COSE 215: Theory of Computation

Examples of Turing Machines (2)

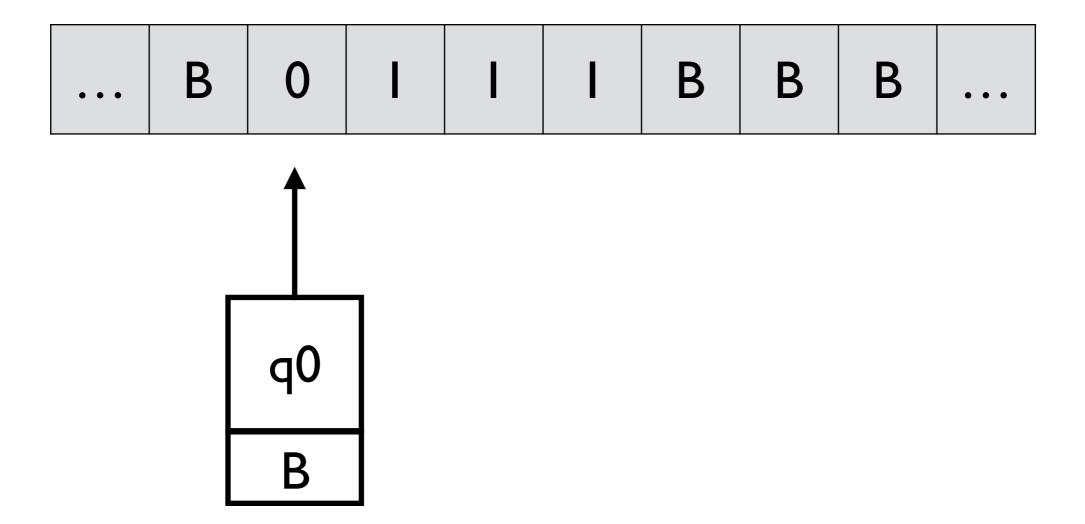
Hakjoo Oh 2017 Spring **Example 1.** Design a Turing machine that accepts $01^* + 10^*$.

 $M = (\{q_0, q_1\} \times \{0, 1, B\}, \{0, 1\}, \{0, 1, B\}, \delta, (q_0, B), B, \{(q_1, B)\})$

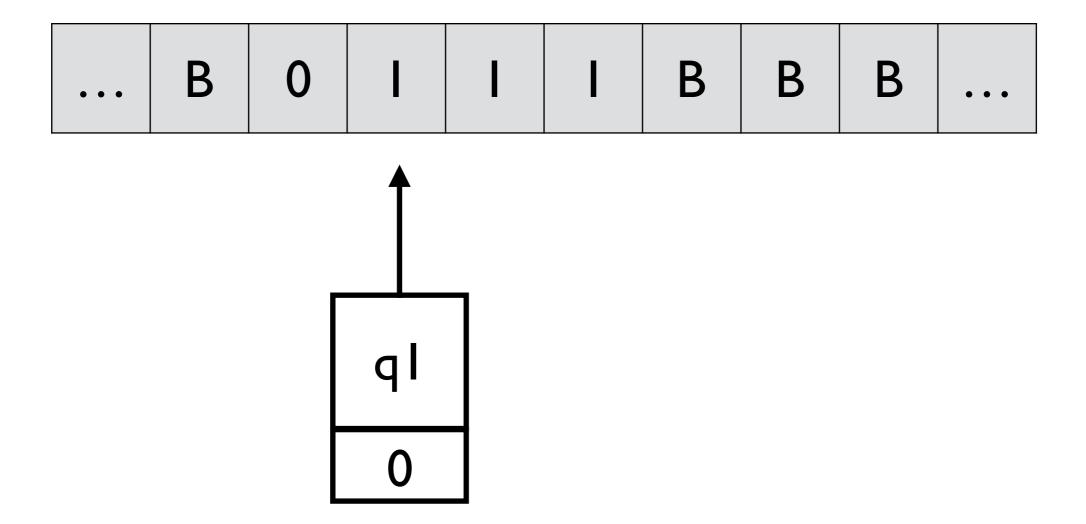
Example 1. Design a Turing machine that accepts $01^* + 10^*$.

