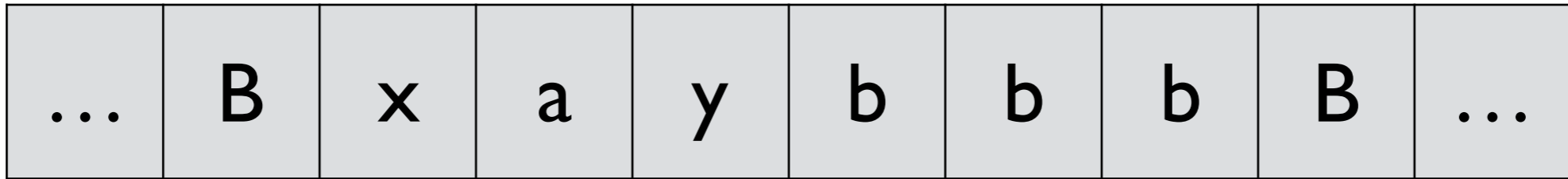
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

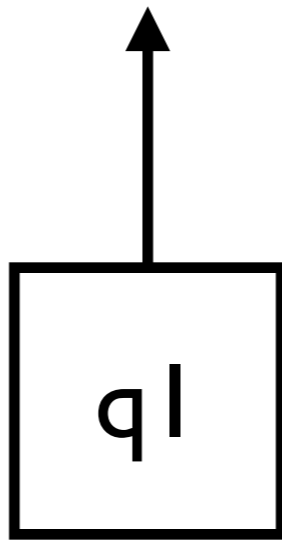
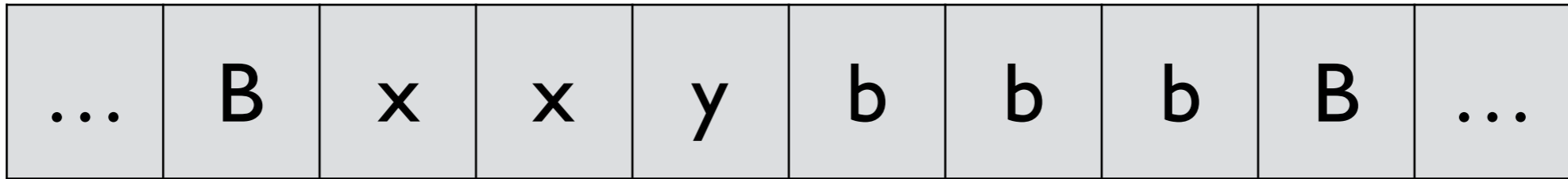
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

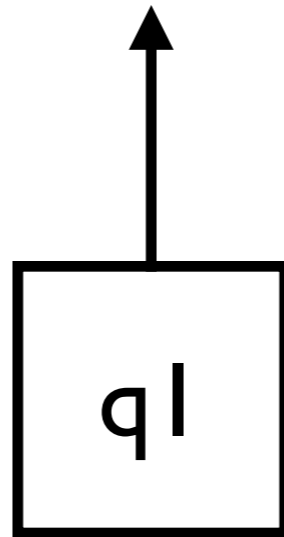
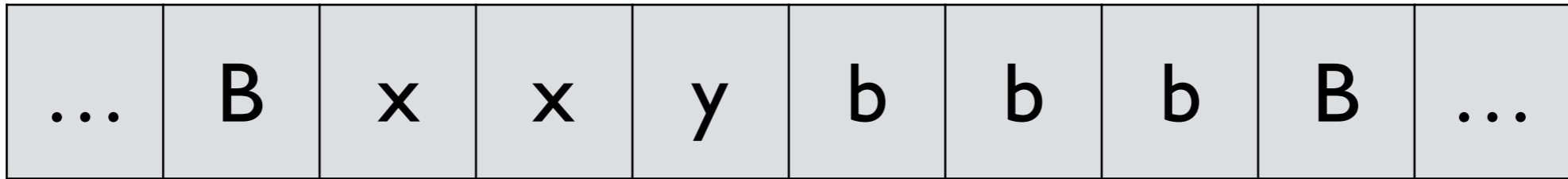
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

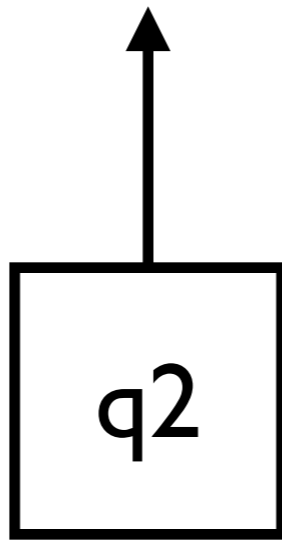
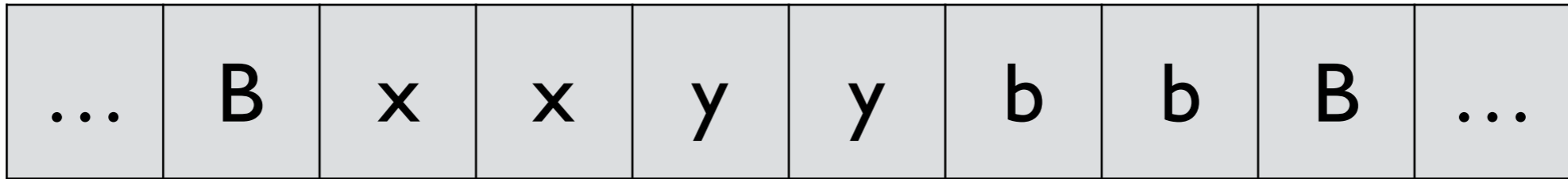
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

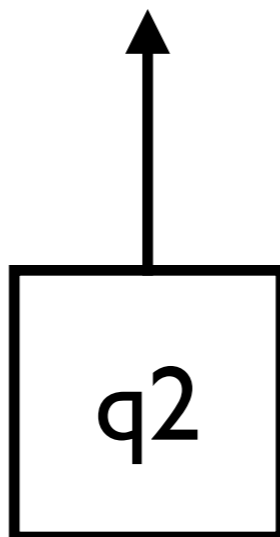
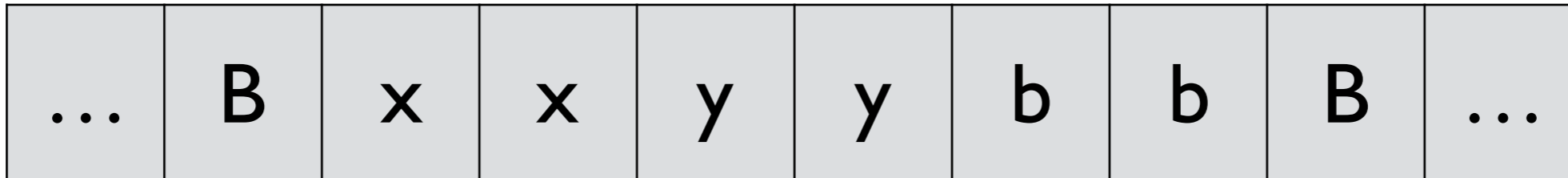
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

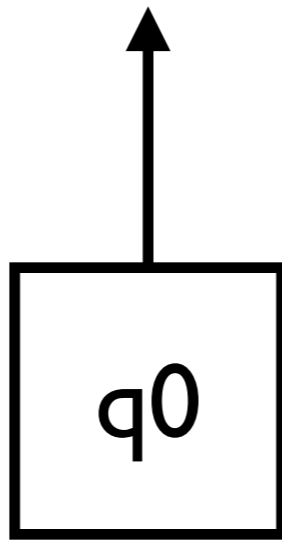
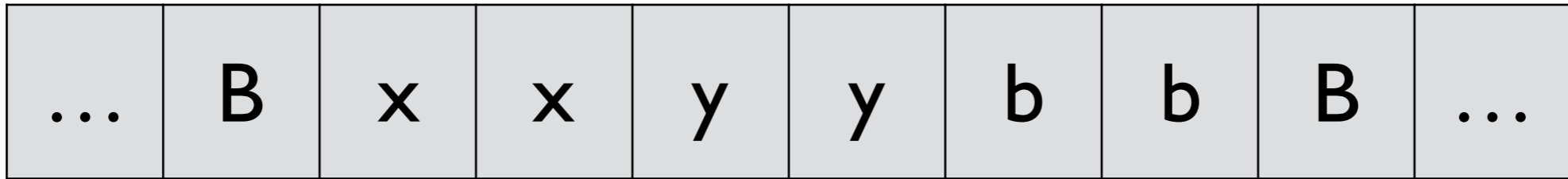
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

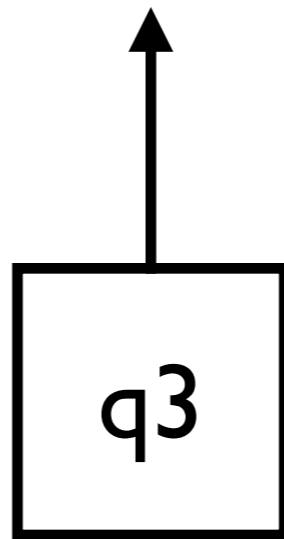
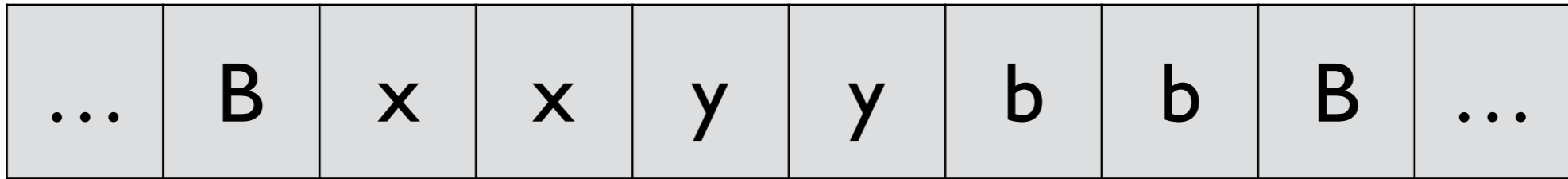
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

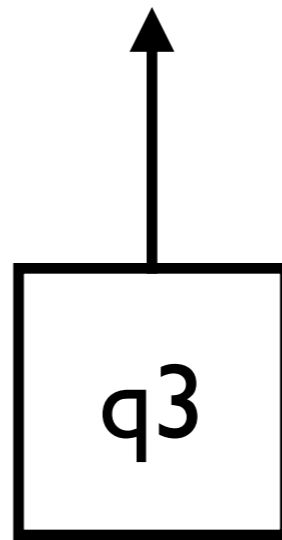
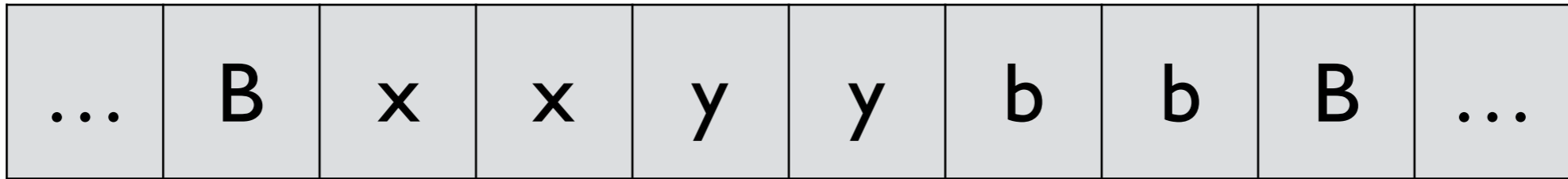
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



(undefined, halt)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

Example 2. Given x and y , design a Turing machine that computes $x + y$.

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_4\})$$

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

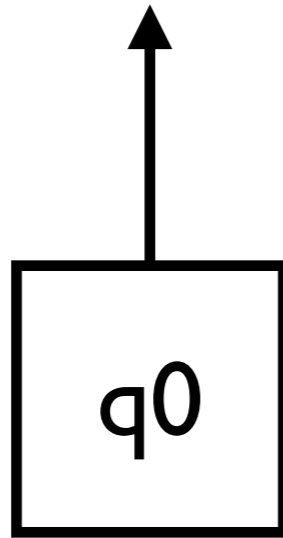
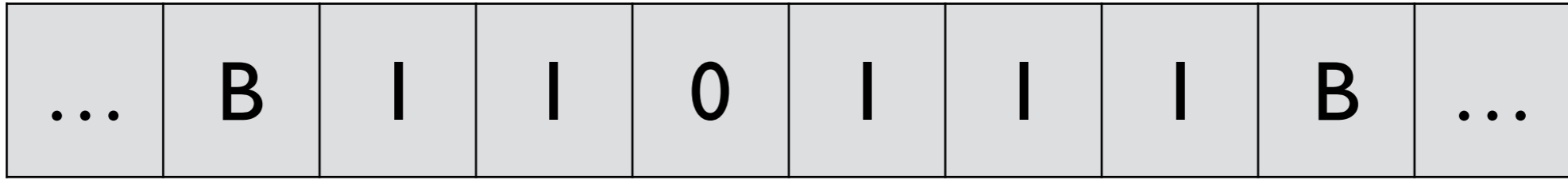
$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

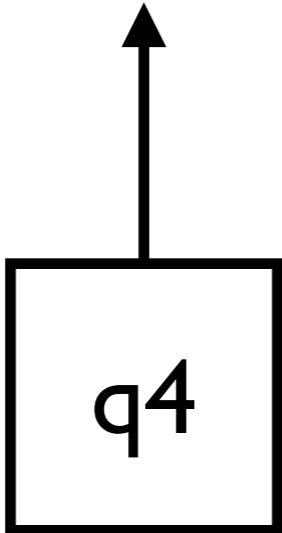
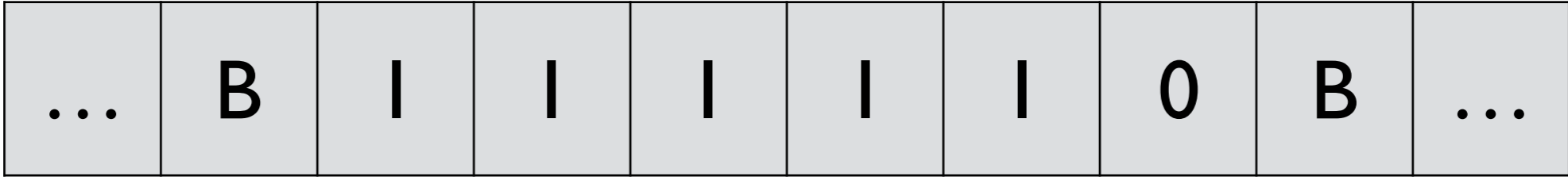
$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$

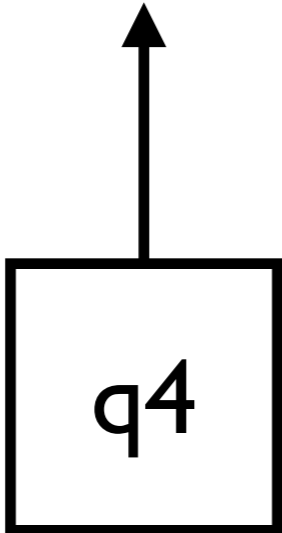
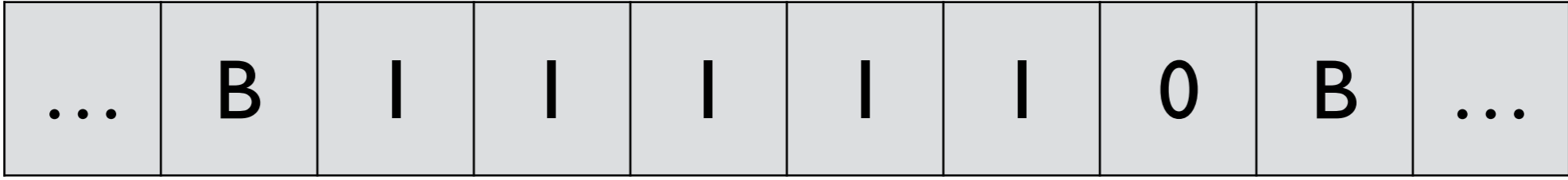
Initial machine configuration:



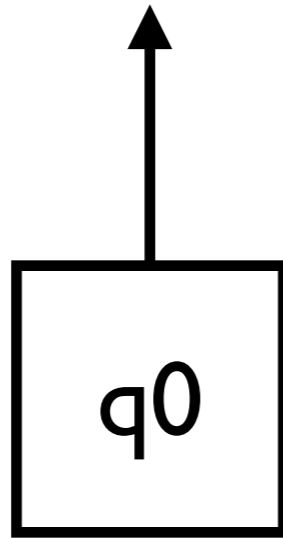
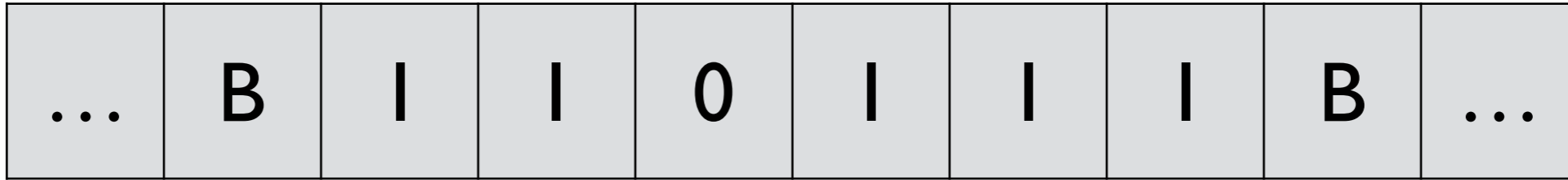
Final machine configuration:



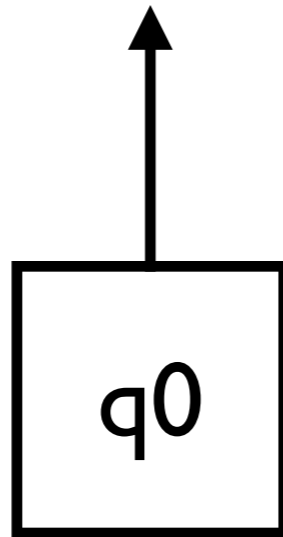
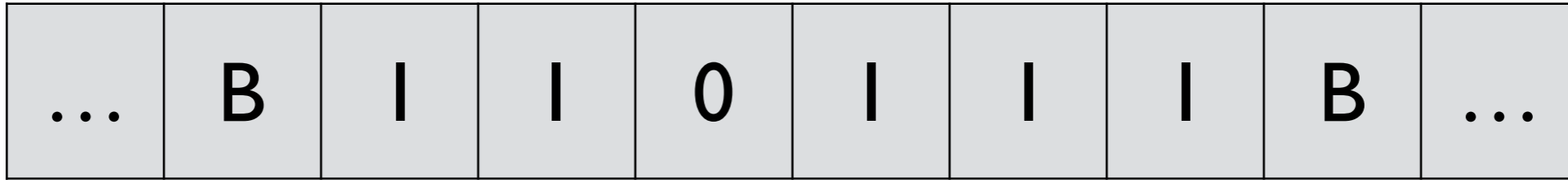
Final machine configuration:



Addition in math is to move 0 to the right end



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

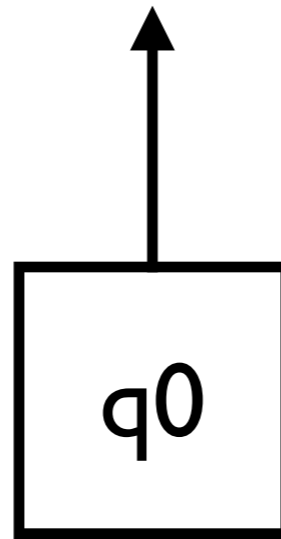
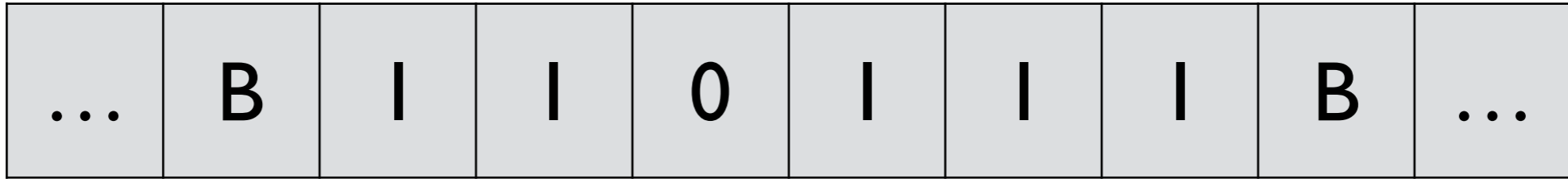
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

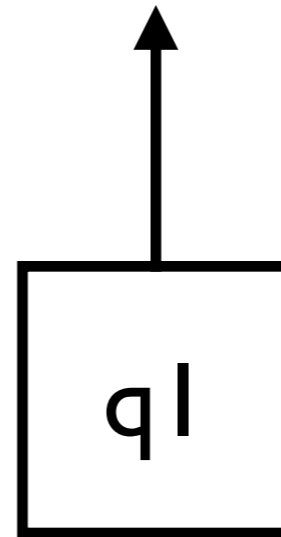
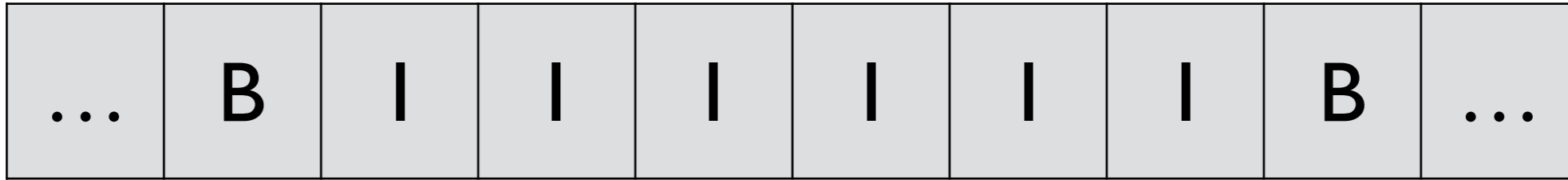
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



(In q_1 , search for the right end of y)

