Final Exam
COSE215, Spring 2017

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Problem 1. (6pts) Draw a Venn-diagram to illustrate the relationships between the following classes of languages:

- *RE*: the set of recursively-enumerable languages
- *Decidable*: the set of decidable languages
- *Regular*: the set of regular languages
- *CFL*: the set of context-free languages

Problem 2. (14pts) Define the languages accepted by the PDA:

1. (7pts) $P(\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, \{q_3\})$

$$\begin{array}{rcl} \delta(q_0, a, Z_0) & = & \{(q_1, aZ_0)\} \\ \delta(q_1, a, a) & = & \{(q_1, aa)\} \\ \delta(q_1, b, a) & = & \{(q_2, \epsilon)\} \\ \delta(q_2, b, a) & = & \{(q_2, \epsilon)\} \\ \delta(q_2, b, Z_0) & = & \{(q_3, Z_0)\} \\ \delta(q_3, b, Z_0) & = & \{(q_3, Z_0)\} \end{array}$$

2. (7pts) $P = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, Z_0\}, \delta, q_0, Z_0, \{q_2\})$

$$\begin{array}{rcl} \delta(q_0, a, Z_0) & = & \{(q_0, aaZ_0)\} \\ \delta(q_0, a, a) & = & \{(q_0, aaa)\} \\ \delta(q_0, b, a) & = & \{(q_1, \epsilon)\} \\ \delta(q_1, b, a) & = & \{(q_1, \epsilon)\} \\ \delta(q_1, \epsilon, Z_0) & = & \{(q_2, Z_0)\} \end{array}$$

Problem 3. (20pts) Define the languages accepted by the Turing machines.

1. (10pts) $M = (Q, \Sigma, \Gamma, \delta, q_0, B, \{q_f\})$ where
   - $Q = \{q_0, q_a, q_b, q_c, q_Y, q_Z, q_f\}$
   - $\Sigma = \{a, b, c\}$
   - $\Gamma = \{a, b, c, X, Y, Z, B\}$
   - δ:

$$\begin{array}{rclrcl} \delta(q_0, a) & = & (q_a, X, R) & \delta(q_0, Y) & = & (q_Y, Y, R) \\ \delta(q_a, a) & = & (q_a, a, R) & \delta(q_a, b) & = & (q_b, Y, R) \\ \delta(q_a, Y) & = & (q_a, Y, R) & \delta(q_b, b) & = & (q_b, b, R) \\ \delta(q_b, c) & = & (q_c, Z, L) & \delta(q_b, Z) & = & (q_b, Z, R) \\ \delta(q_c, a) & = & (q_c, a, L) & \delta(q_c, b) & = & (q_c, b, L) \\ \delta(q_c, X) & = & (q_0, X, R) & \delta(q_c, Y) & = & (q_c, Y, L) \\ \delta(q_c, Z) & = & (q_c, Z, L) & \delta(q_Y, Y) & = & (q_Y, Y, R) \\ \delta(q_Y, Z) & = & (q_Z, Z, R) & \delta(q_Z, Z) & = & (q_Z, Z, R) \\ \delta(q_Z, B) & = & (q_f, B, R) & & & \end{array}$$

2. (10pts) $M = (Q, \Sigma, \Gamma, \delta, q_i, B, \{q_f\})$ where
   - $Q = \{q_i, q_0, q_1, q_2, q_3, q_b, q_f\}$
   - $\Sigma = \{0, 1\}$
   - $\Gamma = \{0, 1, B\}$
   - δ:

$$\begin{array}{rclrcl} \delta(q_i, 0) & = & (q_0, B, R) & \delta(q_i, 1) & = & (q_1, B, R) \\ \delta(q_i, B) & = & (q_f, B, R) & \delta(q_0, 0) & = & (q_0, 0, R) \\ \delta(q_0, 1) & = & (q_0, 1, R) & \delta(q_0, B) & = & (q_2, B, L) \\ \delta(q_1, 0) & = & (q_1, 0, R) & \delta(q_1, 1) & = & (q_1, 1, R) \\ \delta(q_1, B) & = & (q_3, B, L) & \delta(q_2, 0) & = & (q_b, B, L) \\ \delta(q_3, 1) & = & (q_b, B, L) & \delta(q_b, 0) & = & (q_b, 0, L) \\ \delta(q_b, 1) & = & (q_b, 1, L) & \delta(q_b, B) & = & (q_i, B, R) \end{array}$$

Problem 4. (60pts) True/False questions. (Do not answer when you are uncertain; A correct answer gets you 2 points but you lose 2 points for each wrong answer.)

1. $L = \{a^n b^m \mid n, m \geq 0\}$ is regular.
2. $L = \{a^n b^n \mid n \geq 0\}$ is context-free but not regular.
3. $L = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free but decidable.
4. $L = \{a^m b^m c^n d^n \mid m, n \geq 0\}$ is context-free.
5. $L = \{ww \mid w \in \{0,1\}^*\}$ is context-free.
6. $L = \{ww^R \mid w \in \{0,1\}^*\}$ is context-free.
7. $L = \{ww^R \mid w \in \{0,1\}^*\}$ is deterministic context-free.
8. $L = \{wcw \mid w \in \{0,1\}^+\}$ is context-free.
9. $L = \{wcw^R \mid w \in \{0,1\}^+\}$ is context-free.
10. $L = \{wcw^R \mid w \in \{0,1\}^+\}$ is deterministic context-free.
11. $L = \{w \mid |w| = 2k \text{ for some } k\}$ is recursive.
12. $L = \{w \in \{a,b,c\}^* \mid n_a(w) = n_b(w) = n_c(w)\}$ is context-free.
13. $L = \{a^{n^2} \mid n \geq 0\}$ is context-free.
14. Intersection with a context-free language and a regular language always produces a regular language.
15. For any CFL L, L^* is context-free.
16. We can use the pumping lemma for context-free languages to show that some language not to be regular.
17. Pushdown automata with two stacks are as powerful as Turing machines.
18. Pushdown automata with a single queue is as powerful as Turing machines.
19. The grammar
$$S \to TA \mid bA \mid Ab \mid b, \qquad A \to Aa \mid a, \qquad T \to Ab$$
is in Chomsky Normal Form.
20. The grammar
$$S \to aXbX, \qquad X \to aY \mid bY \mid \epsilon, \qquad Y \to X \mid c$$
has two nullable variables.
21. Let $G = (V, T, S, P)$ be a CFG. The set of unit pairs is defined as the smallest set X such that $X = F(X)$, where $F(X) = \{(A, A) \mid A \in V\} \cup \{(A, C) \mid (A, B) \in X, B \to C \in P\}$.
22. The problem of removing ambiguity from a context-free language is in the NP class.
23. With multitape Turing machine, it is possible to decide the language $L = \{a^n b^n c^n \mid n \geq 1\}$ in linear time.
24. Nondeterministic Turing machines have the same expressiveness as the standard Turing machines.
25. There is a problem solvable by Turing machines with two tapes but unsolvable by Turing machines with a single tape.
26. The language $L = \{\langle M, w\rangle \mid M \text{ halts on input } w\}$ is recursively enumerable.
27. The language $L = \{\langle M, w\rangle \mid M \text{ halts on input } w\}$ is recursive.
28. The boolean formula $x \wedge \neg(y \vee z)$ is satisfiable.
29. NP-Complete problems cannot be solved by Turing machines.
30. When there is a polynomial-time reduction from L_1 to L_2 (i.e. $L_1 \leq_P L_2$), L_1 is harder than L_2 to solve.