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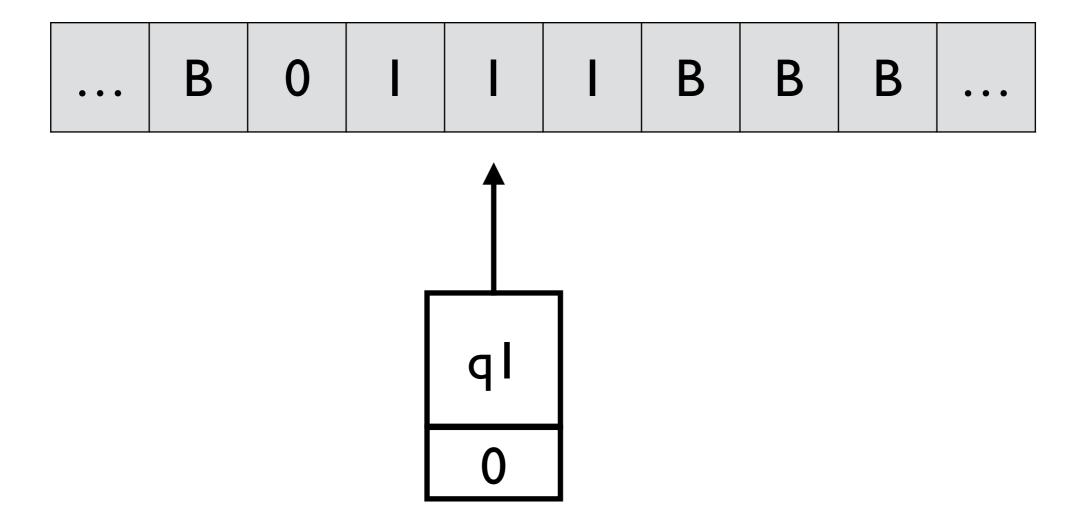
1.
$$\delta((q_0, B), a) = ((q_1, a), a, R)$$
 for $a = 0$
2. $\delta((q_1, a), \bar{a}) = ((q_1, a), \bar{a}, R)$
3. $\delta((q_1, a), B) = ((q_1, B), B, R)$