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

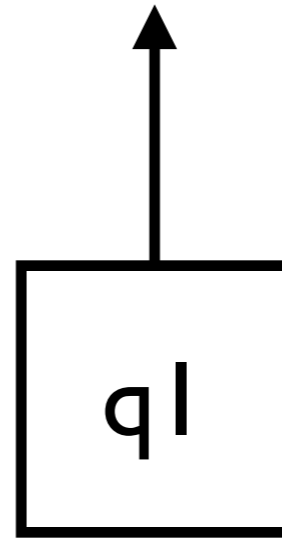
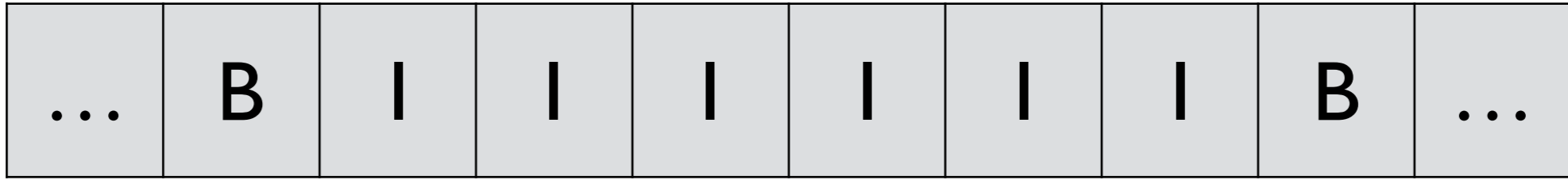
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

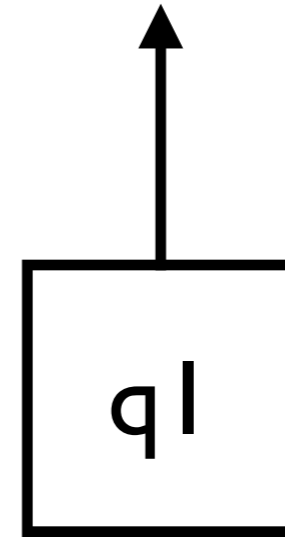
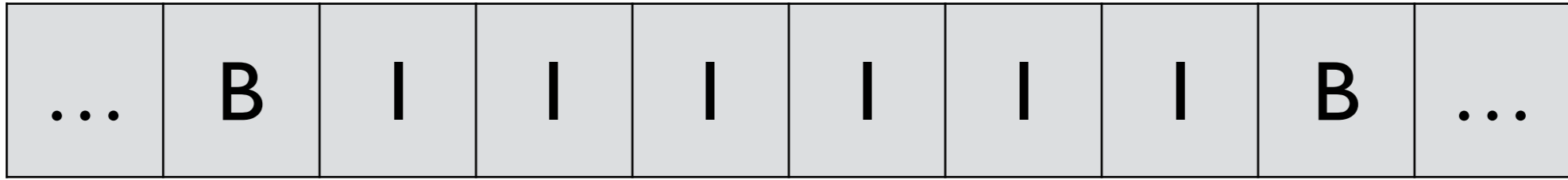
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

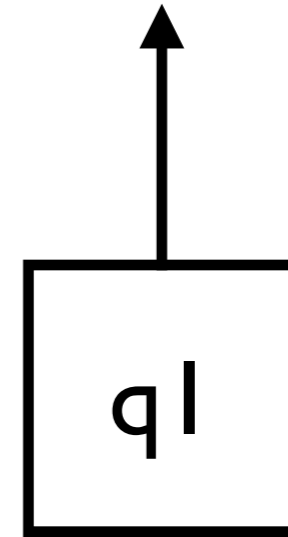
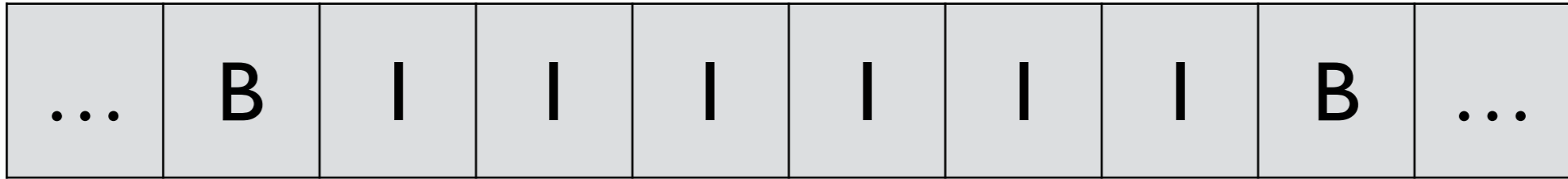
$$\delta(q_3, B) = (q_4, B, R)$$



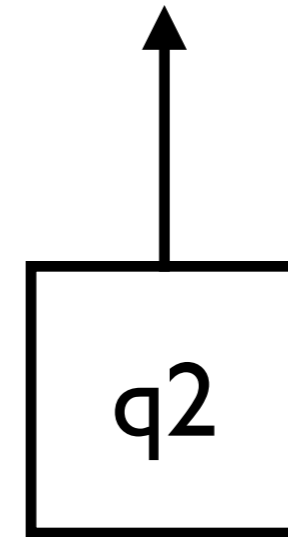
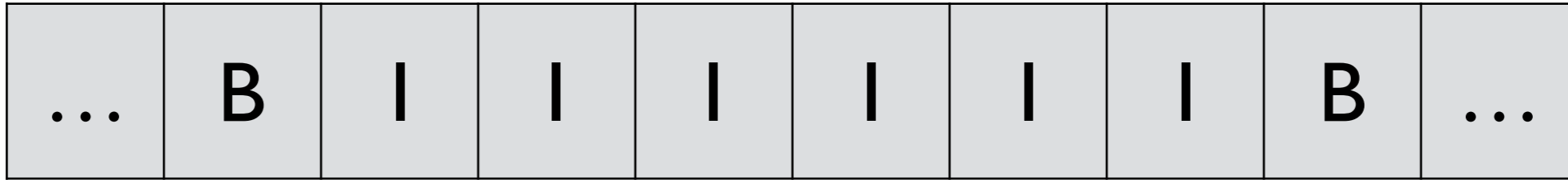
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



(In q_2 , replace the rightmost I by 0)

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

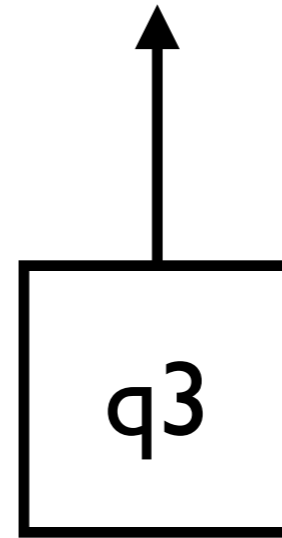
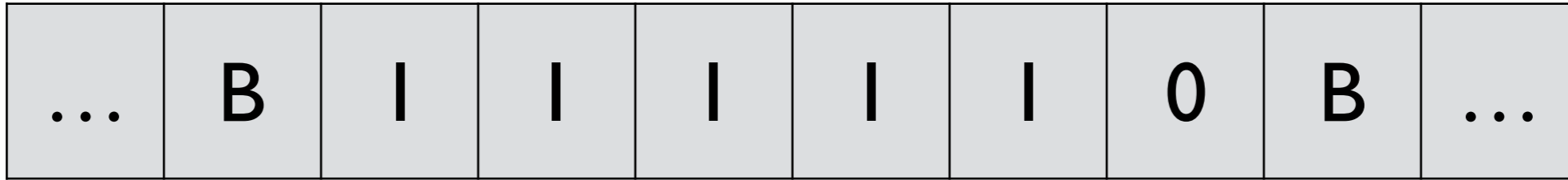
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$



(In q_3 , look for the leftmost I)

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

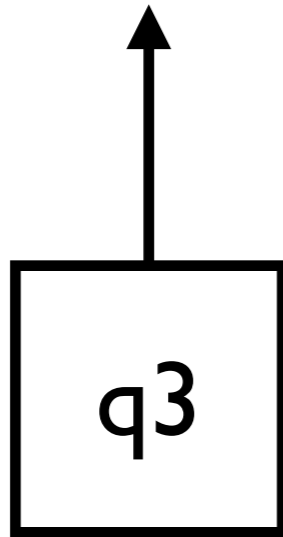
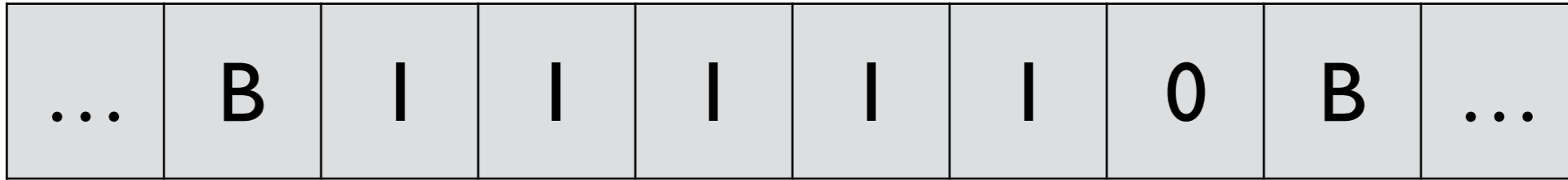
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

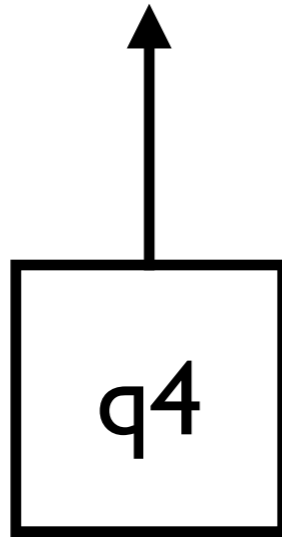
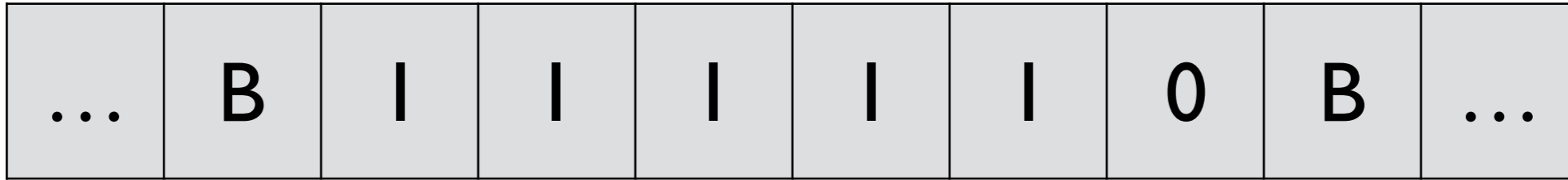
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



“final state”

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$

Example 3. Design a Turing machine that transforms w into ww .

$$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$$

$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

$$\delta(q_1, 1) = (q_1, 1, L)$$

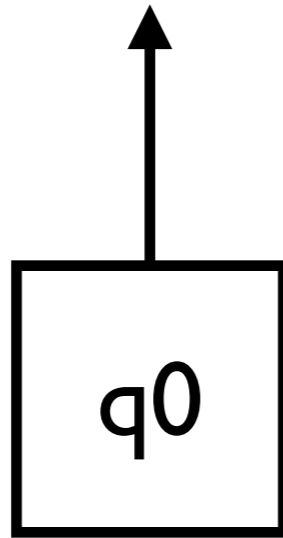
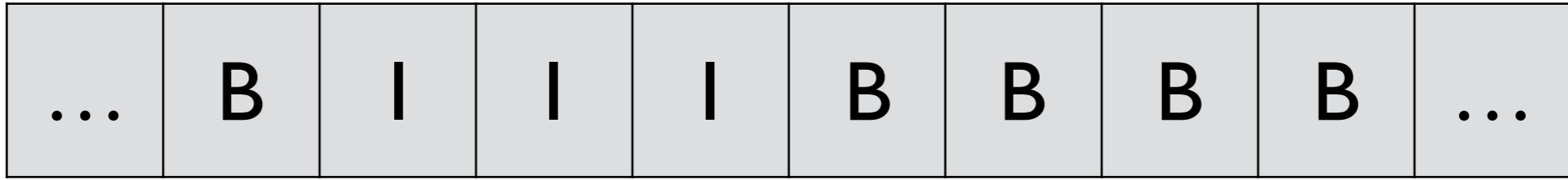
$$\delta(q_1, x) = (q_2, 1, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

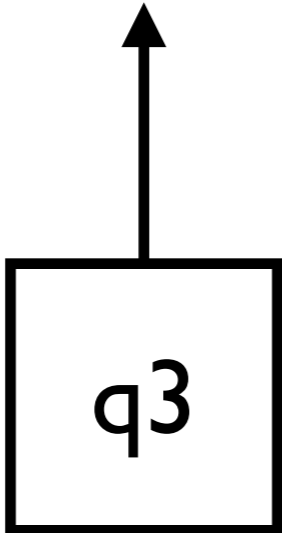
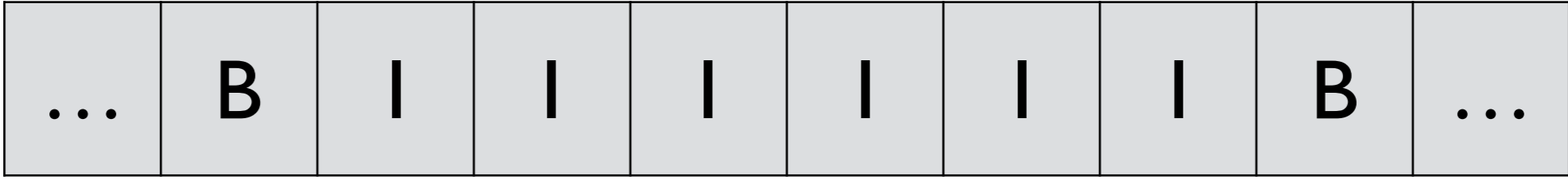
$$\delta(q_2, B) = (q_1, 1, L)$$

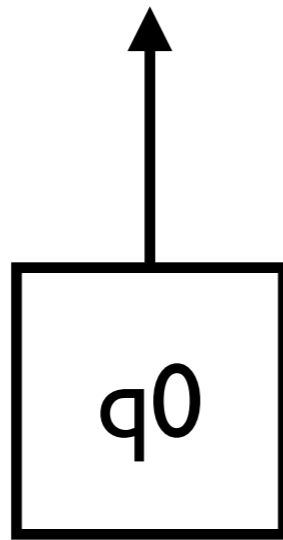
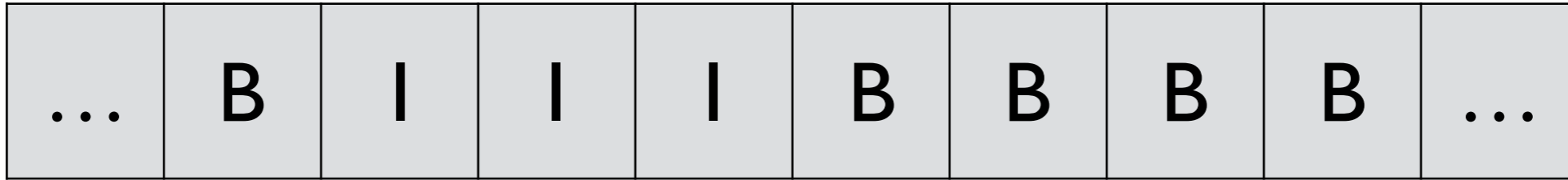
$$\delta(q_1, B) = (q_3, B, R)$$

Initial machine configuration:



Final machine configuration:





$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

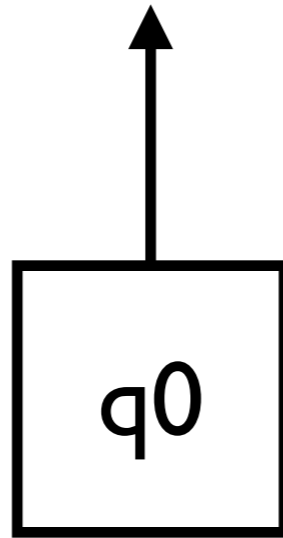
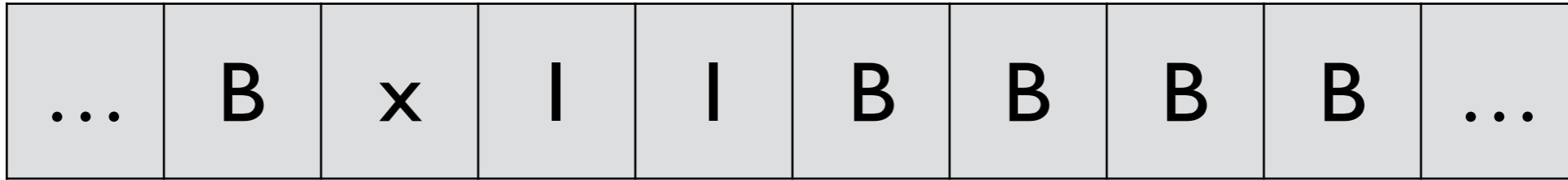
$$\delta(q_1, 1) = (q_1, 1, L)$$

$$\delta(q_1, x) = (q_2, 1, R)$$

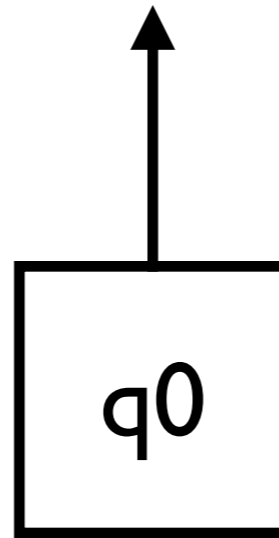
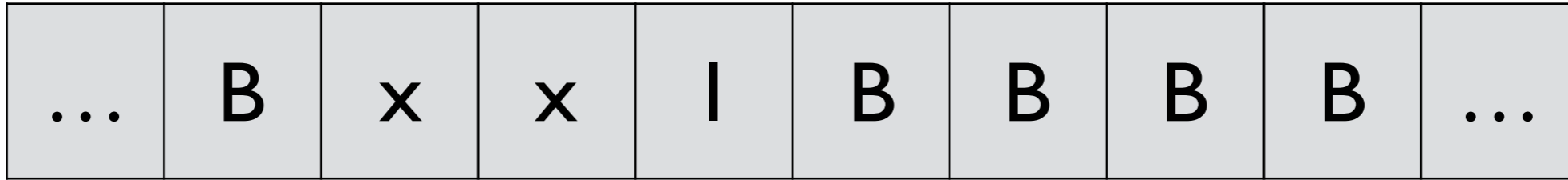
$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

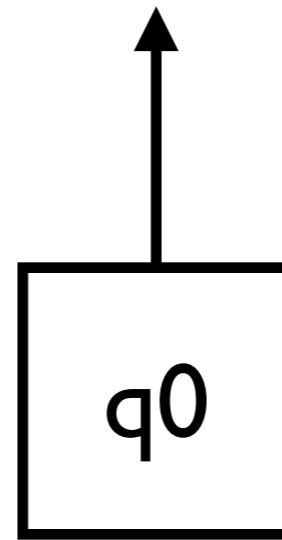
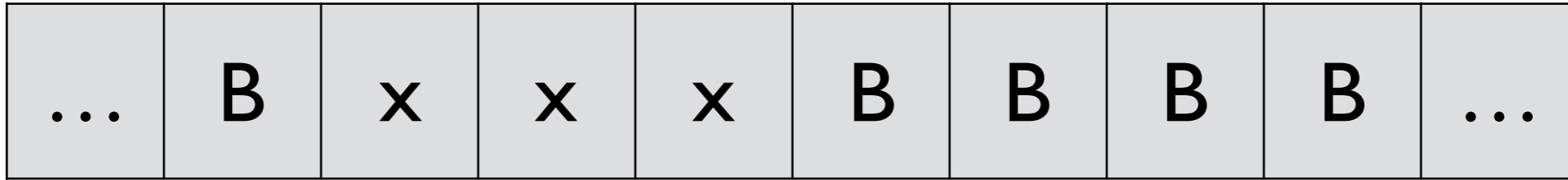
$$\delta(q_1, B) = (q_3, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

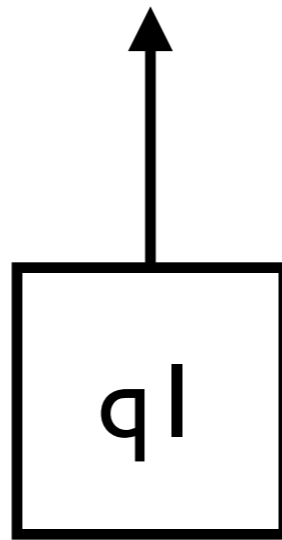


$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



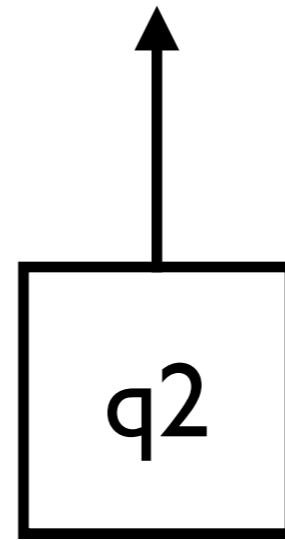
Initially, replace every 1 by x

$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$



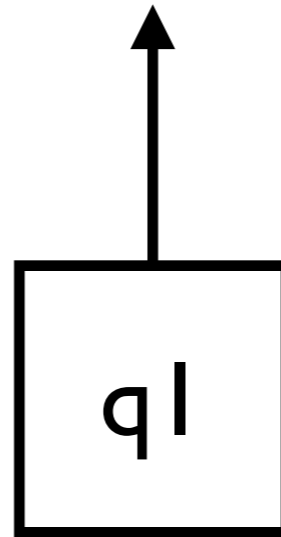
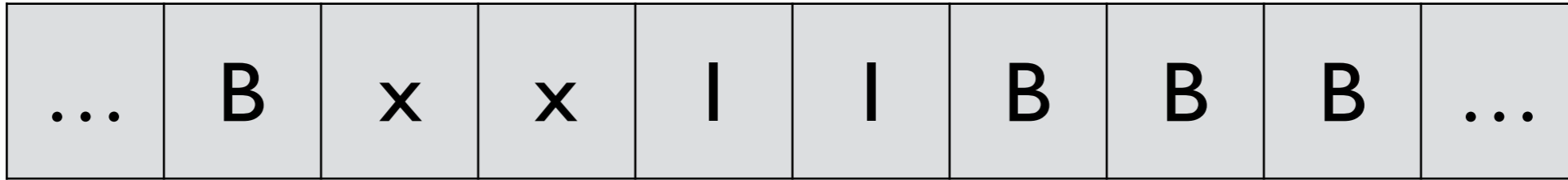
In q_1 , look for the rightmost x

