Homework 2
COSE215, Spring 2019

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Due: 4/17 (in class)

Problem 1 (25pts, 5pts each) Find regular expressions for the following languages.

1. $L = \{w \in \{a, b, c\}^* \mid w \text{ contains at least one } a \text{ and at least one } b\}$
2. $L = \{a^n b^m \mid n \geq 1, m \geq 1, nm \geq 3\}$
3. $L = \{w \in \{a, b, c\}^* \mid w \text{ has no more than three } a\text{'s}\}$
4. $L = \{w \in \{0, 1\}^* \mid w \text{ begins and ends with 0 and contains at least one 1}\}$
5. $L = \{w \in \{0, 1\}^* \mid w \text{ does not contain 111}\}$

Problem 2 (20pts) Consider a DFA represented by a transition table:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

Give all the regular expressions $R_{ij}^{(0)}, R_{ij}^{(1)}, R_{ij}^{(2)}$. Try to simplfy the expressions as much as possible. Think of state q_i as if it were the state with number i.

Problem 3 (10pts) Convert the following regular expressions to finite automata (ϵ-NFA):

1. $ab^*aa + bba^*ab$
2. $(a + b)^*b(a + bb)^*$

Problem 4 (10pts) Find an ϵ-NFA that accepts language $L(ab^*a^*) \cap L(a^*b^*a)$.

Problem 5 (15pts) Suppose h is a homomorphism. Are the following statements true? If so, prove it, otherwise give a counter-example.

1. $h(L_1 \cup L_2) = h(L_1) \cup h(L_2)$
2. $h(L_1 \cap L_2) = h(L_1) \cap h(L_2)$
3. $h(L_1 L_2) = h(L_1)h(L_2)$

Problem 6 (30pts) Use the pumping lemma to prove that the following languages are not regular.

1. $L = \{0^i \mid i \text{ is a prime}\}$:
2. $L = \{ww \mid w \in \{0, 1\}^*\}$
3. $L = \{w \in \{a, b\}^* \mid n_a(w) = n_b(w)\}$ ($n_a(w)$ and $n_b(w)$ denote the number of a's and b's in w, respectively)