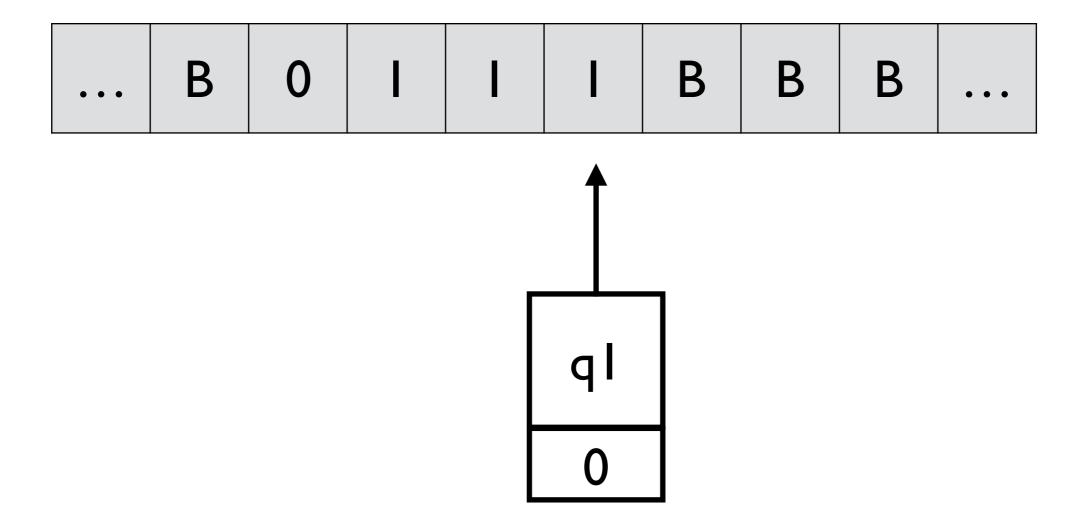
1.
$$\delta((q_0, B), a) = ((q_1, a), a, R)$$
 for $a = 0$ or $a = 1$
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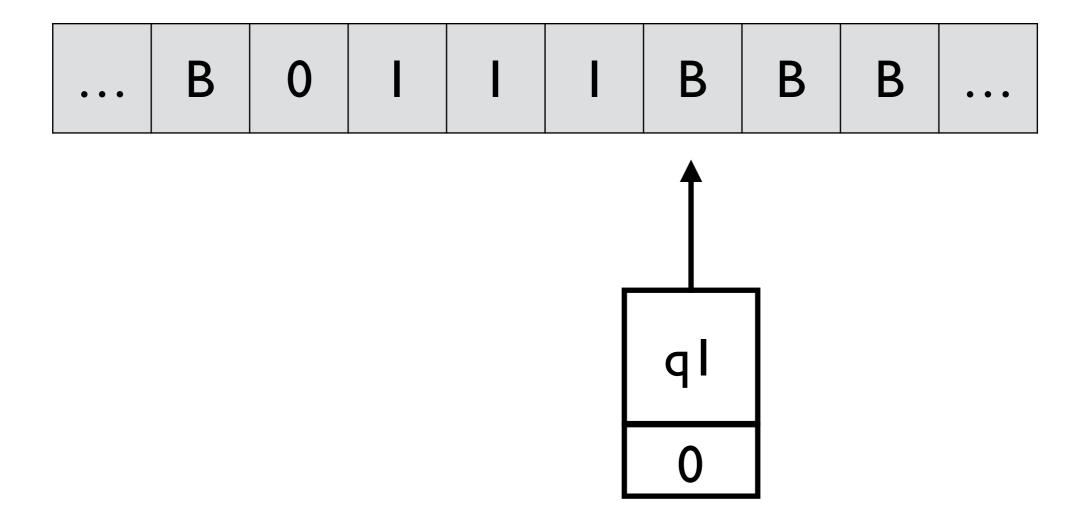
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