$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$

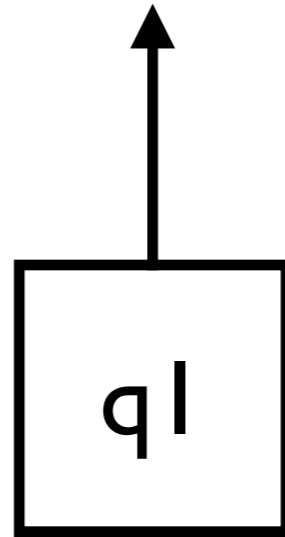
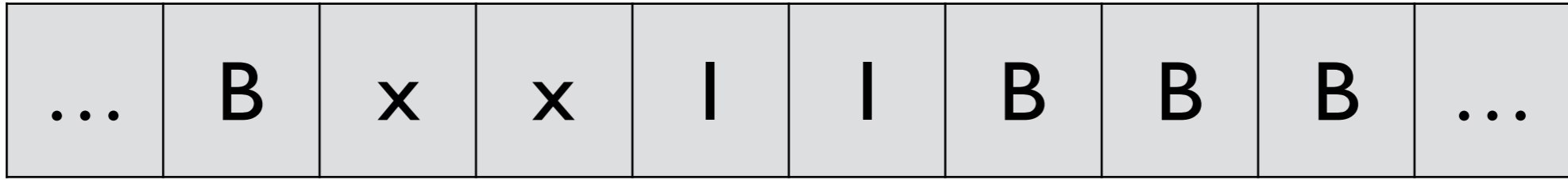


In q_2 , look for the first blank and write I

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

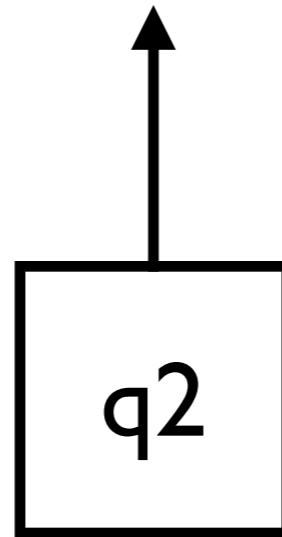
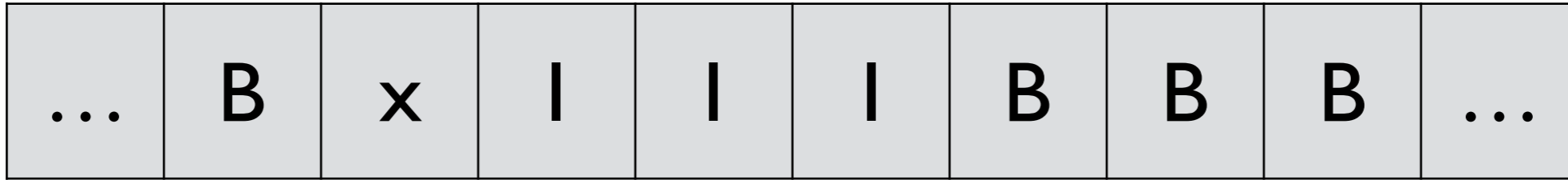
$$\delta(q_1, 1) = (q_1, 1, L)$$

$$\delta(q_1, x) = (q_2, 1, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

$$\delta(q_1, B) = (q_3, B, R)$$



$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

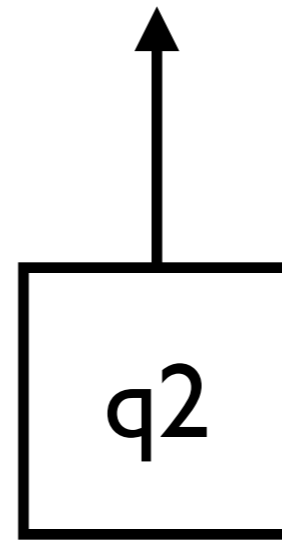
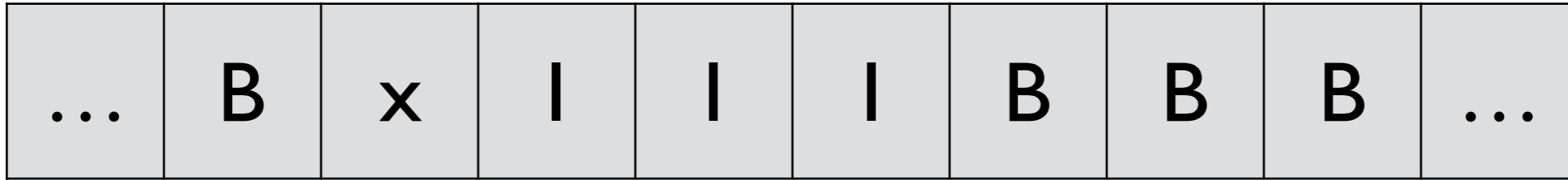
$$\delta(q_1, 1) = (q_1, 1, L)$$

$$\delta(q_1, x) = (q_2, 1, R)$$

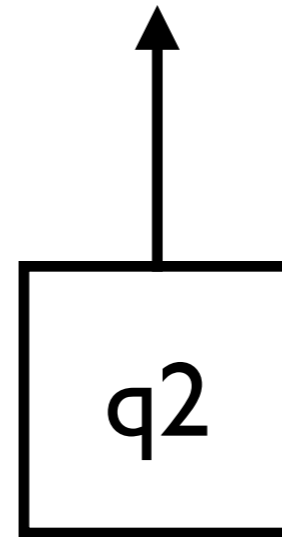
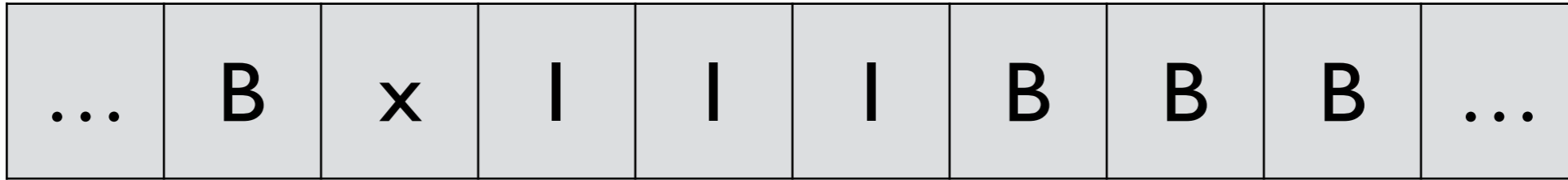
$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

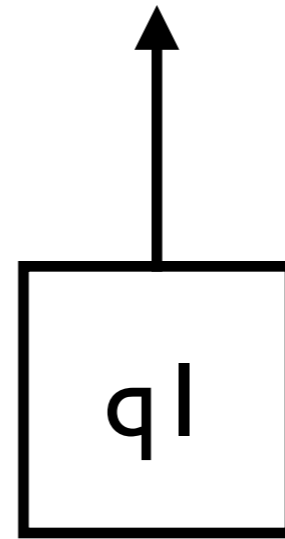
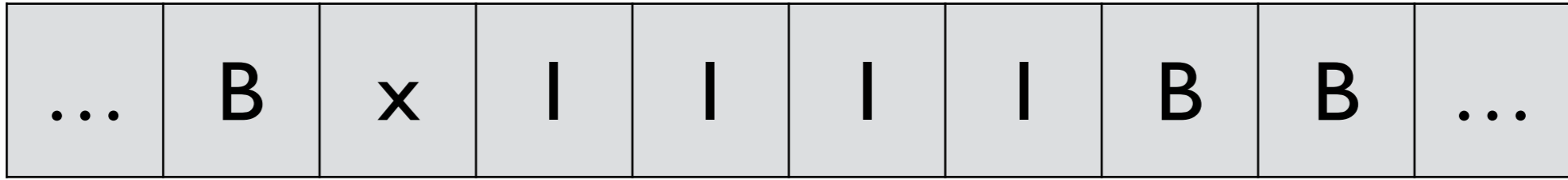
$$\delta(q_1, B) = (q_3, B, R)$$



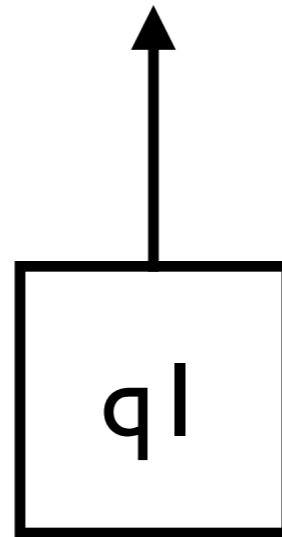
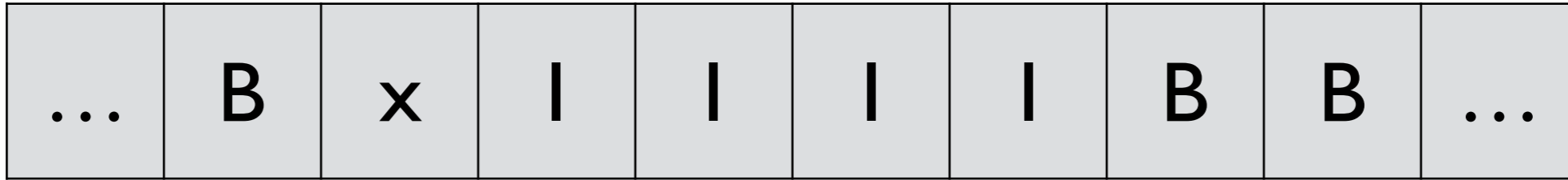
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



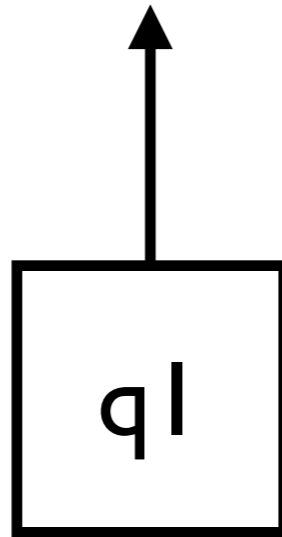
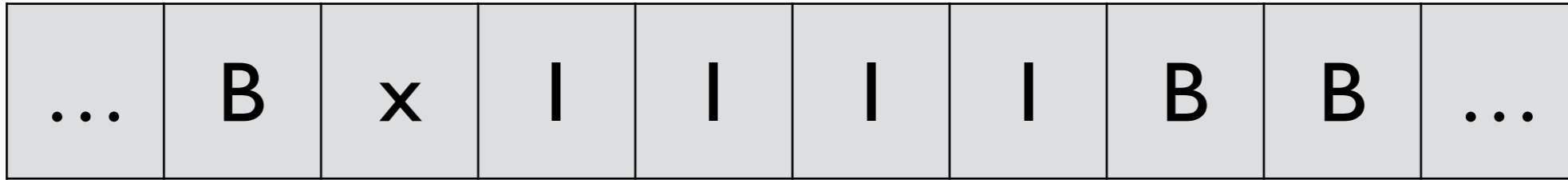
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

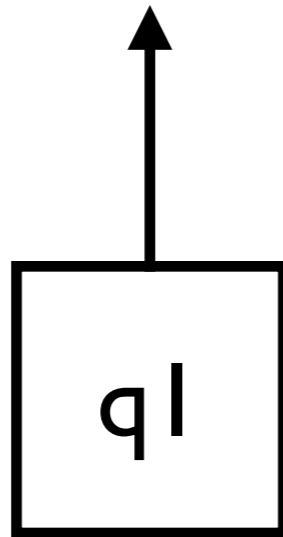
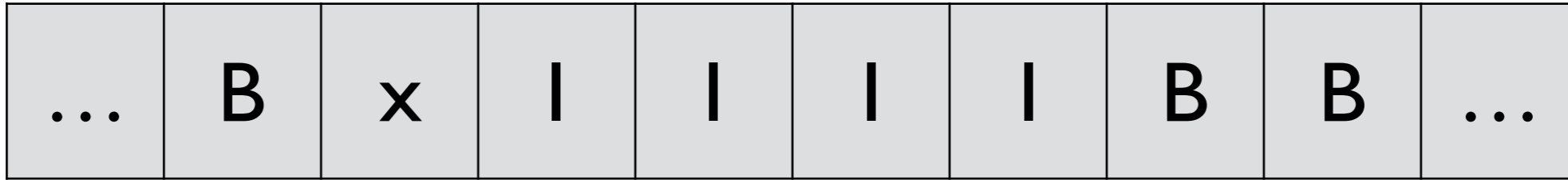
$$\delta(q_1, 1) = (q_1, 1, L)$$

$$\delta(q_1, x) = (q_2, 1, R)$$

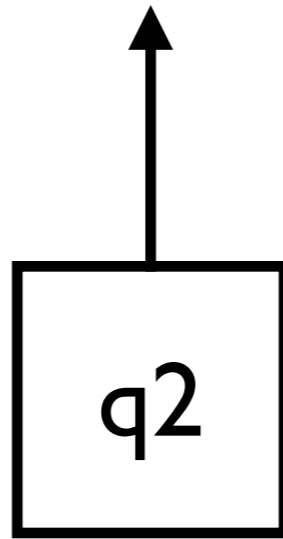
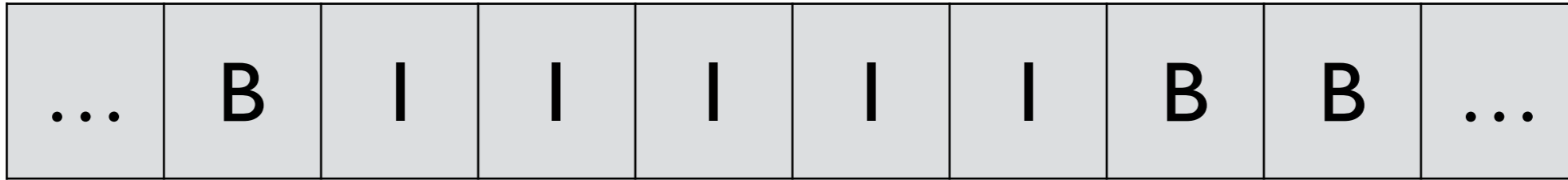
$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

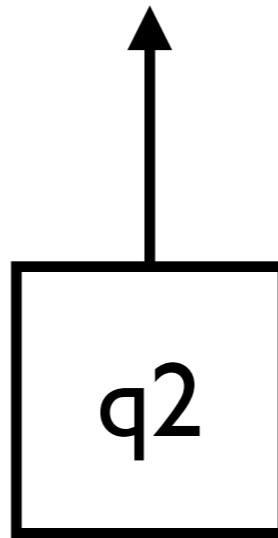
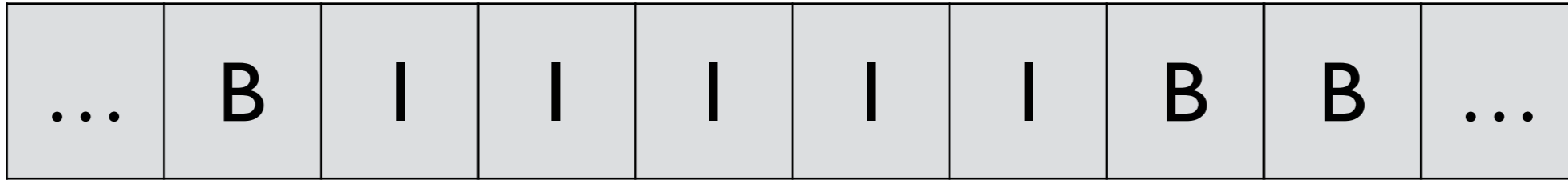
$$\delta(q_1, B) = (q_3, B, R)$$



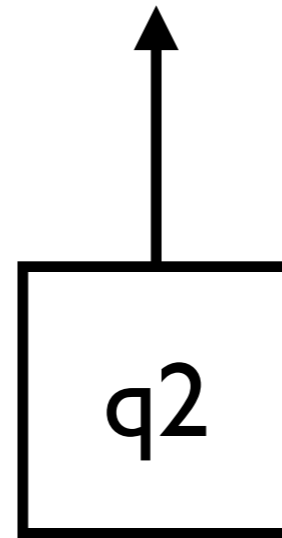
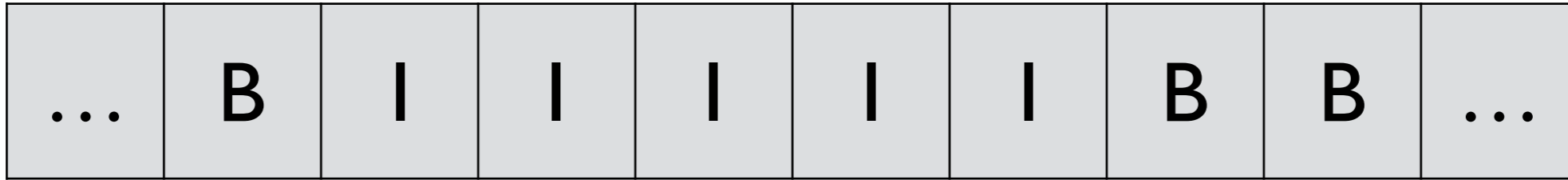
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



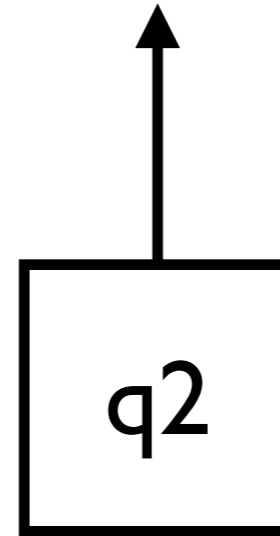
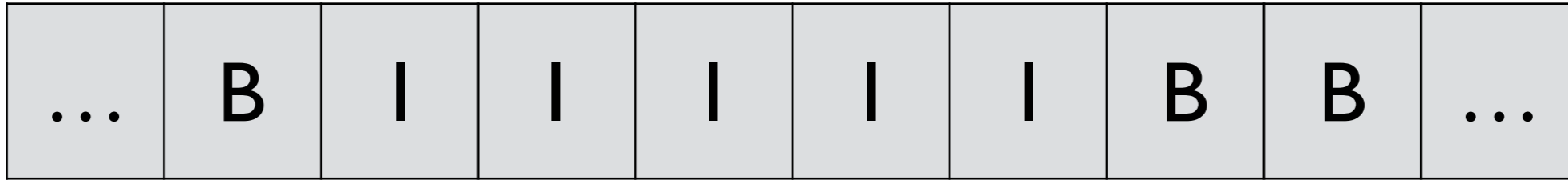
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



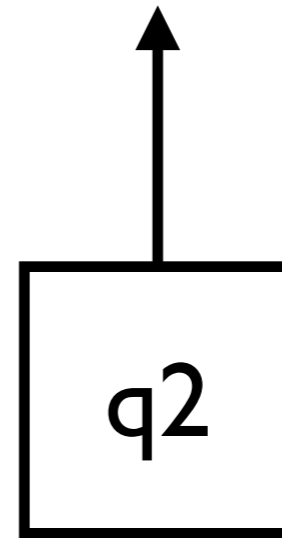
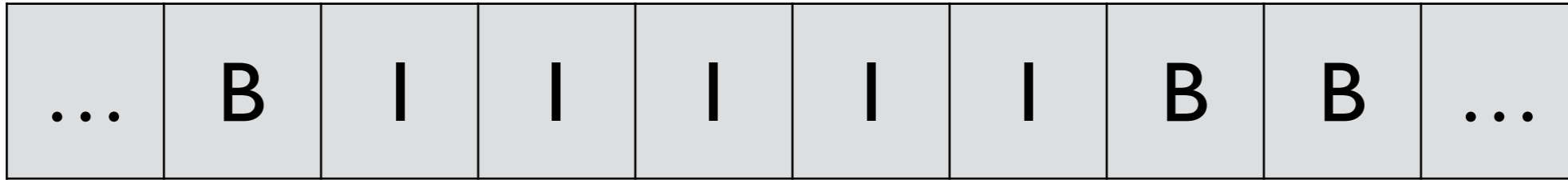
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



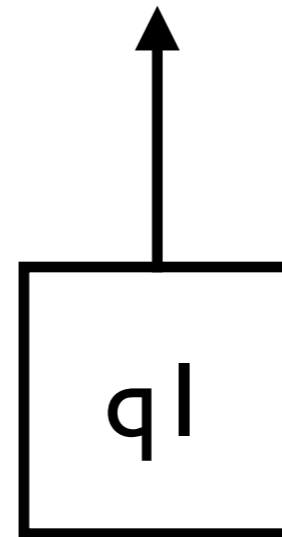
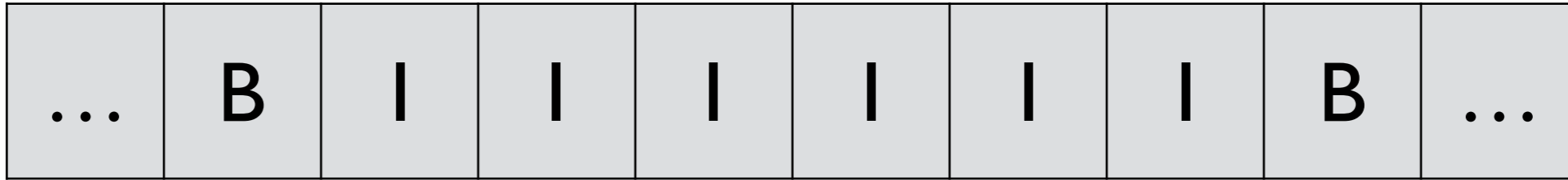
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



