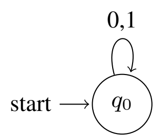
Mid-term Exam
COSE215, Spring 2016

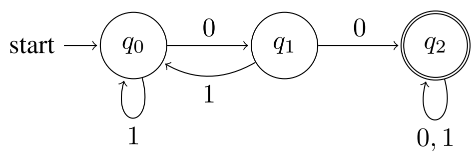
Instructor: Hakjoo Oh

Problem 1. (20pts) Write regular expressions for the following languages. ($\Sigma = \{0, 1\}$)

1. (4pts) The language of the DFA:



2. (4pts) The language of the DFA:



3. (4pts) $\{0^n 1^m \mid n \geq 2, m \text{ is odd}\}$
4. (4pts) $\{w \in \Sigma^* \mid w \text{ has an even number of 0's}\}$
5. (4pts) $\{w \in \Sigma^* \mid \text{every } 0 \text{ in } w \text{ is followed by at least one } 1\}$

Problem 2. (10pts) Recall that an NFA is a tuple of five components:

$$M = (Q, \Sigma, \delta, q_0, F)$$

where Q is a finite set of states, Σ is a finite set of input symbols, $\delta : Q \times \Sigma \to 2^Q$ is a transition function, $q_0 \in Q$ is the initial state, and $F \subseteq Q$ is a set of final states.

1. (5pts) Define the extended transition function $\delta^* : Q \times \Sigma^* \to 2^Q$.
2. (5pts) Define the language of NFA:

$$L(M) = \{w \in \Sigma^* \mid \qquad\qquad\qquad\qquad \}$$

Problem 3. (10pts) Consider the following transition table of an ϵ-NFA:

	ϵ	a	b	c
$\to p$	$\emptyset$	$\{p\}$	$\{q\}$	$\{r\}$
q	$\{p\}$	$\{q\}$	$\{r\}$	$\emptyset$
$*r$	$\{q\}$	$\{r\}$	$\emptyset$	$\{p\}$

where $\to$ indicates the initial state and $*$ the final state.

1. (5pts) Compute the ϵ-closure(ECLOSE) of each state.

$$\text{ECLOSE}(p) =$$
$$\text{ECLOSE}(q) =$$
$$\text{ECLOSE}(r) =$$

2. (5pts) Convert the automaton to a DFA. Express the DFA by a transition table.

Problem 4. (10pts) Consider the language

$$L = \{0^i 1^j \mid i > j\}.$$

Is L a regular language? If so, design a finite automaton or a regular expression for the language. Otherwise, prove that L is non-regular using the following pumping lemma.

For any regular language L there exists an integer n, such that for all $x \in L$ with $|x| \geq n$, there exist $u, v, w \in \Sigma^*$, such that

1. $x = uvw$
2. $|uv| \leq n$
3. $|v| \geq 1$
4. for all $i \geq 0$, $uv^i w \in L$.

Problem 5. (10pts) Consider the context-free grammar:

$$S \to aS \mid aSbS \mid \epsilon$$

Show that the grammar is ambiguous.

Problem 6. (10pts) Consider the context-free grammar:

$$E \to +EE \mid *EE \mid -EE \mid x \mid y$$

and consider the string $+*-xyxy$.

1. (5pts) Find the leftmost derivation of the string.
2. (5pts) Find the rightmost derivation of the string.

Problem 7. (30pts) True/false questions. (Leave a blank when you are uncertain: a correct answer gets you 2 points but you lose 2 points for each wrong answer.)

1. There is a language whose closure is not infinite.
2. Every regular language can be defined by a context-free grammar.
3. If a language is infinite, then it is non-regular.
4. Every DFA can be transformed into an equivalent NFA.
5. There is some language that can be specified by a regular expression but not by a finite automaton.
6. There is a language that satisfies the pumping lemma but is not regular.
7. We can design a finite automaton whose language equals to the C language.
8. Suppose D_1 and D_2 are any DFAs. We can always construct a DFA D such that $L(D) = L(D_1) \cap L(D_2)$.
9. Some context-free languages are recognizable by finite automata.
10. Let L be any language whose alphabet is $\{0\}$ (L is not necessarily regular). Then, L^* is always regular.
11. Recall that regular expressions are constructed by the grammar:

    $$e \to a \in \Sigma \mid \epsilon \mid \emptyset \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^*$$

    The set of all regular expressions is non-regular.
12. If L is regular, so is L^R.
13. A parse tree uniquely defines a derivation.
14. A context-free language is unambiguous iff every context-free grammar of the language is unambiguous.
15. The language of balanced parentheses

    $$\{\epsilon, (), ()(), (()), (()()), \ldots\}$$

    is defined by the following context-free grammar:

    $$S \to \epsilon \mid (S) \mid SS.$$