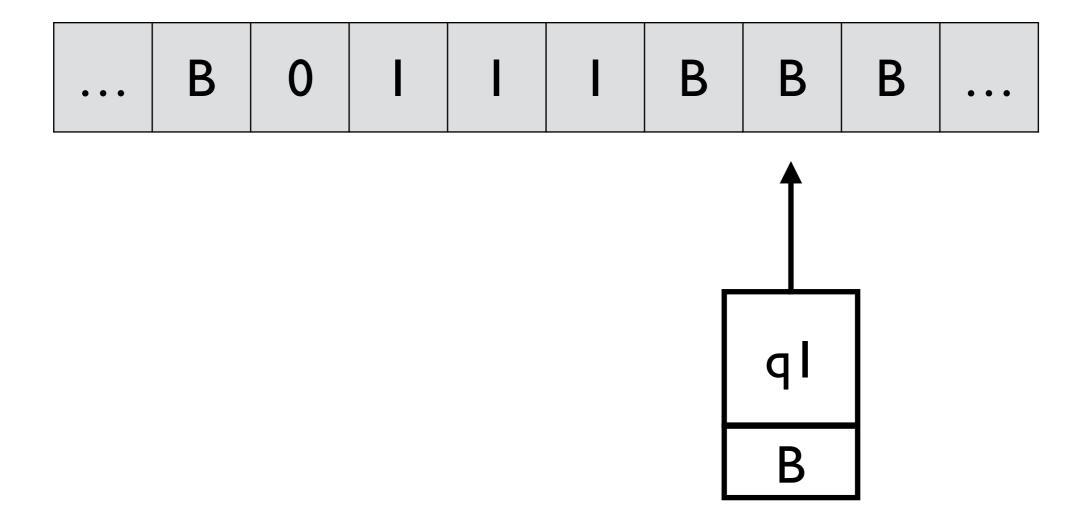
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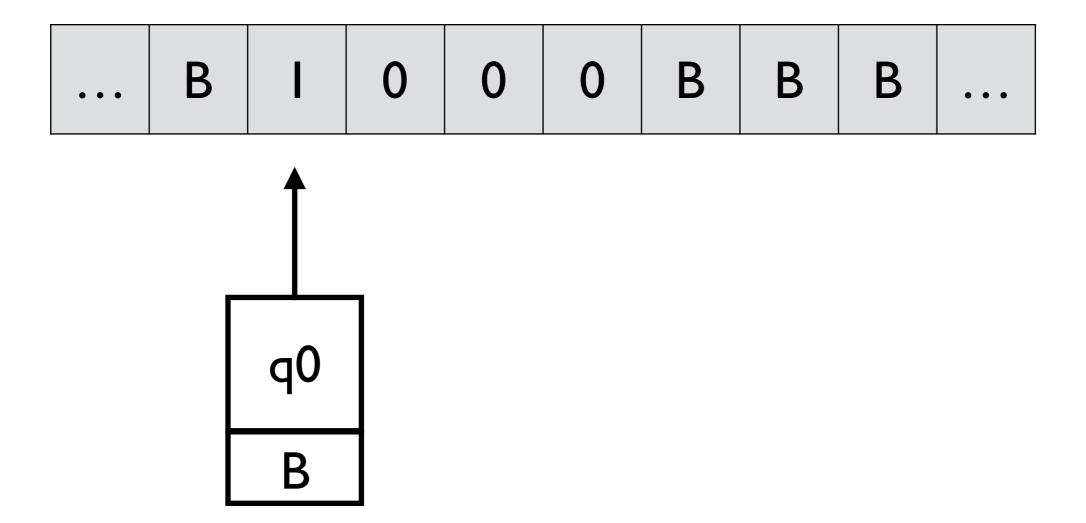
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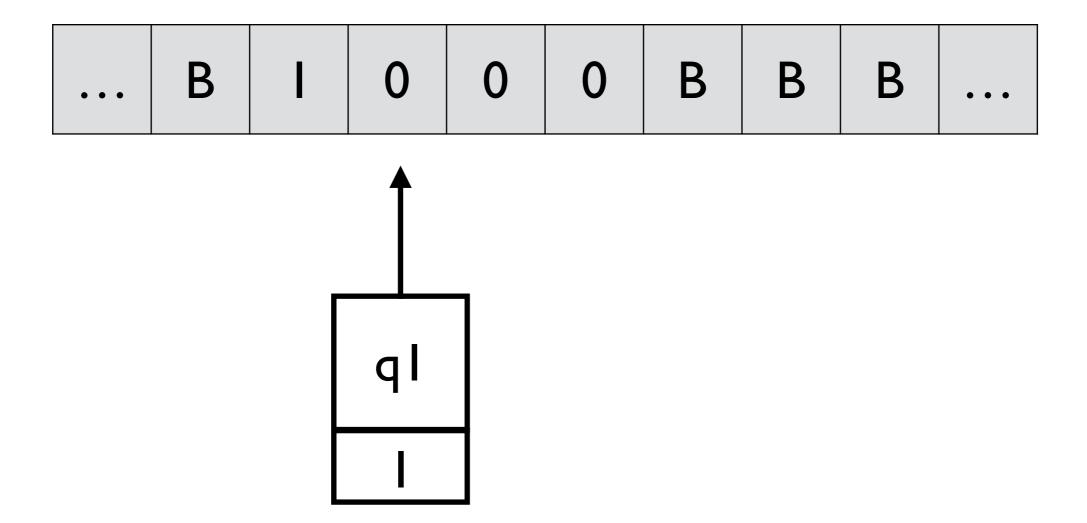