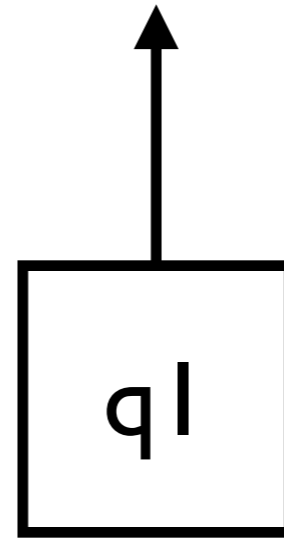
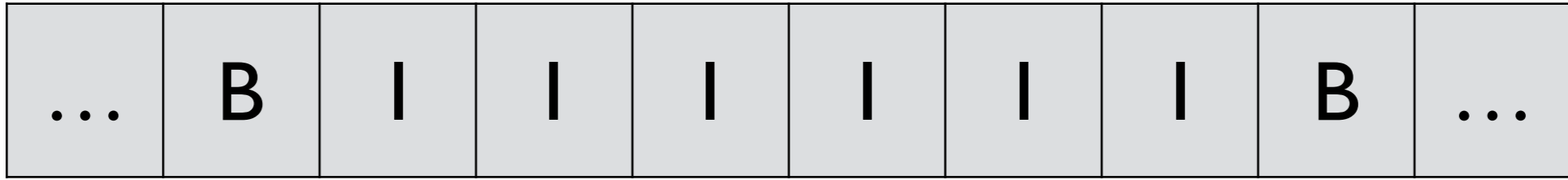
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



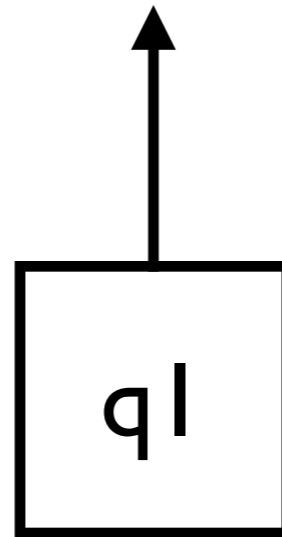
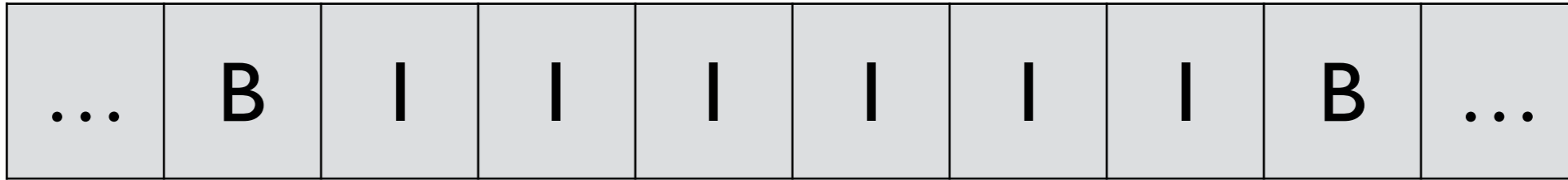
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



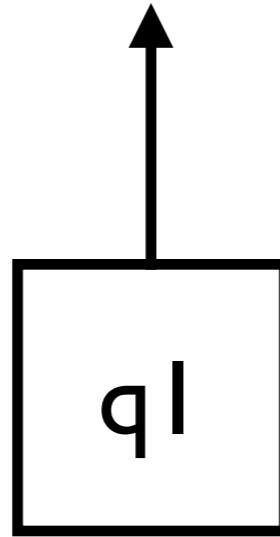
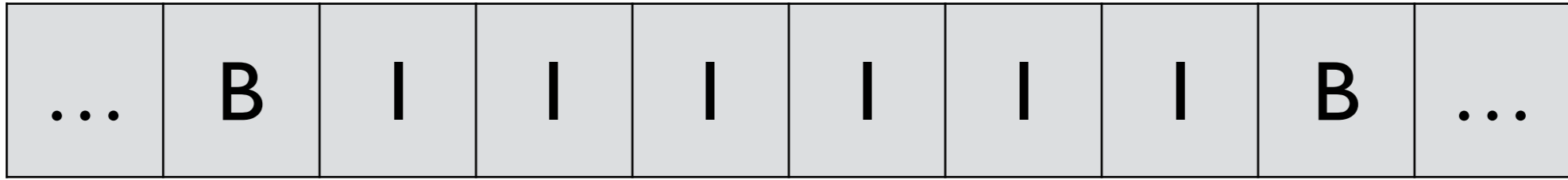
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



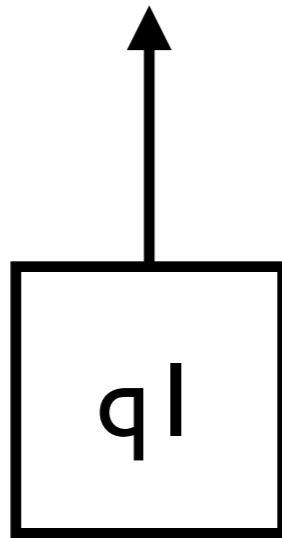
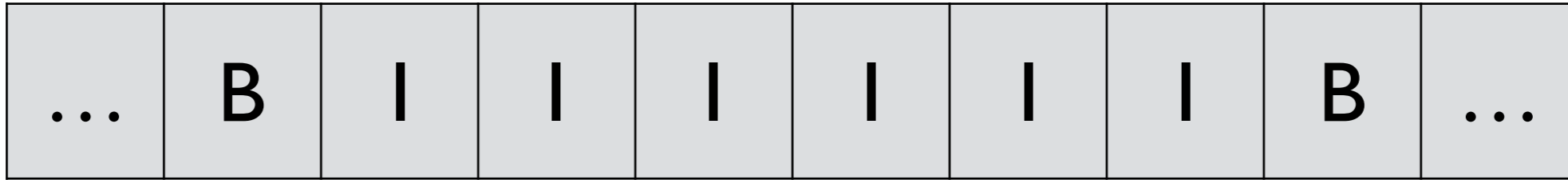
$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$



$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

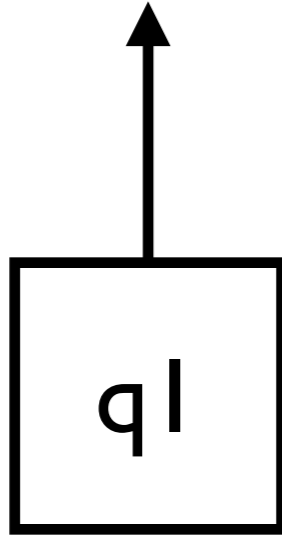
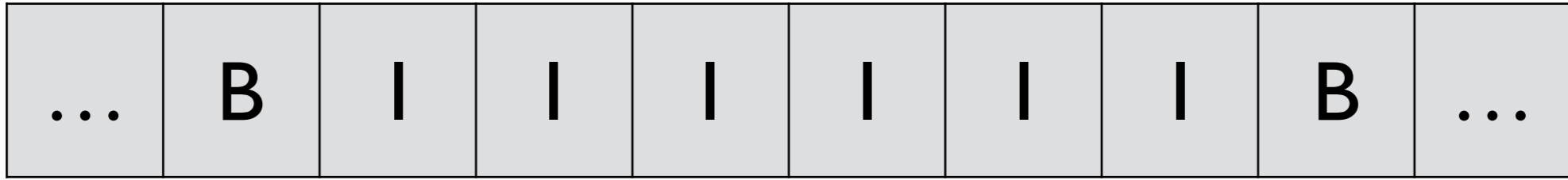
$$\delta(q_1, 1) = (q_1, 1, L)$$

$$\delta(q_1, x) = (q_2, 1, R)$$

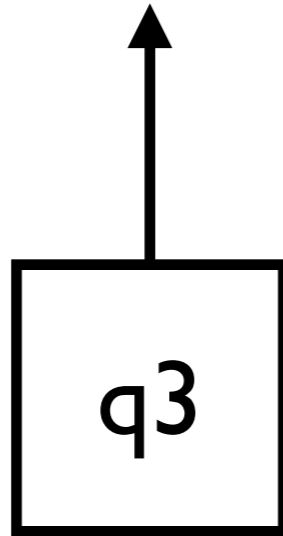
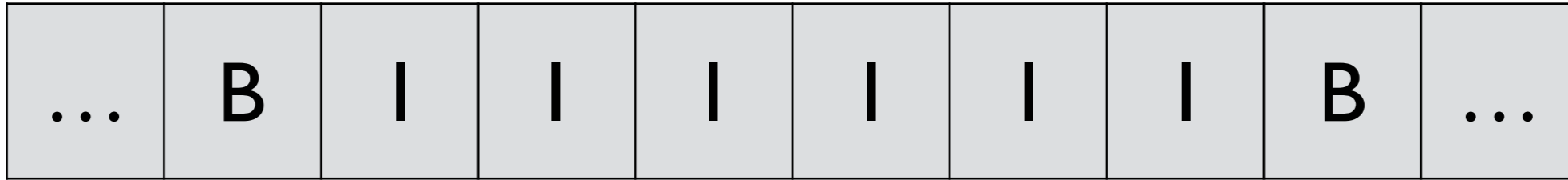
$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

$$\delta(q_1, B) = (q_3, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



“final state”

$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

Example 4. Design a Turing machine that computes $f(m, n)$.

$$f(m, n) = \max(m - n, 0) = \text{if } m \geq n \text{ then } m - n \text{ else } 0$$

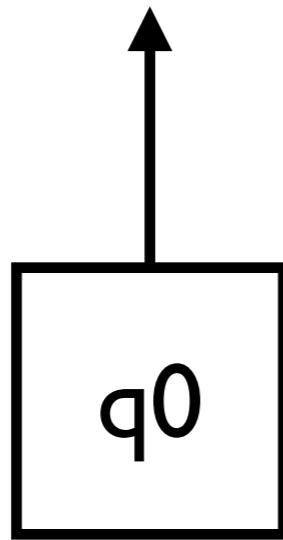
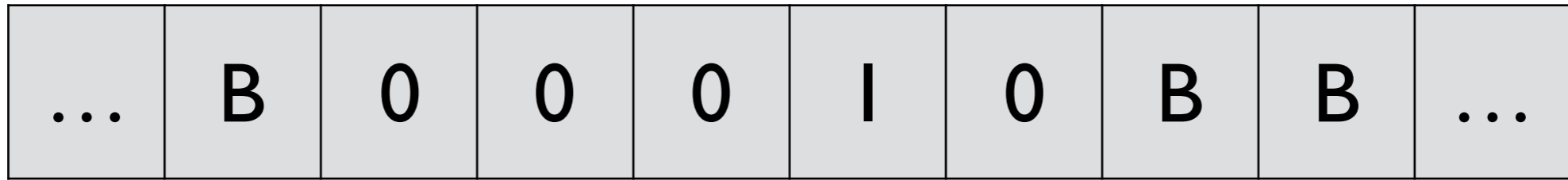
Example 4. Design a Turing machine that computes $f(m, n)$.

$$f(m, n) = \max(m - n, 0) = \text{if } m \geq n \text{ then } m - n \text{ else } 0$$

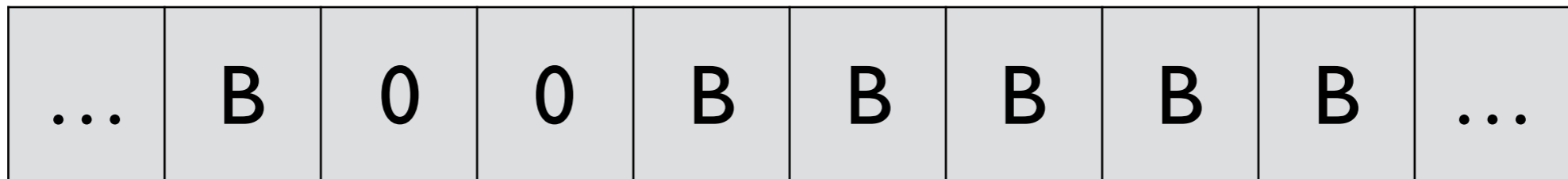
$$M = (\{q_0, q_1, \dots, q_6\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_6\})$$

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

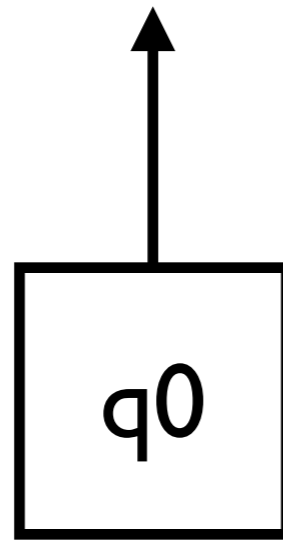
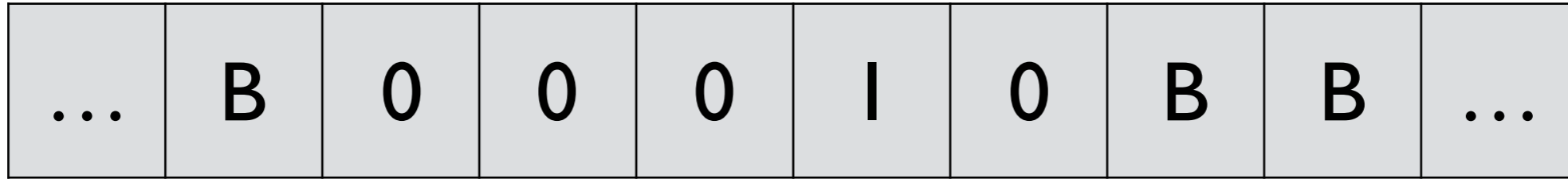
Initial machine configuration: e.g., $f(3, 1)$



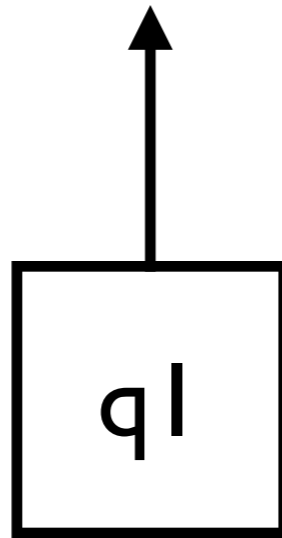
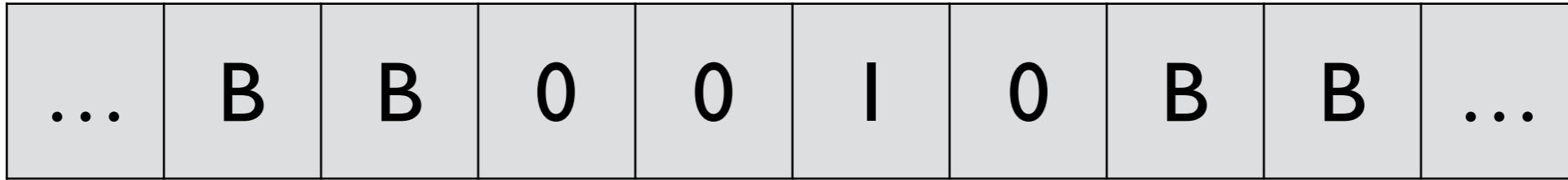
When the machine halts:



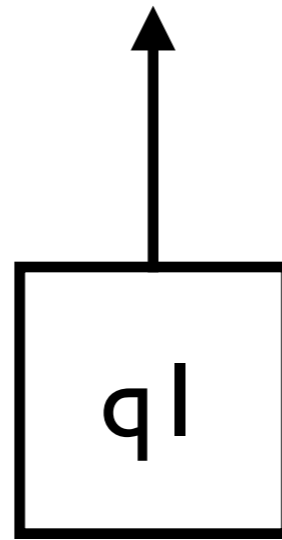
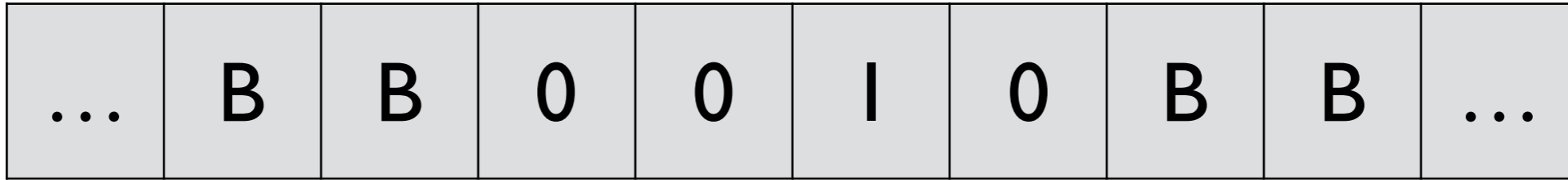
e.g., $f(3, 1)$



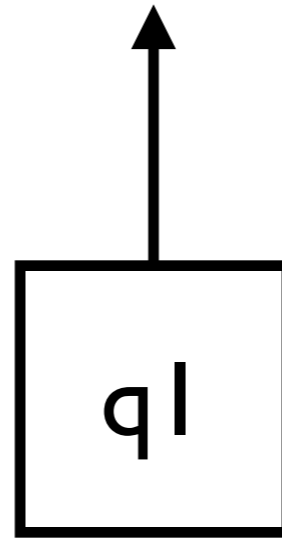
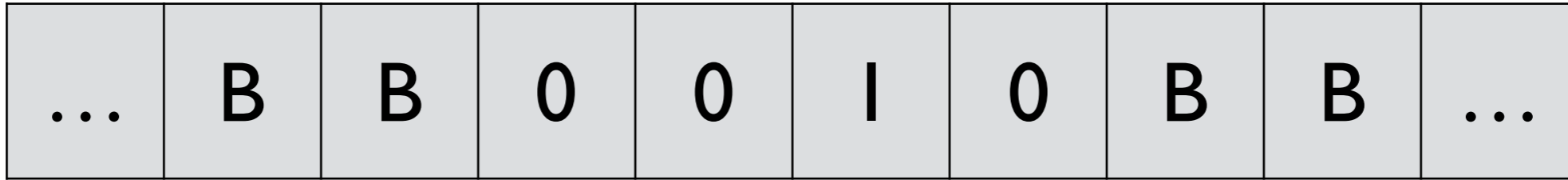
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



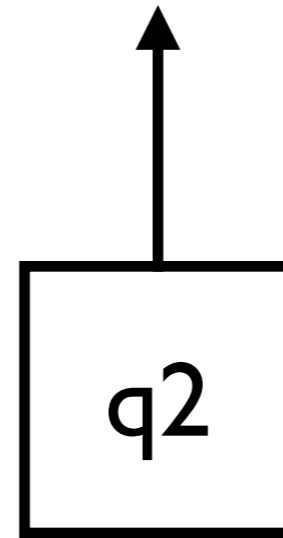
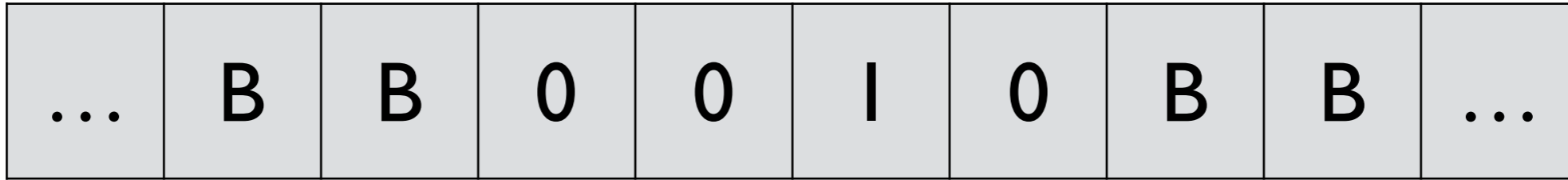
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



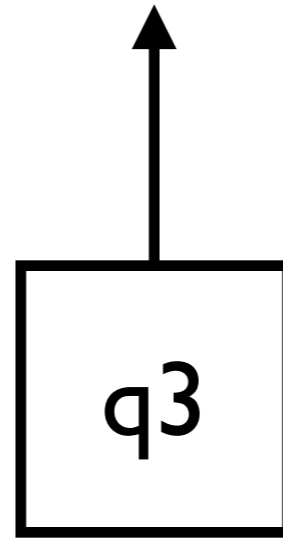
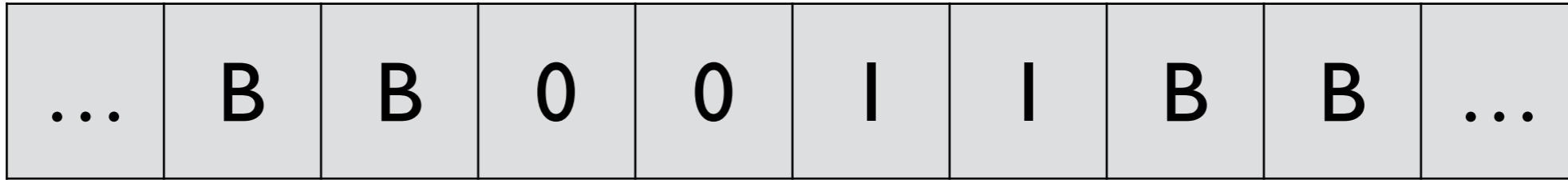
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



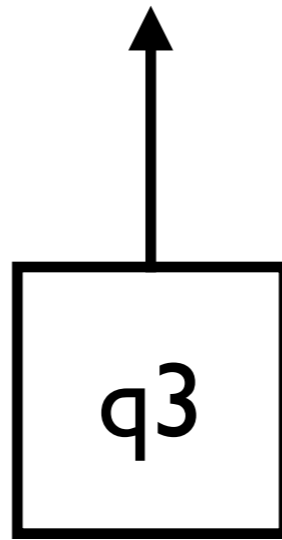
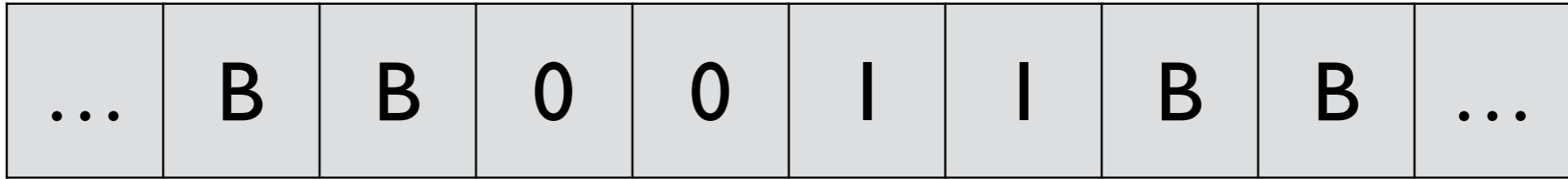
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



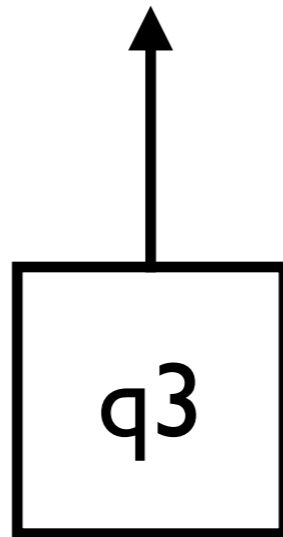
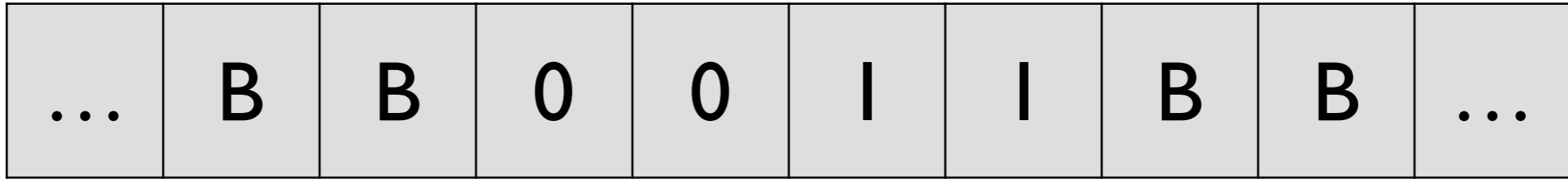
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



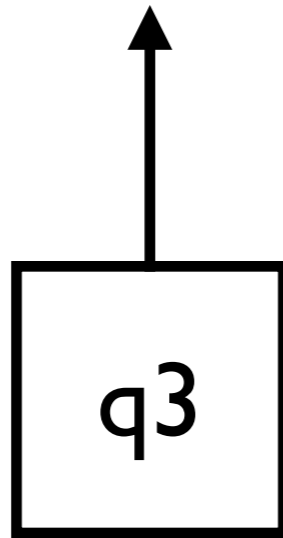
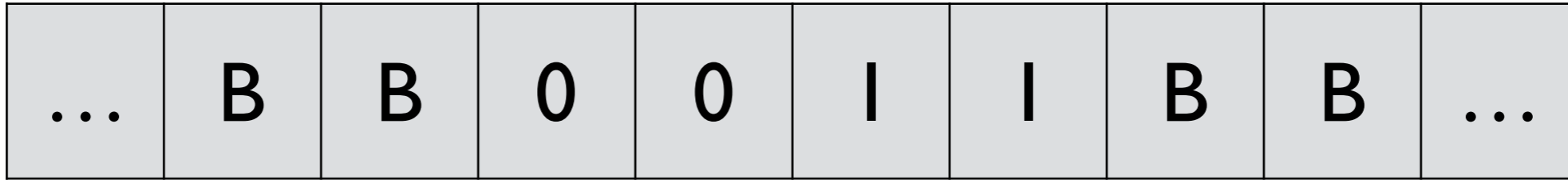
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



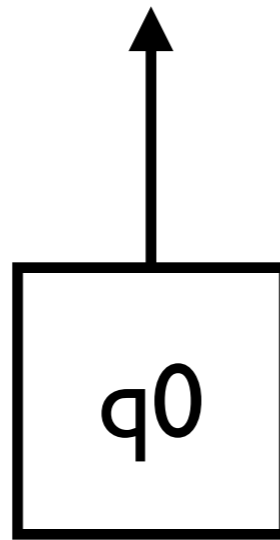
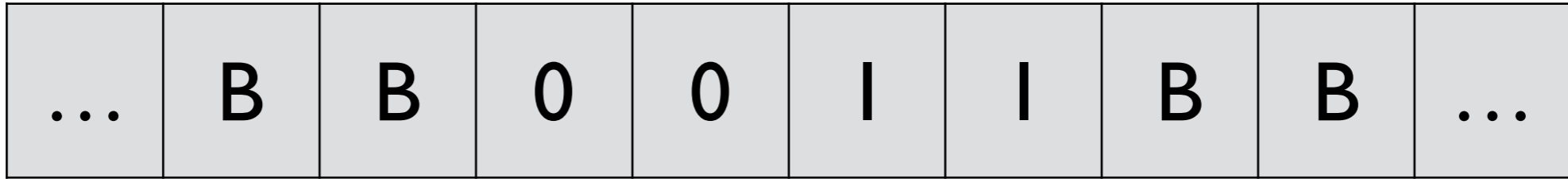
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



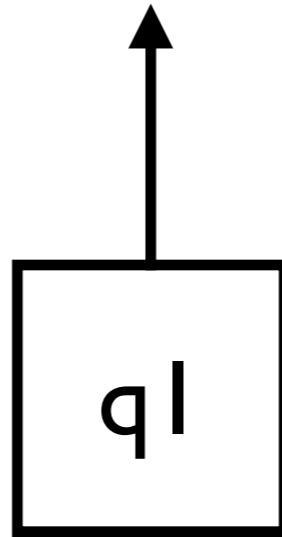
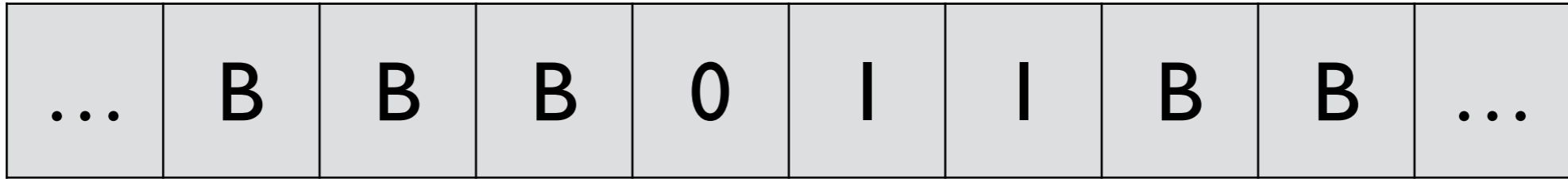
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



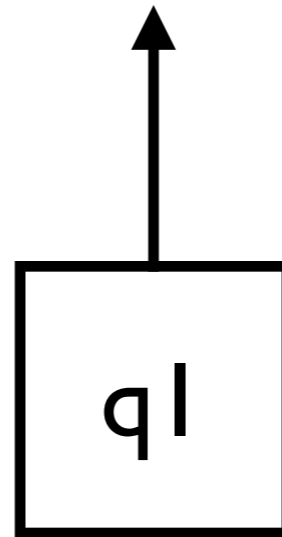
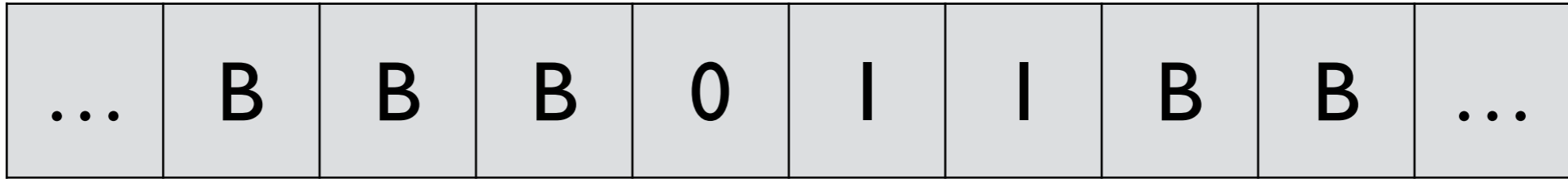
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



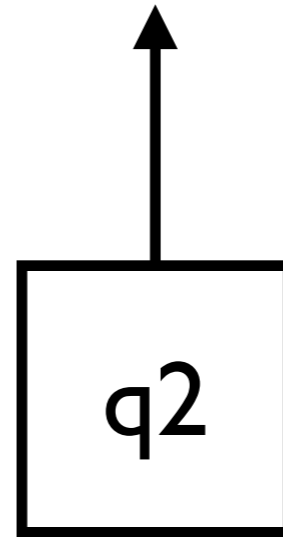
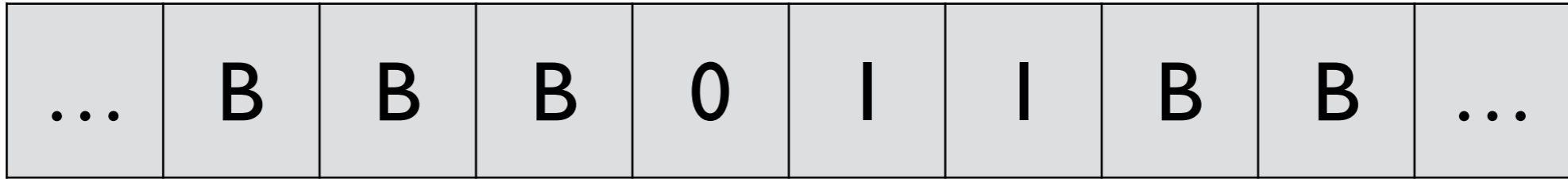
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



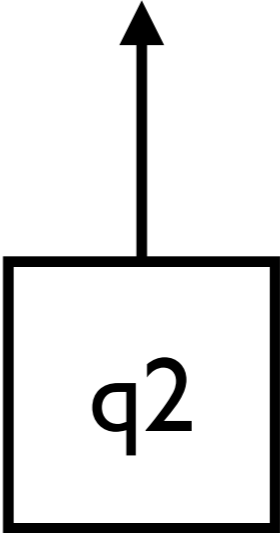
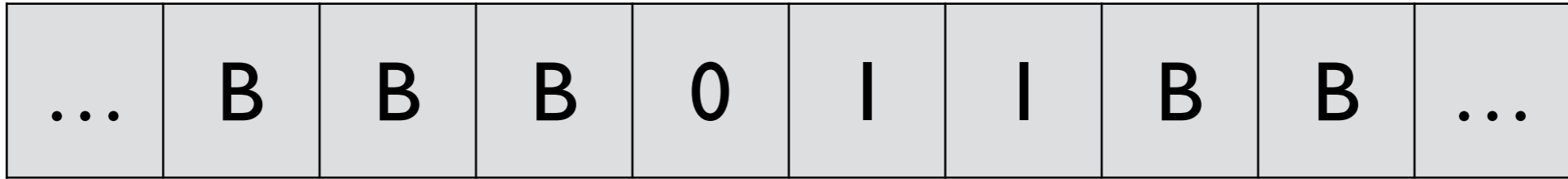
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



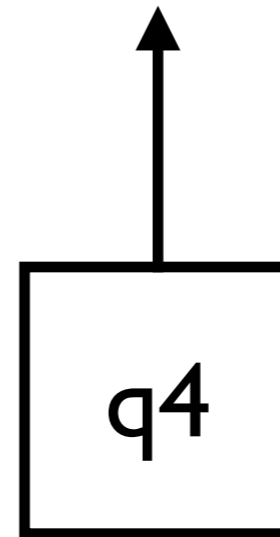
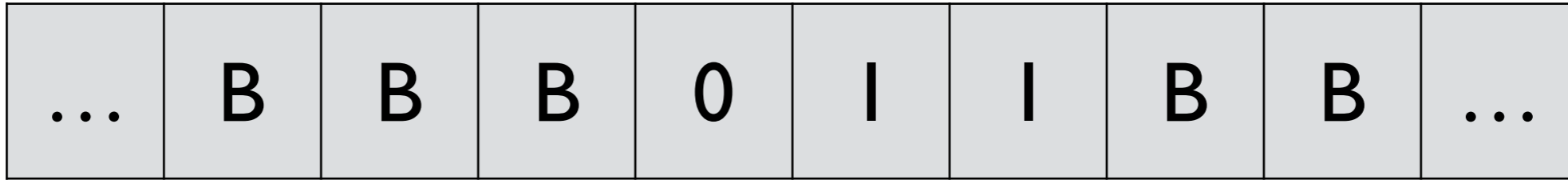
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



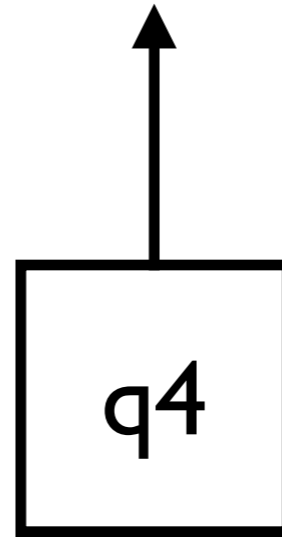
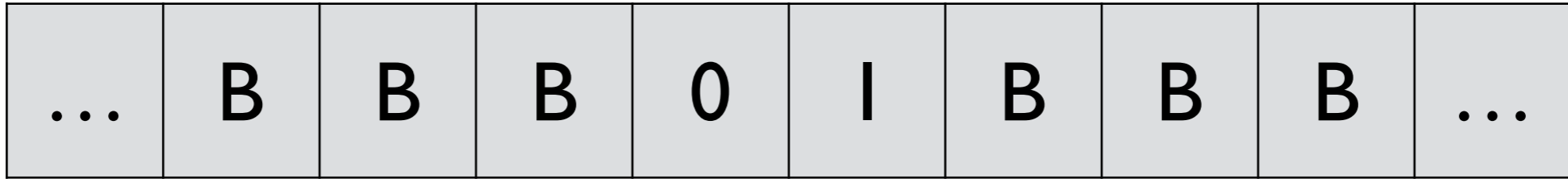
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



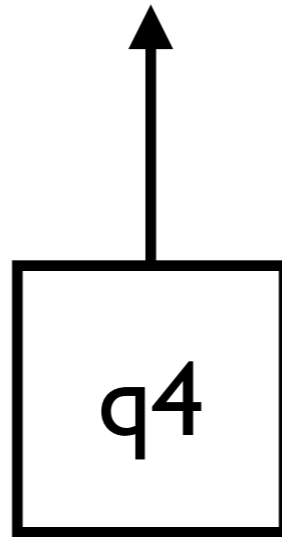
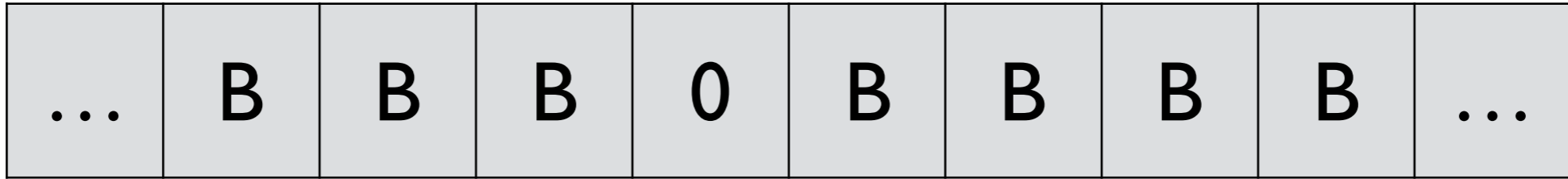
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



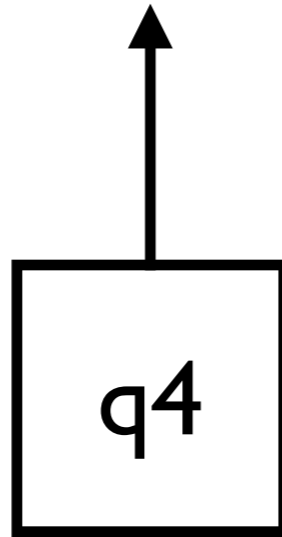
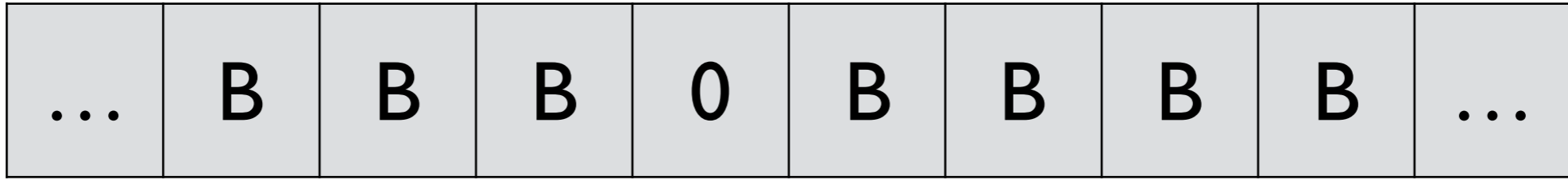
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



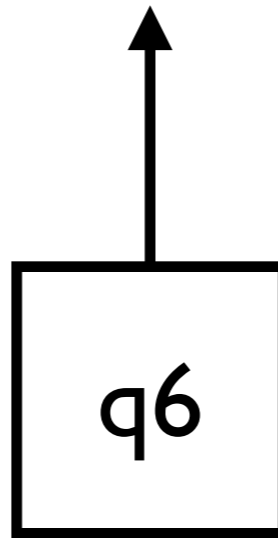
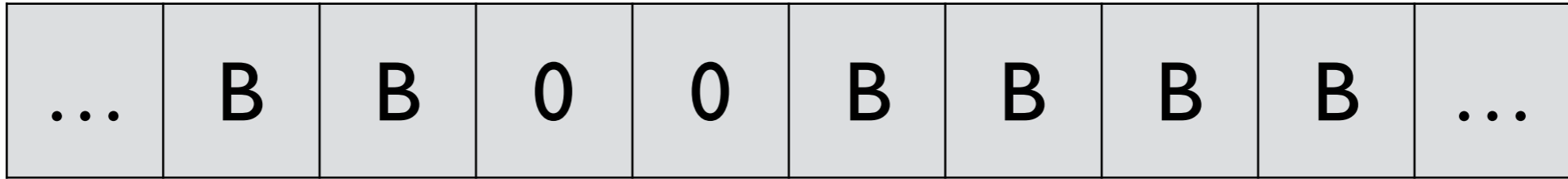
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

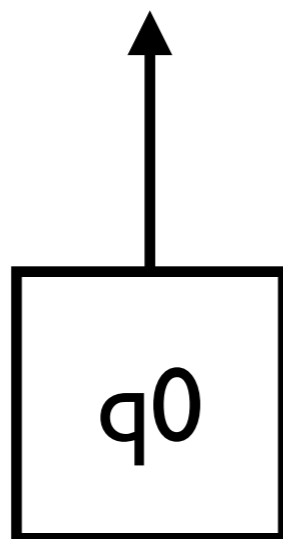


	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

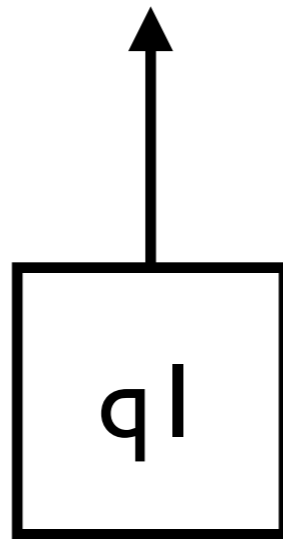
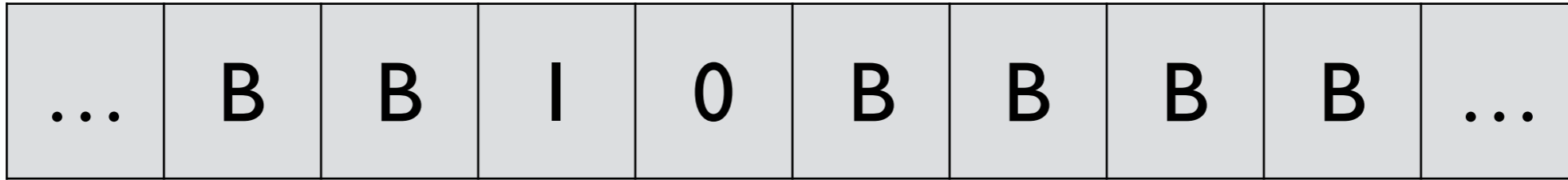


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

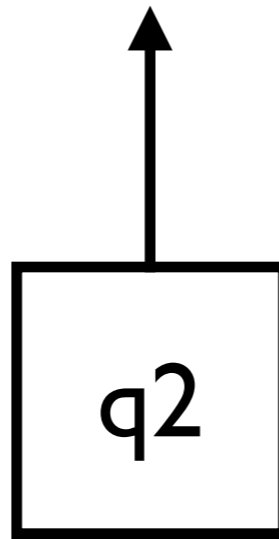
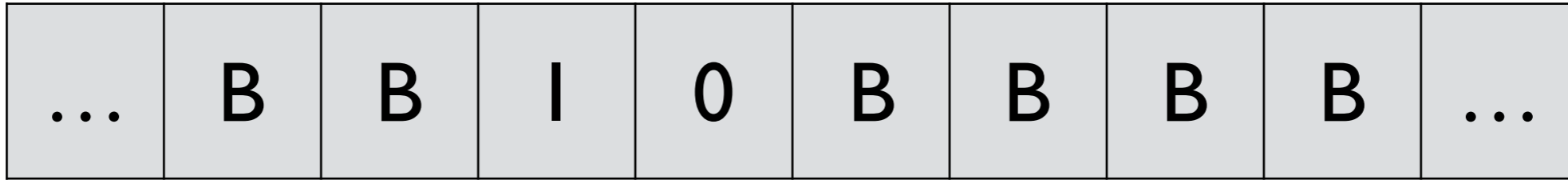
e.g., $f(1, 1)$