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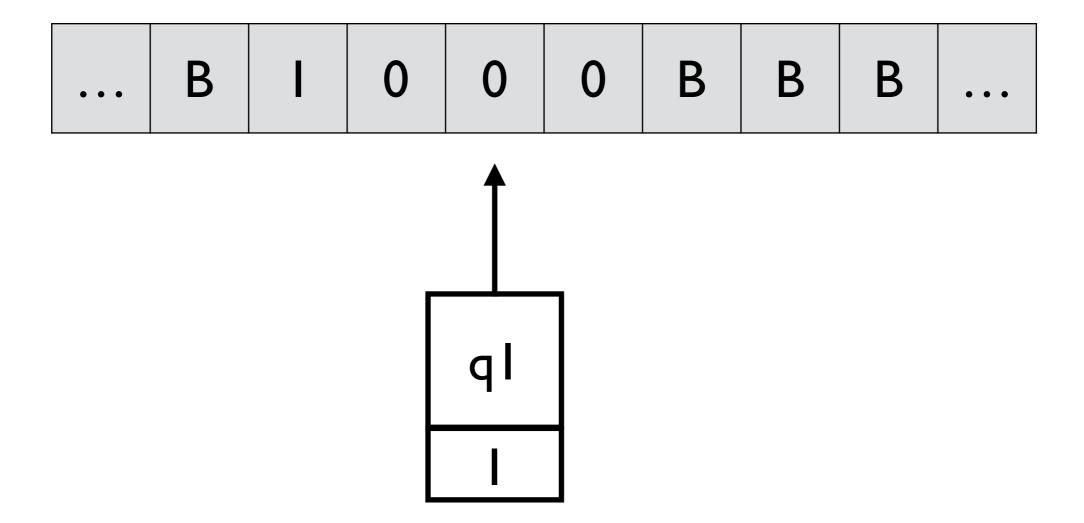
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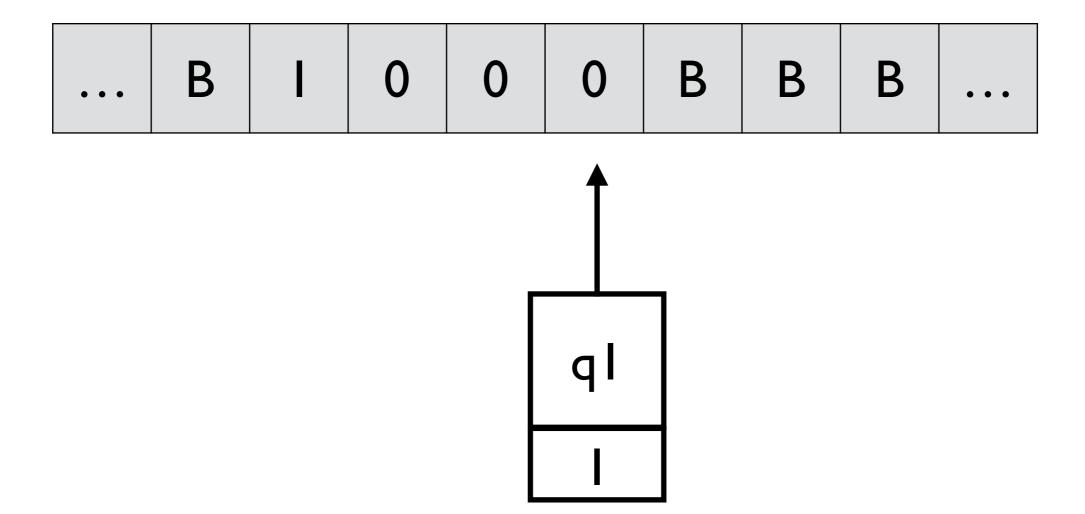
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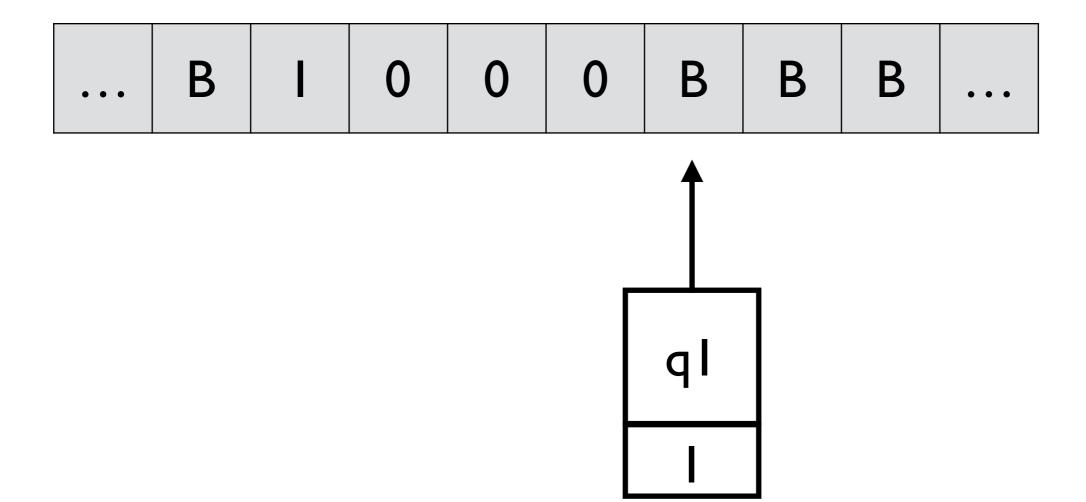