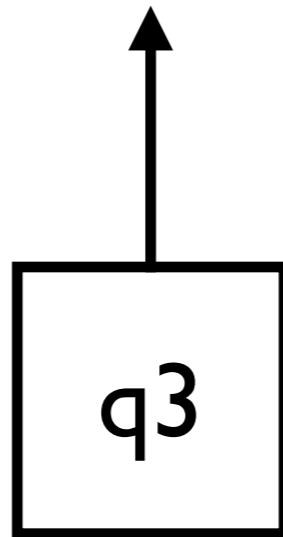
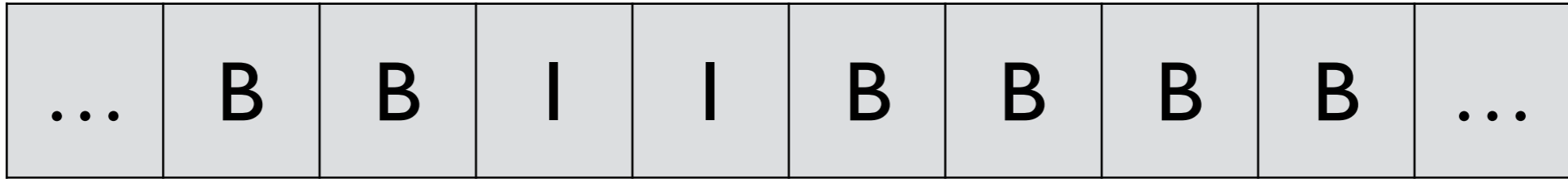
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



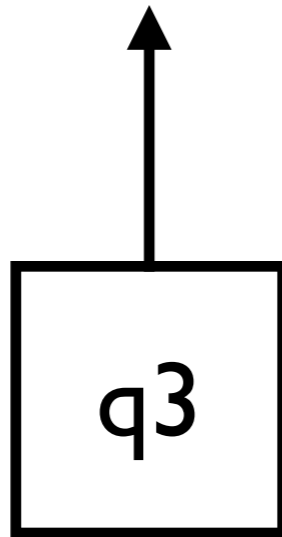
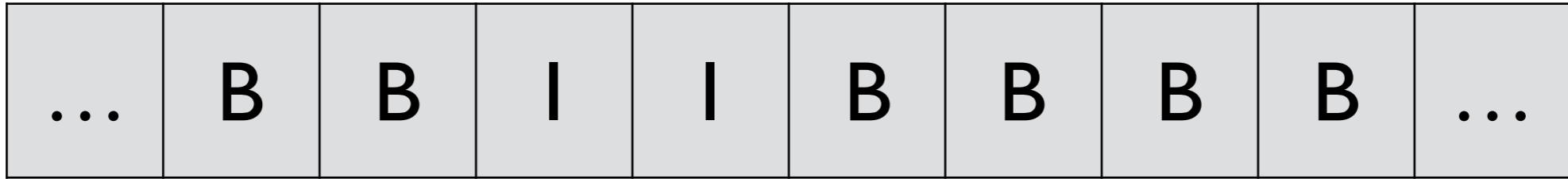
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



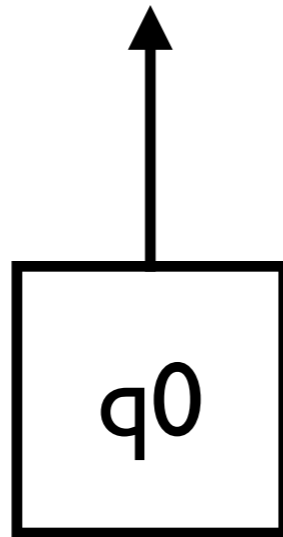
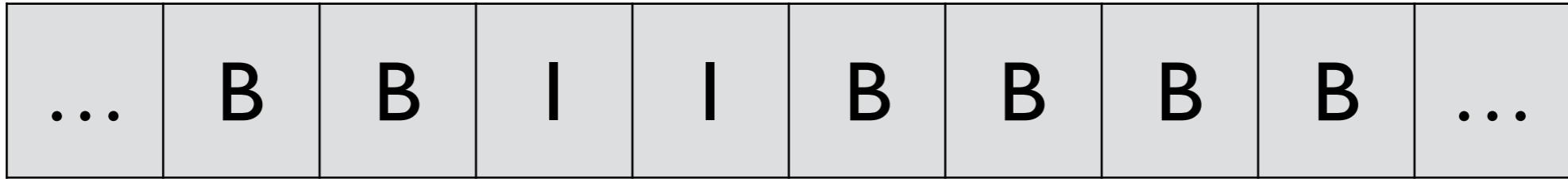
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



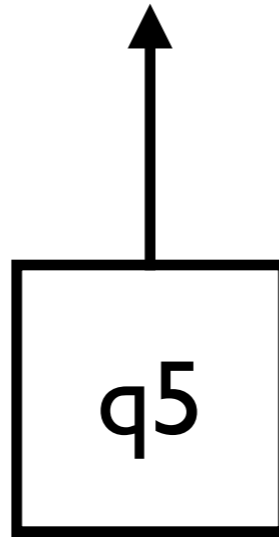
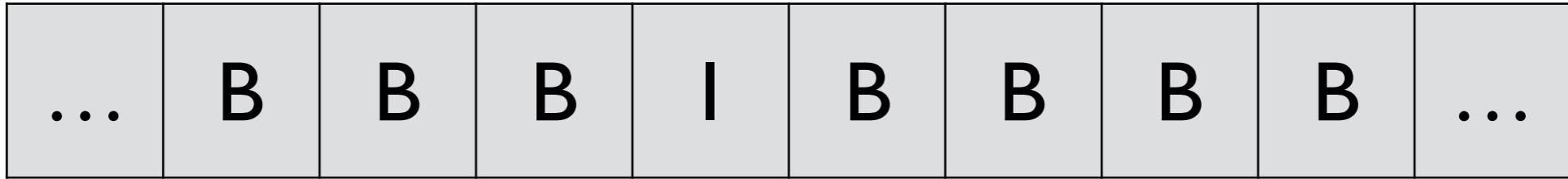
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



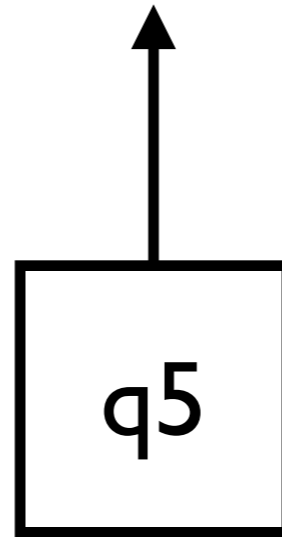
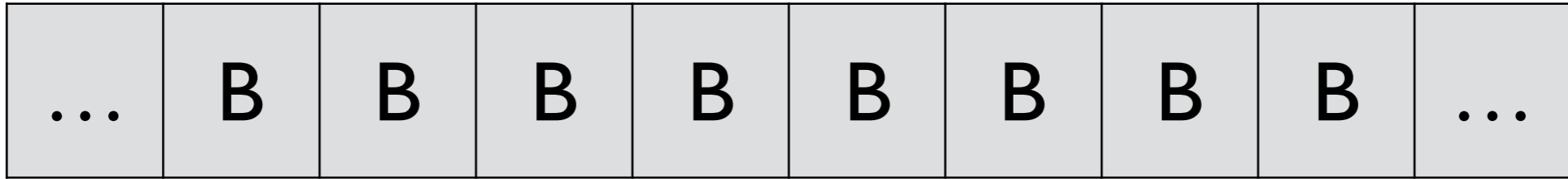
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



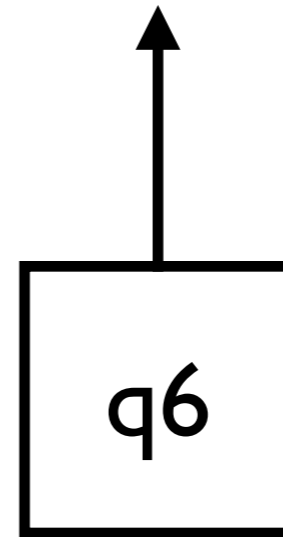
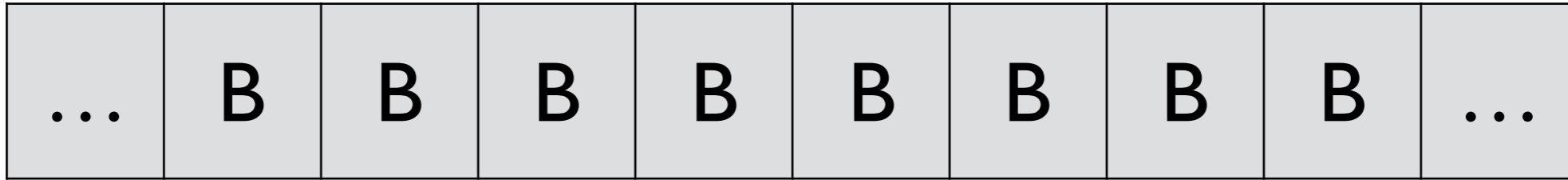
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

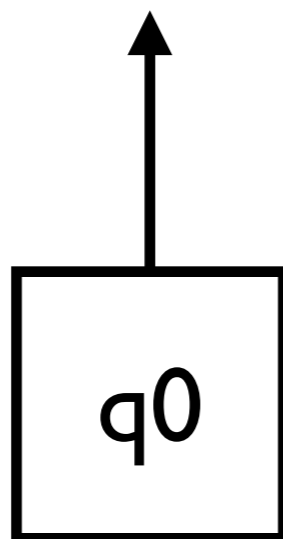
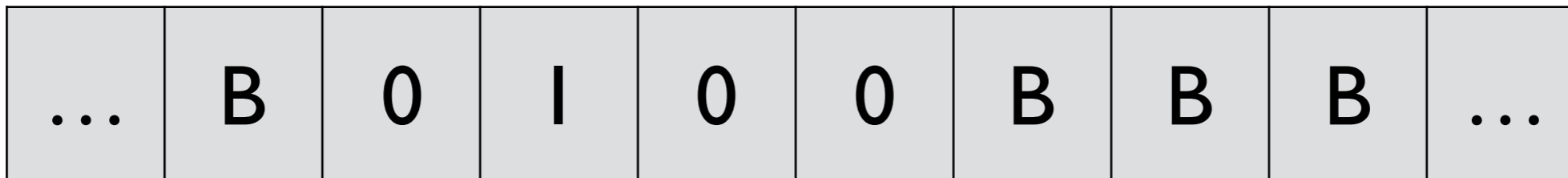


	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

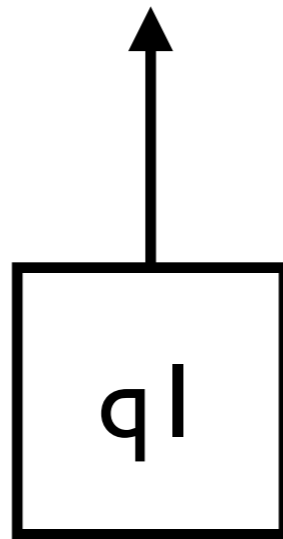
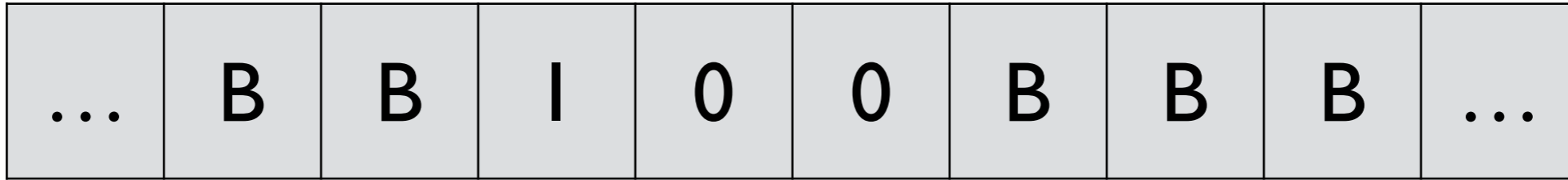


	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

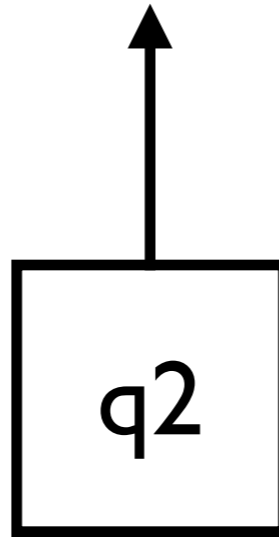
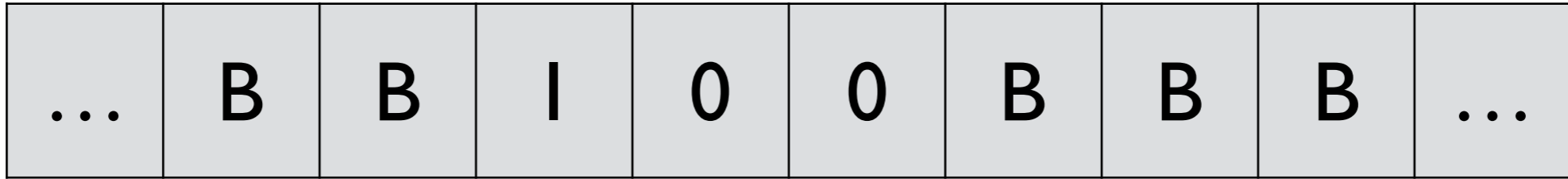
e.g., $f(1,2)$



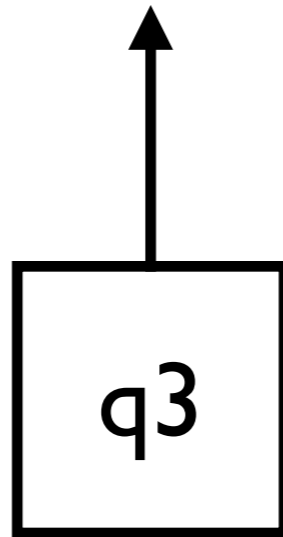
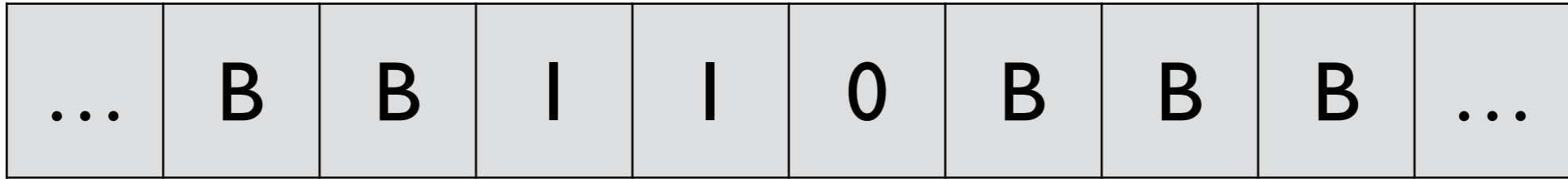
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



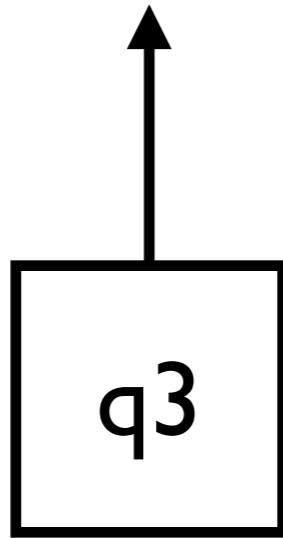
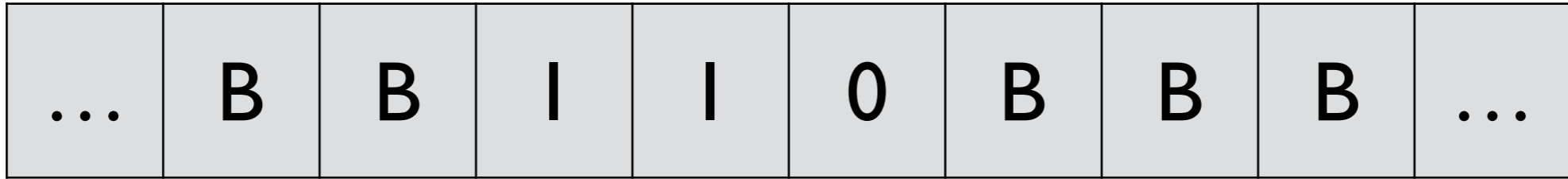
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



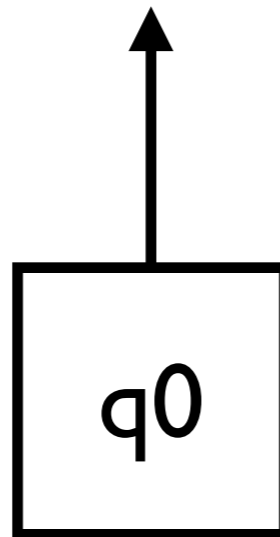
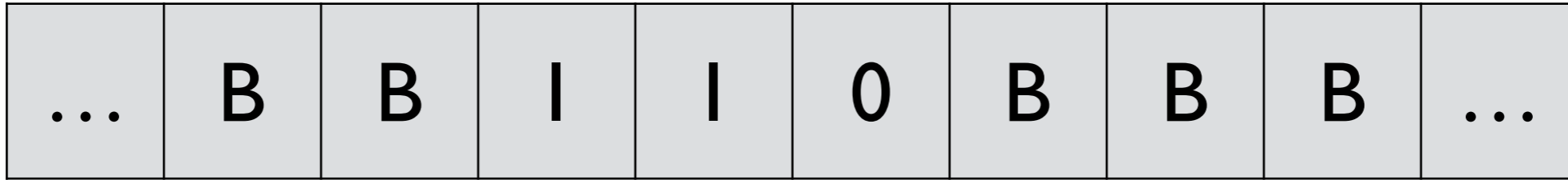
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



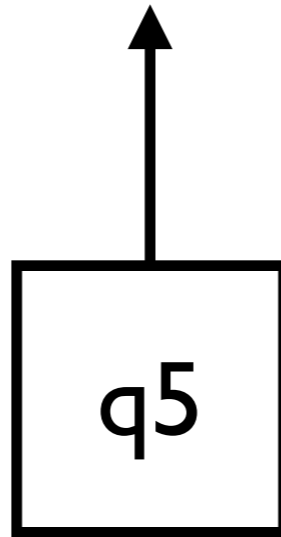
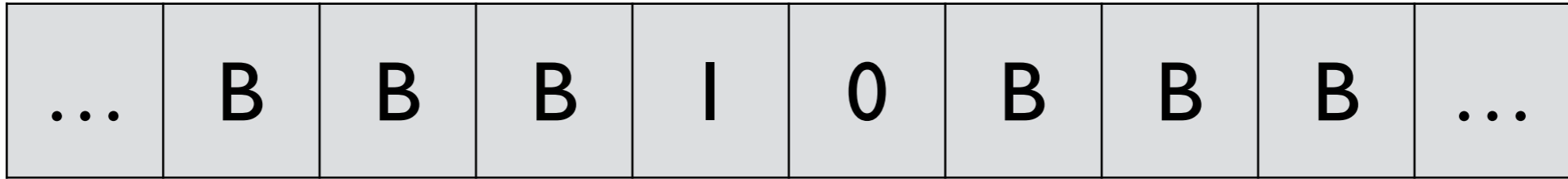
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



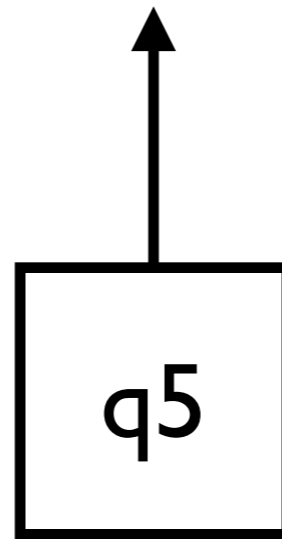
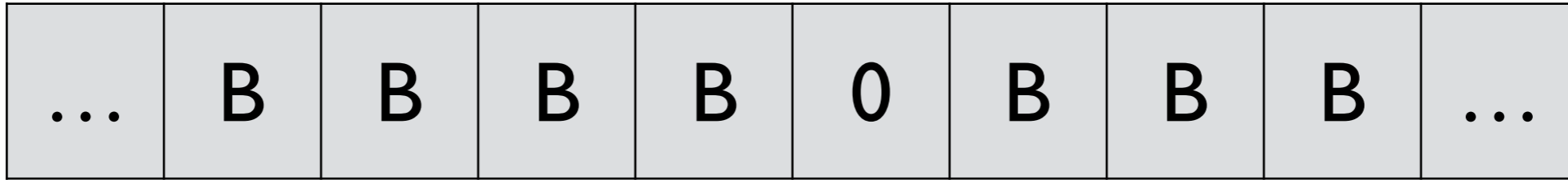
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



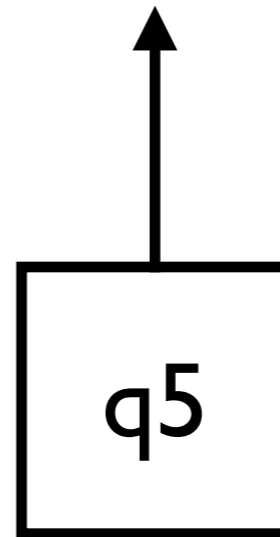
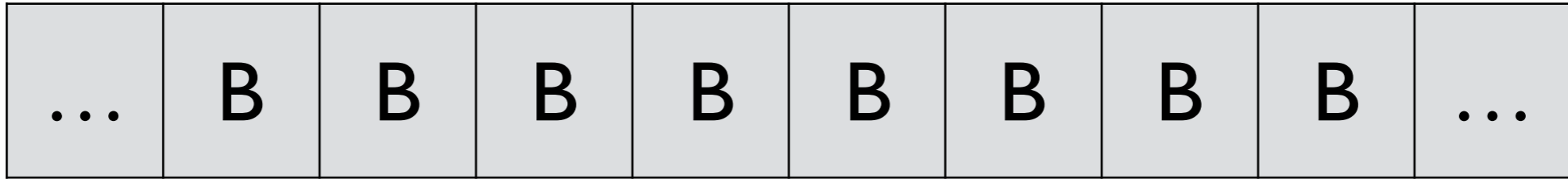
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