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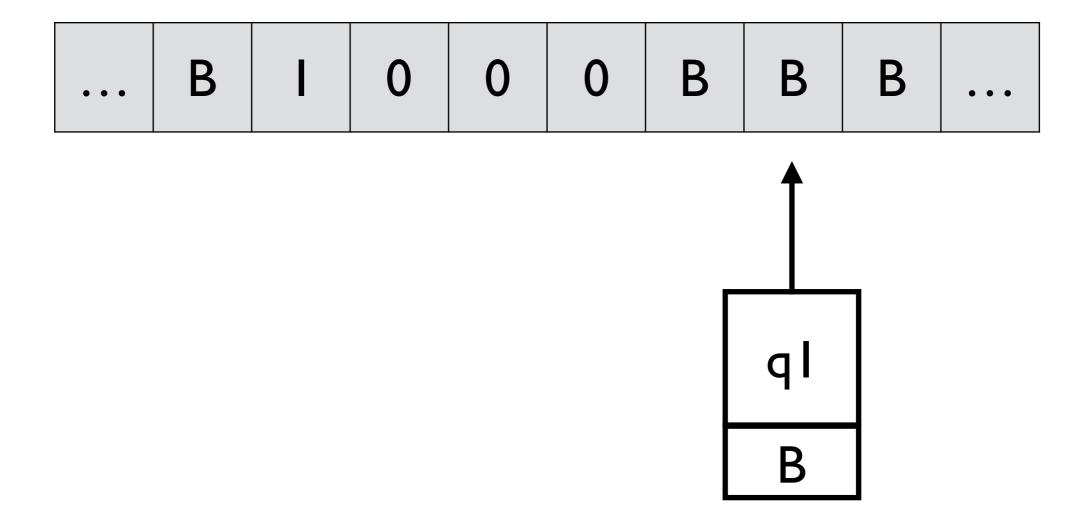
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Example 2. Design a Turing machine that accepts $L = \{wcw \mid w \in \{0, 1\}^+\}$.

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$$M = (Q, \Sigma, \Gamma, \delta, (q_1, B), (B, B), \{q_9, B\})$$

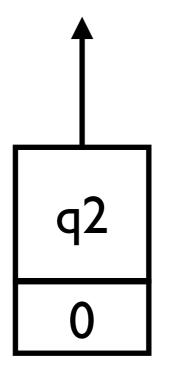
- $\{q_1, q_2, \dots, q_9\} \times \{0, 1, B\}$
- $\Gamma = \{B, *\} \times \{0, 1, c, B\}$
- $\Sigma = \{(B, 0), (B, 1), (B, c)\}$

 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R)$ $\delta((q_4, B), (B, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (*, a)) = ((q_5, B), (B, c), L)$

• • •	В	В	В	В	В	В	В	В	•••
• • •	В	0		С	0		В	В	•••

$$\begin{split} &\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R) \\ &\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R) \\ &\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R) \\ &\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \\ &\delta((q_3, a), (B, a)) = ((q_4, B), (*, a), L) \\ &\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L) \\ &\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L) \end{split}$$

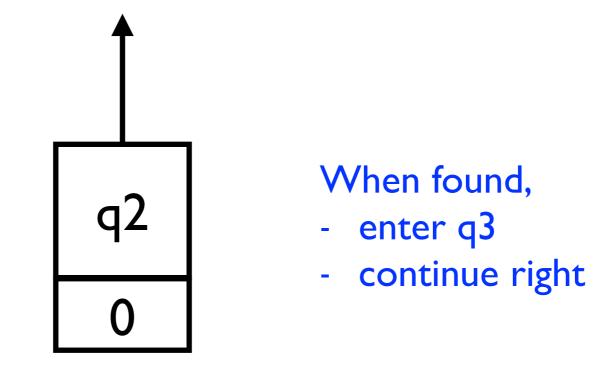
•••	В	*	В	В	В	В	В	В	•••
•••	В	0		С	0		В	В	•••



In q2, moves right, looking for c

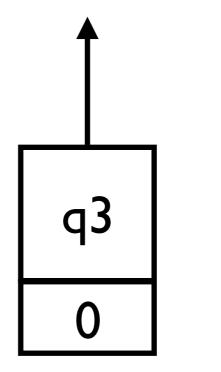
$$\begin{split} \delta((q_1, B), (B, a)) &= ((q_2, a), (*, a), R) \\ \delta((q_2, a), (B, b)) &= ((q_2, a), (B, b), R) \\ \delta((q_2, a), (B, c)) &= ((q_3, a), (B, c), R) \\ \delta((q_3, a), (*, b)) &= ((q_3, a), (*, b), R) \\ \delta((q_3, a), (B, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (*, a)) &= ((q_5, B), (B, c), L) \end{split}$$

•••	В	*	В	В	В	В	В	В	• • •
• • •	В	0		С	0		В	В	• • •



 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

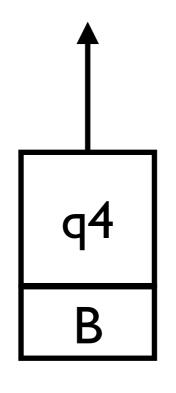
•••	В	*	В	В	В	В	В	В	• • •
•••	В	0		С	0		В	В	•••



In q3, look for the first unchecked symbol

$$\begin{split} \delta((q_1, B), (B, a)) &= ((q_2, a), (*, a), R) \\ \delta((q_2, a), (B, b)) &= ((q_2, a), (B, b), R) \\ \delta((q_2, a), (B, c)) &= ((q_3, a), (B, c), R) \\ \delta((q_3, a), (*, b)) &= ((q_3, a), (*, b), R) \\ \delta((q_3, a), (B, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (*, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (B, c)) &= ((q_5, B), (B, c), L) \end{split}$$