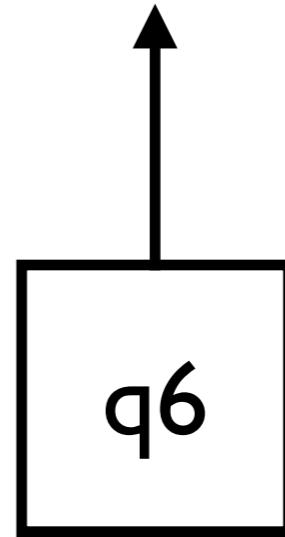
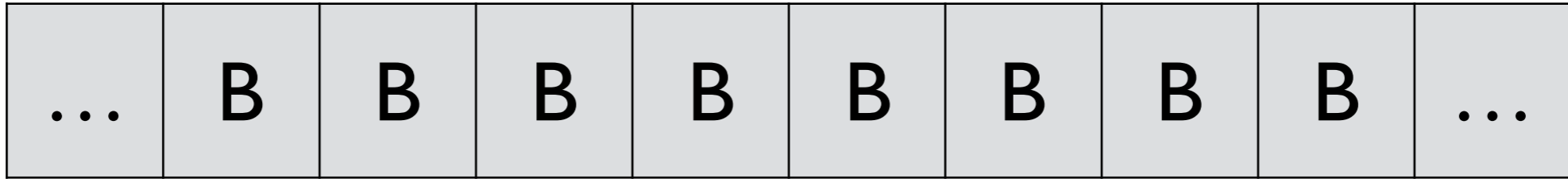
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



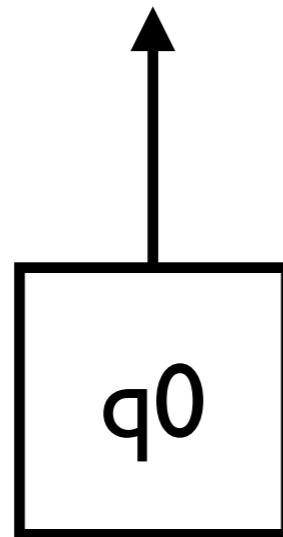
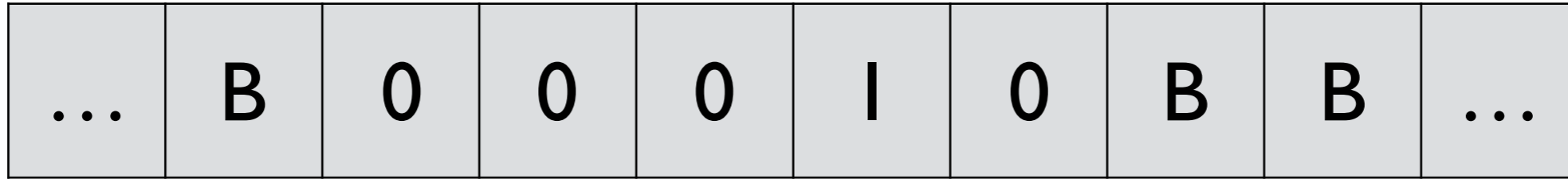
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

Example, revisited

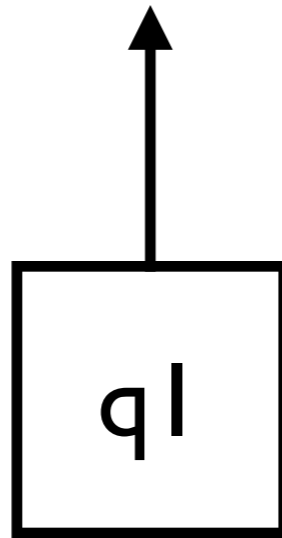
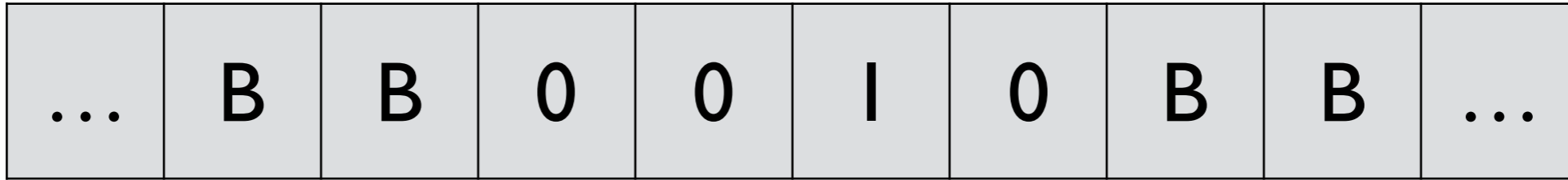
e.g., $f(3, 1)$



Scanned 0 in q_0 , the cycle must repeat:

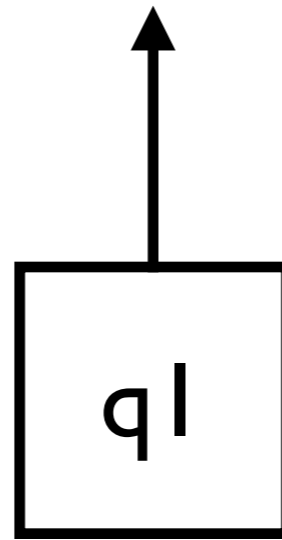
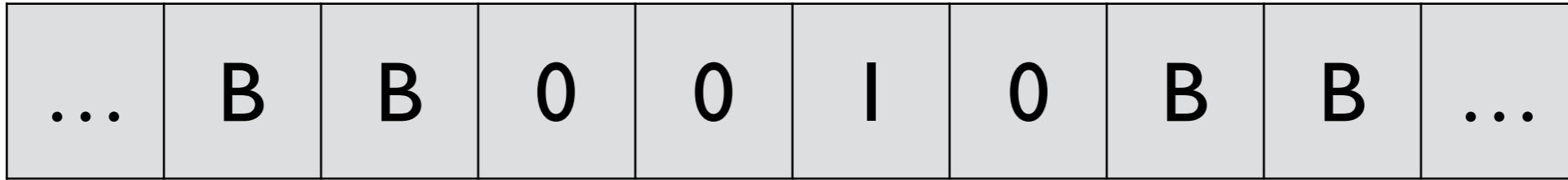
1. replace 0 by B
2. move right
3. enter q_1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



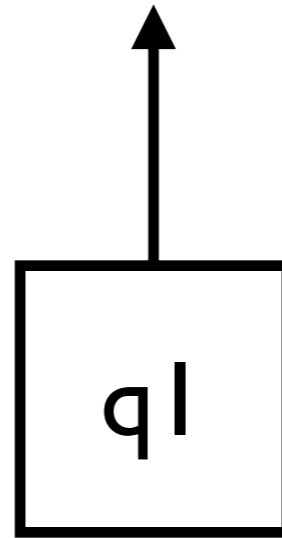
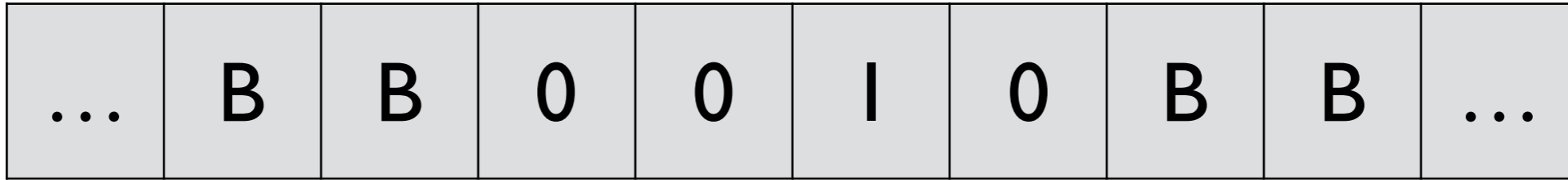
In q_1 , search right,
looking for leftmost 1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



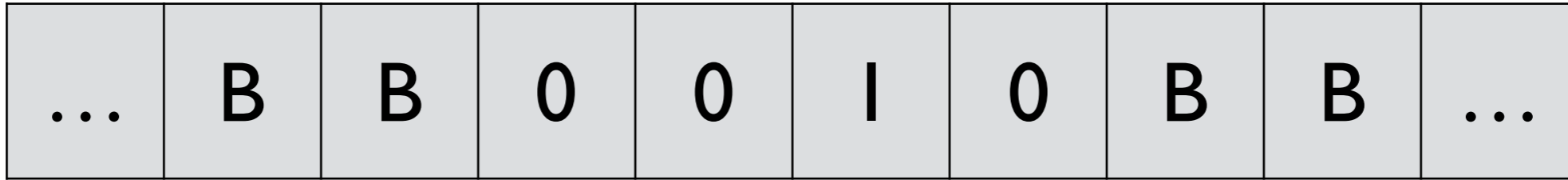
In q_1 , search right,
looking for leftmost 1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

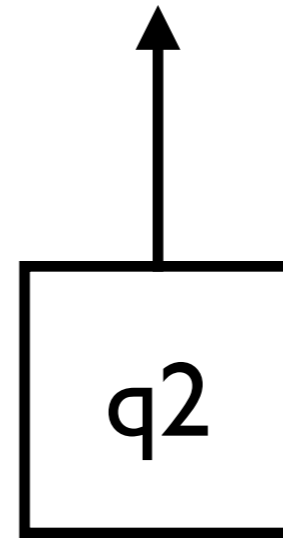


When found, enter q_2

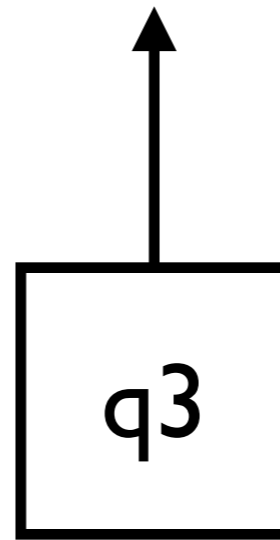
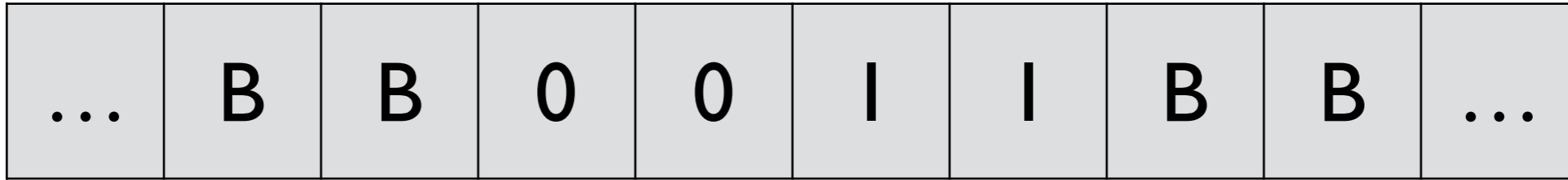
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



- In q_2 , move right until it finds 0
- When found, change it by 1
- enter q_3

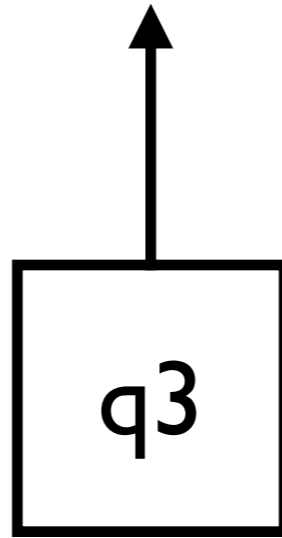
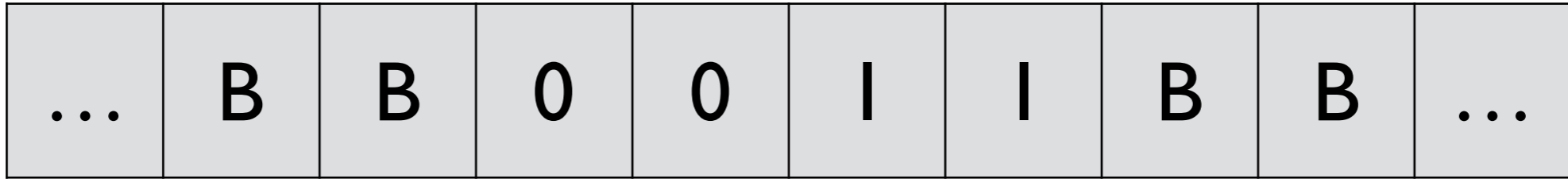


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

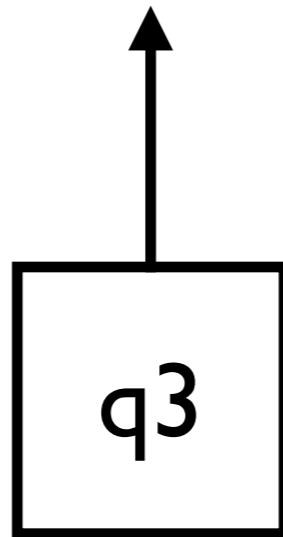
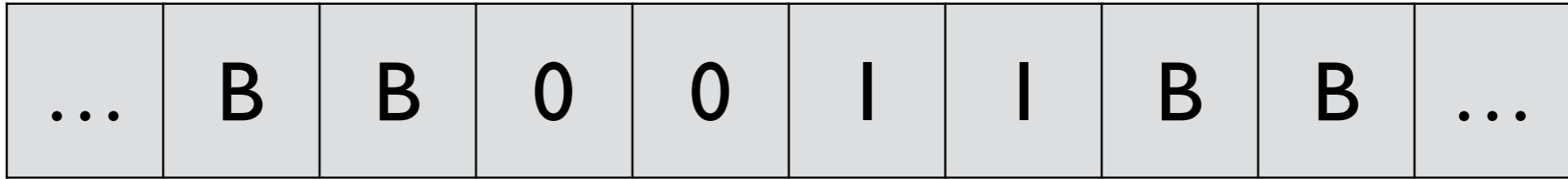


In q_3 , move left until it find B

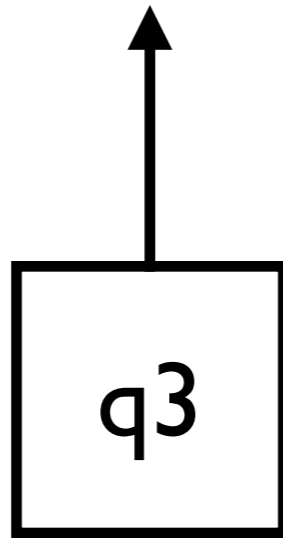
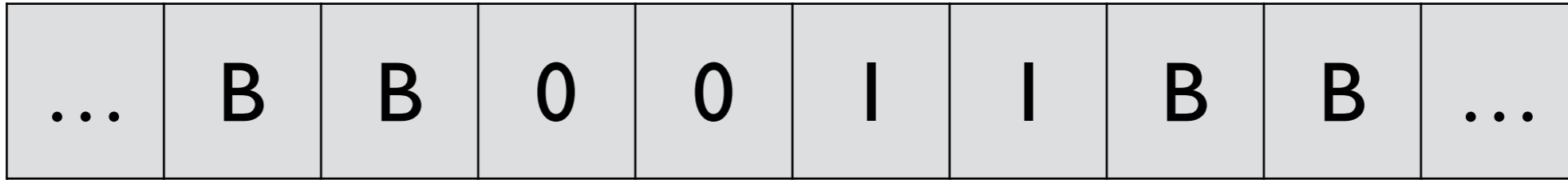
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

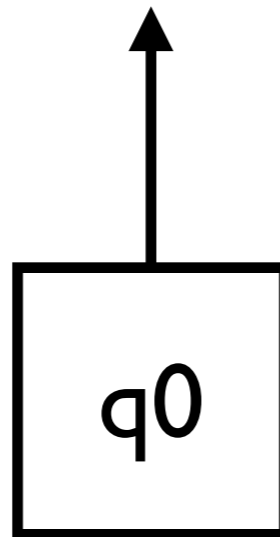
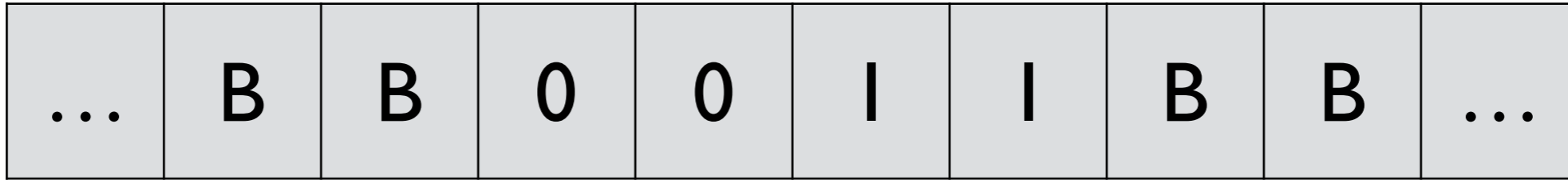


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



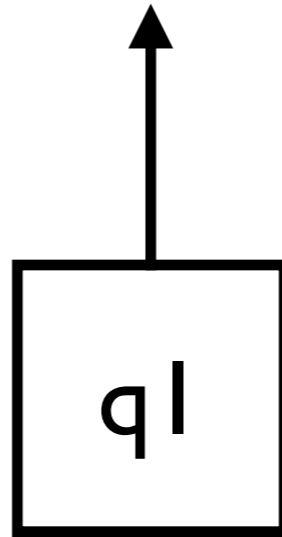
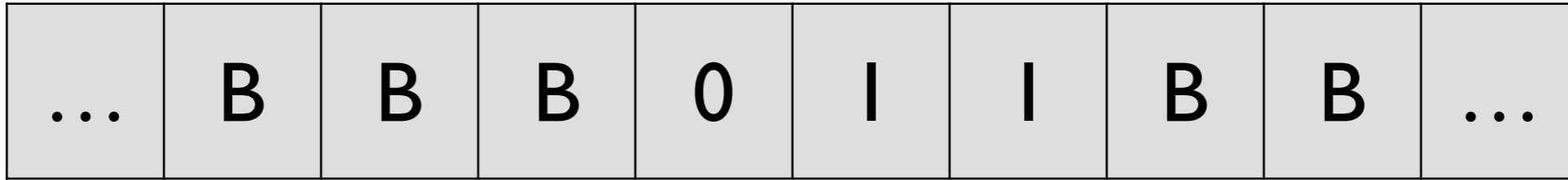
When B found, enter q_0 and begin the cycle

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



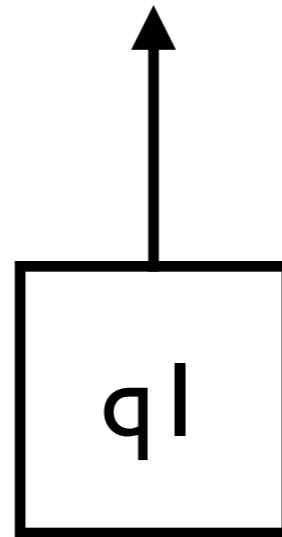
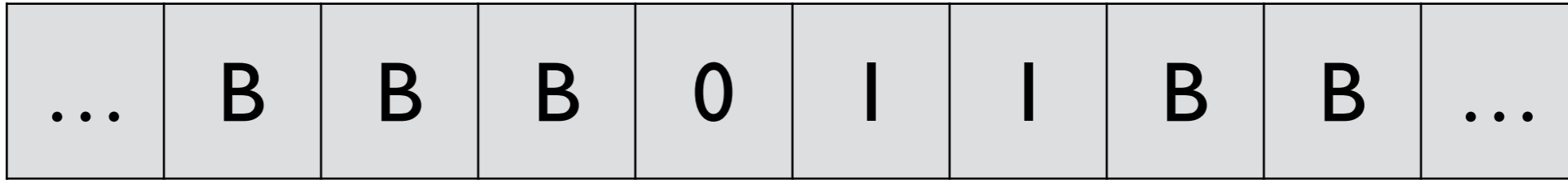
Begin a new cycle.

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



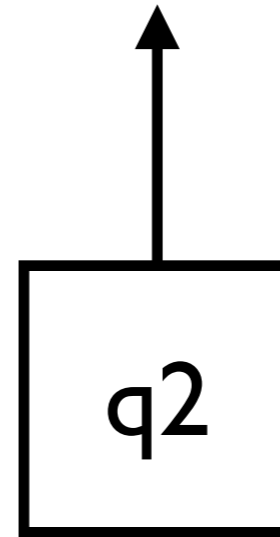
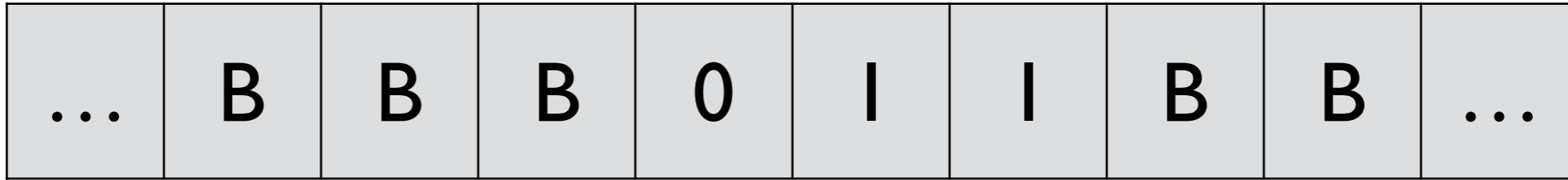
look for leftmost 1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



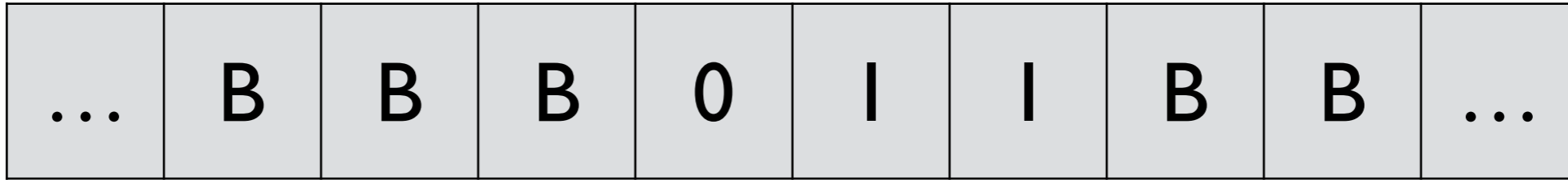
When found, enter q_2

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

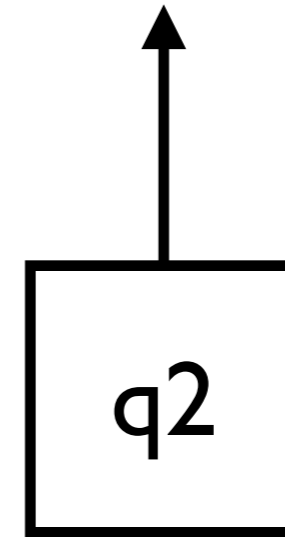


move right until it finds 0

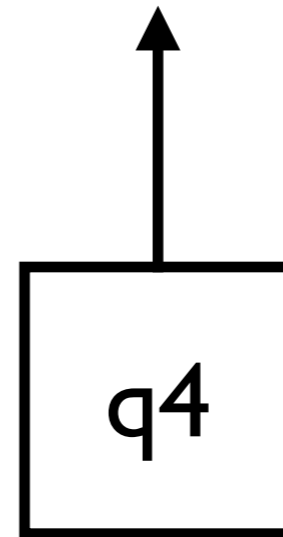
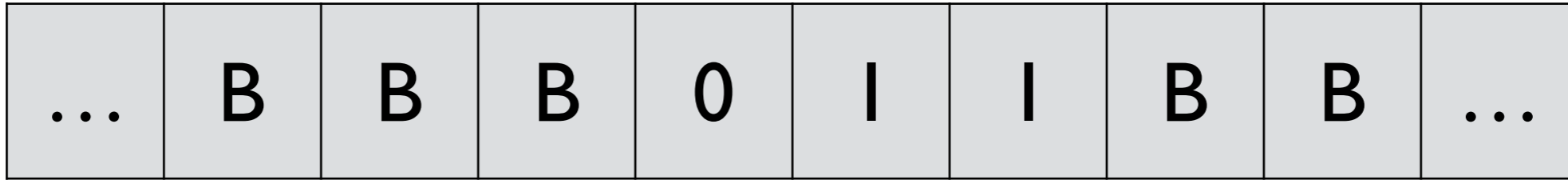
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



- No more 0's.
- All n 0's changed to 1
- $n+1$ 0's changed to B
- $m-(n+1)$ 0's on the tape
- replace all 1's by B and put one 0

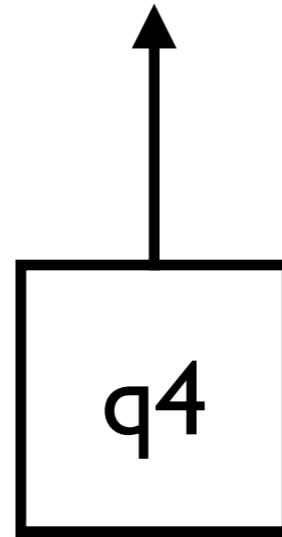
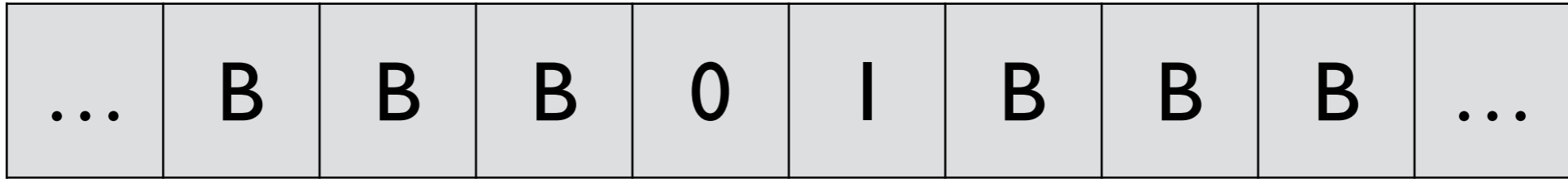


	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

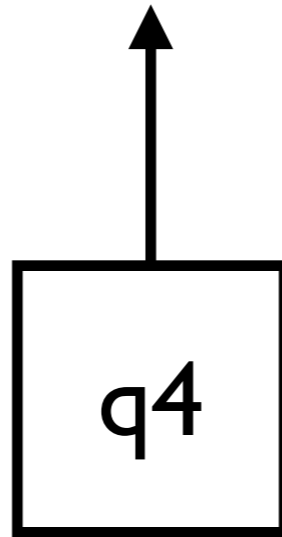
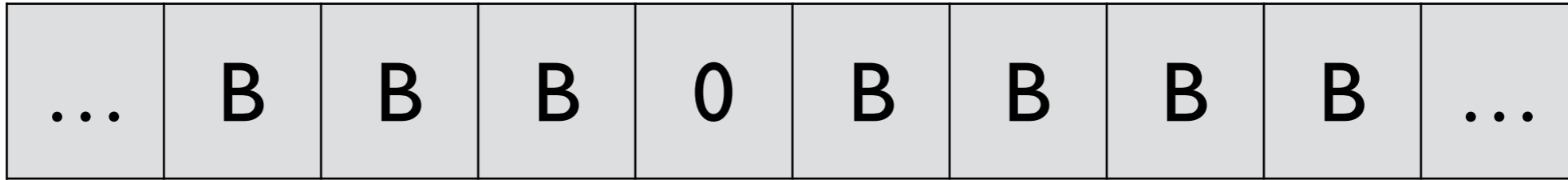


move left, changing I by B, until it finds B

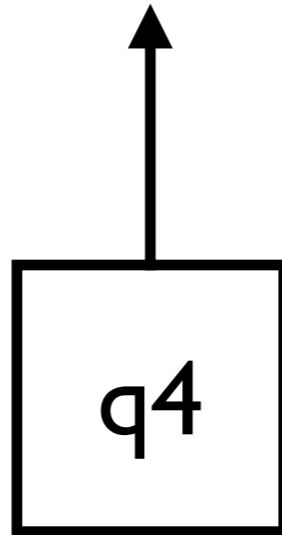
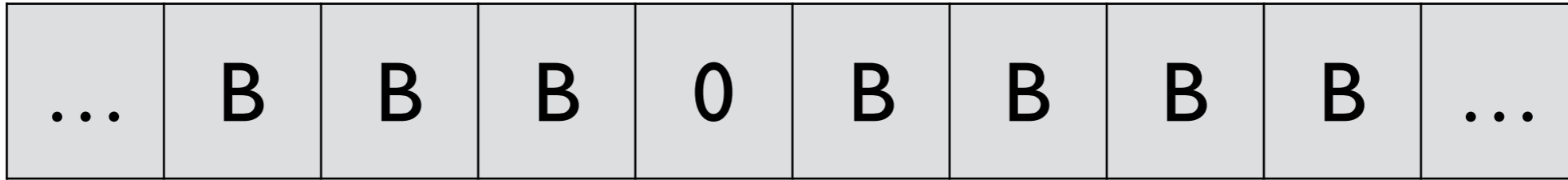
	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

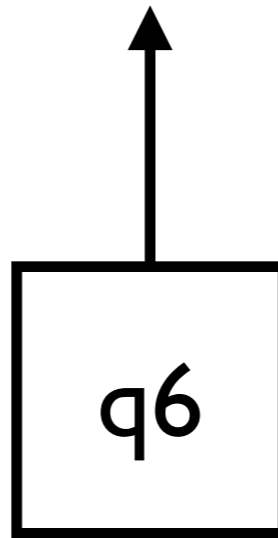
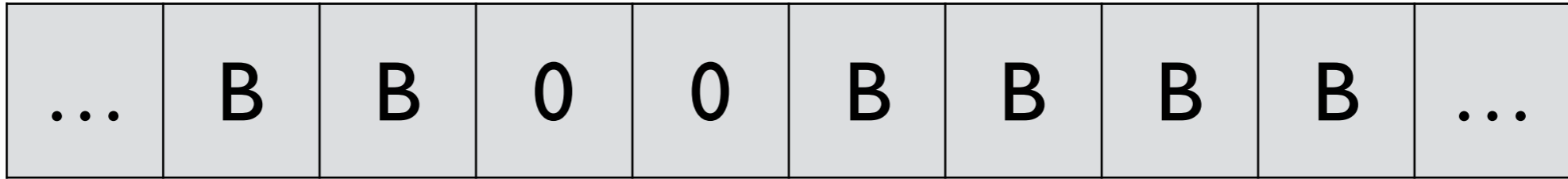


	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



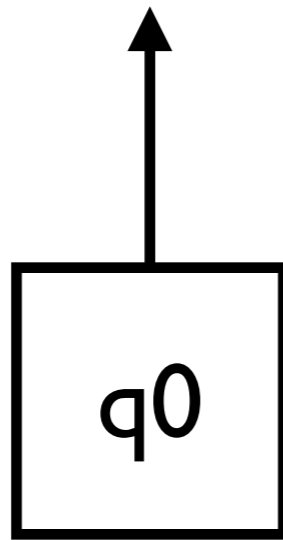
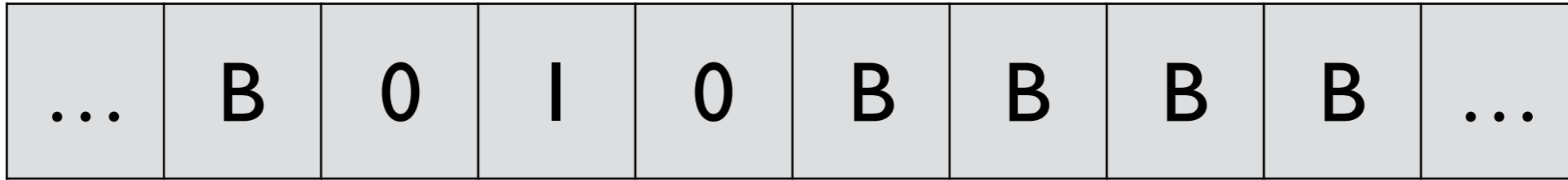
- change that B by 0
- enter the final state

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



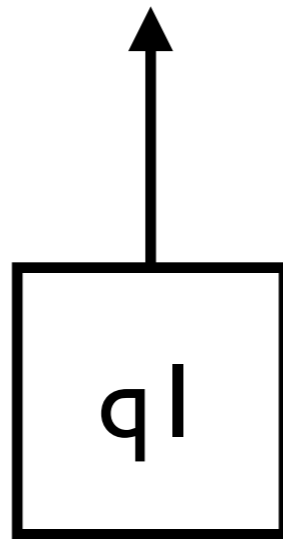
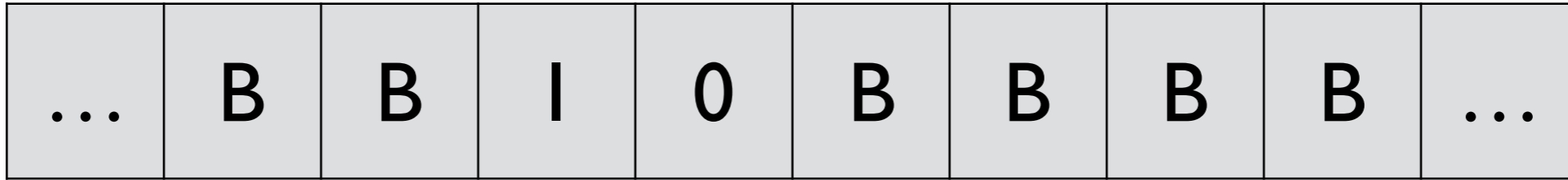
	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			

e.g., $f(1, 1)$



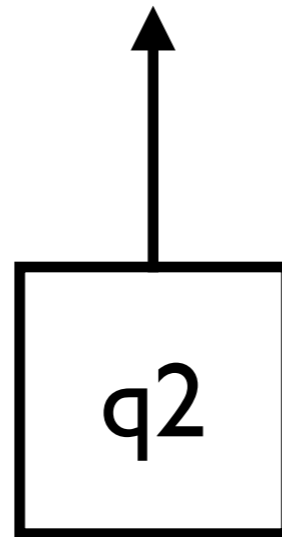
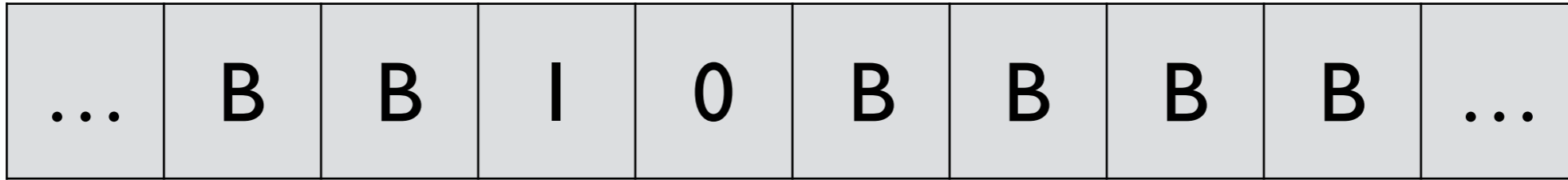
- change 0 by B
- enter q_1

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



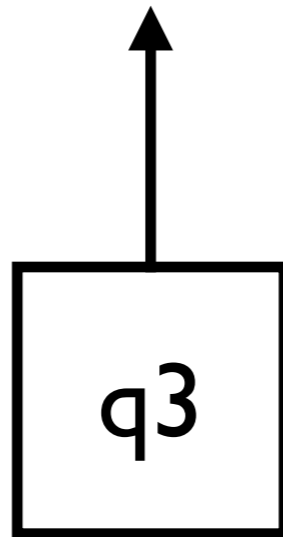
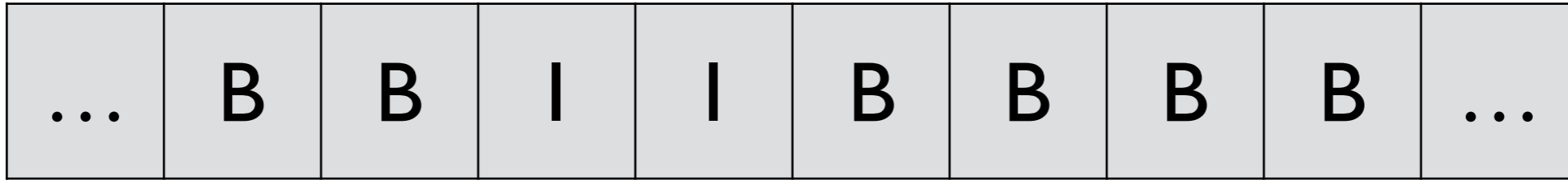
- look for the leftmost I
- when found, enter q2

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



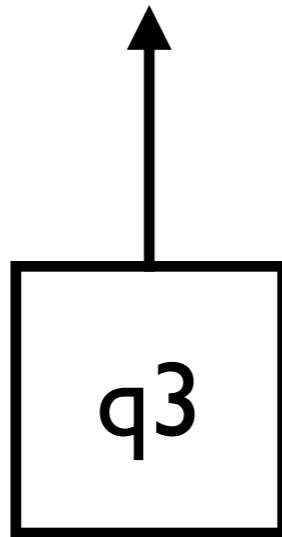
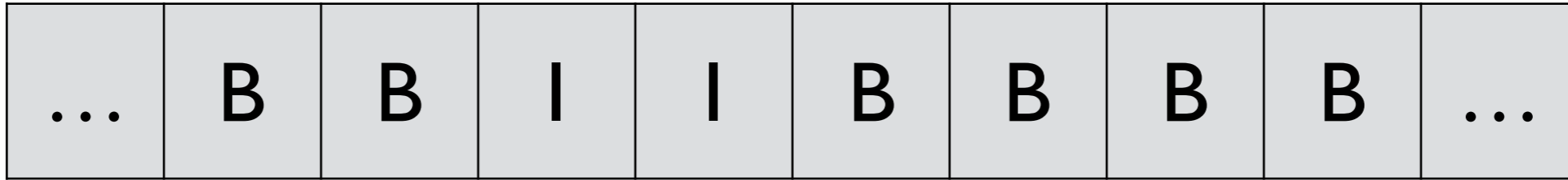
- move right until it finds 0
- when found, that 0 by 1
- enter q3

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



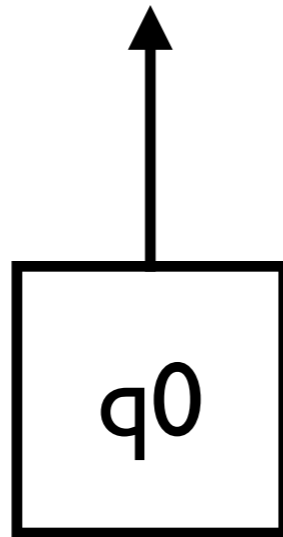
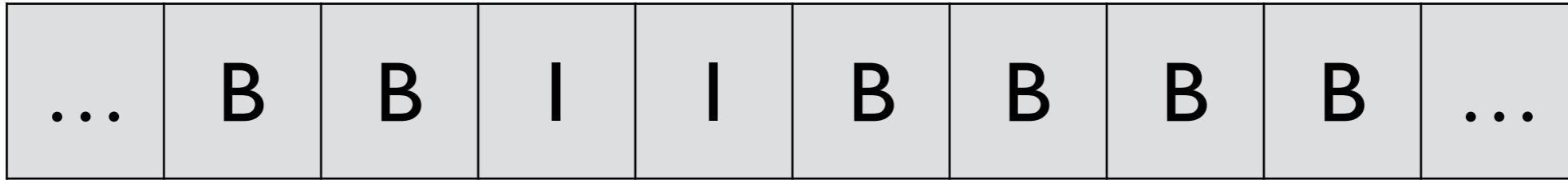
move left until it finds B

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



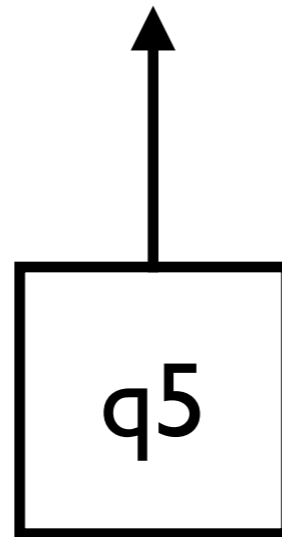
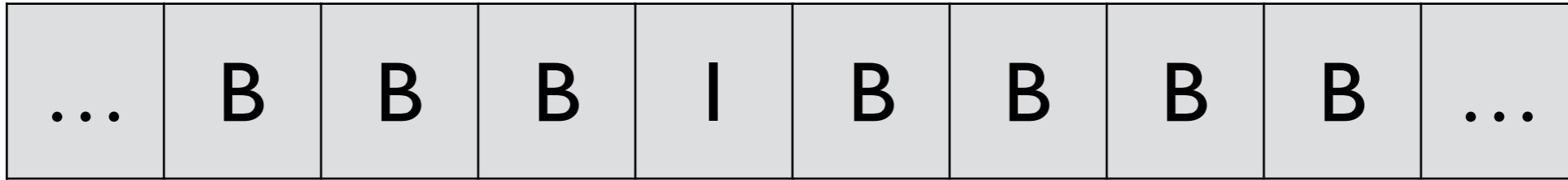
when B found, begin the cycle

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



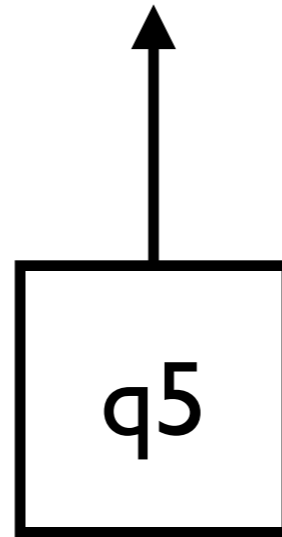
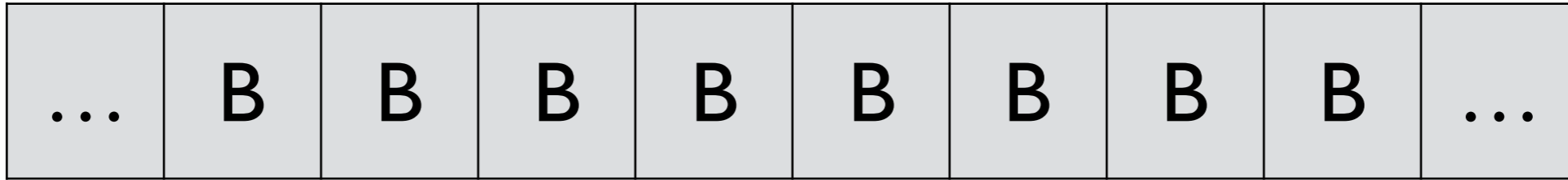
- cannot find 0 in q_0
- $n \geq m$
- replace I by B
- enter q_5

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



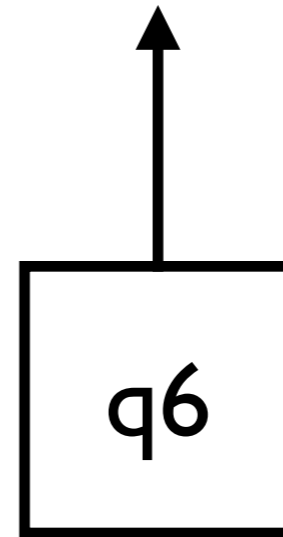
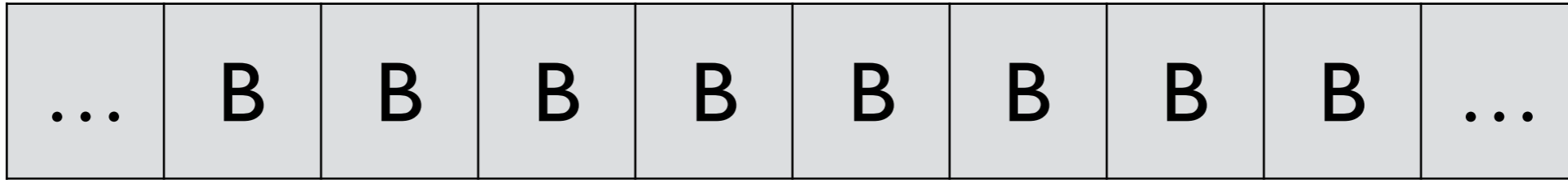
In q_5 , change all 0's and 1's to B

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



enter q6

	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			



	0	1	<i>B</i>
q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6			