•••	В	*	В	В	*	В	В	В	• • •
•••	В	0		С	0		В	В	•••

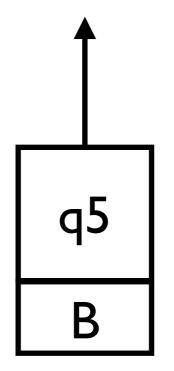


- In q4, move left until it finds c

- When found, enter q5

$$\begin{split} &\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R) \\ &\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R) \\ &\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R) \\ &\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \\ &\delta((q_3, a), (B, a)) = ((q_4, B), (*, a), L) \\ &\delta((q_4, B), (*, a)) = ((q_5, B), (B, c), L) \end{split}$$

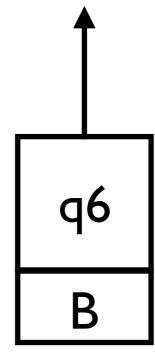
•••	В	*	В	В	*	В	В	В	•••
•••	В	0		С	0		В	В	•••



If there is some unchecked symbol, enter q6

$$\begin{split} &\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R) \\ &\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R) \\ &\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R) \\ &\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \\ &\delta((q_3, a), (B, a)) = ((q_4, B), (*, a), L) \\ &\delta((q_4, B), (*, a)) = ((q_5, B), (B, c), L) \end{split}$$

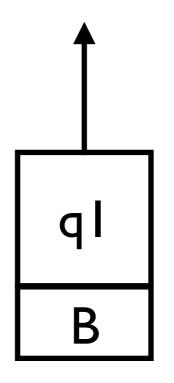
•••	В	*	В	В	*	В	В	В	•••
• • •	В	0		С	0		В	В	•••



In q6, move left and look for the first checked symbol

$$\begin{split} &\delta((q_1,B),(B,a)) = ((q_2,a),(*,a),R) \\ &\delta((q_2,a),(B,b)) = ((q_2,a),(B,b),R) \\ &\delta((q_2,a),(B,c)) = ((q_3,a),(B,c),R) \\ &\delta((q_3,a),(*,b)) = ((q_3,a),(*,b),R) \\ &\delta((q_3,a),(B,a)) = ((q_4,B),(*,a),L) \\ &\delta((q_4,B),(*,a)) = ((q_4,B),(*,a),L) \\ &\delta((q_4,B),(B,c)) = ((q_5,B),(B,c),L) \end{split}$$

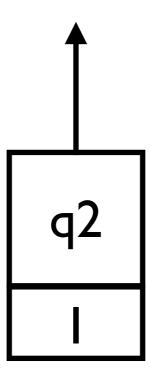
•••	В	*	В	В	*	В	В	В	• • •
• • •	В	0		С	0		В	В	•••



- Repeat the cycle

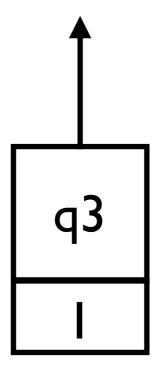
$$\begin{split} &\delta((q_1,B),(B,a)) = ((q_2,a),(*,a),R) \\ &\delta((q_2,a),(B,b)) = ((q_2,a),(B,b),R) \\ &\delta((q_2,a),(B,c)) = ((q_3,a),(B,c),R) \\ &\delta((q_3,a),(*,b)) = ((q_3,a),(*,b),R) \\ &\delta((q_3,a),(B,a)) = ((q_4,B),(*,a),L) \\ &\delta((q_4,B),(*,a)) = ((q_4,B),(*,a),L) \\ &\delta((q_4,B),(B,c)) = ((q_5,B),(B,c),L) \end{split}$$

•••	В	*	*	В	*	В	В	В	• • •
• • •	В	0		С	0		В	В	•••



 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \qquad \delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

•••	В	*	*	В	*	В	В	В	•••
• • •	В	0		С	0		В	В	•••



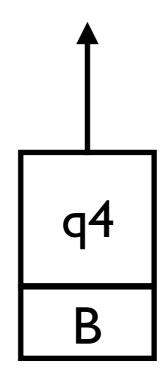
 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

•••	В	*	*	В	*	В	В	В	•••
• • •	В	0		С	0		В	В	•••

q3

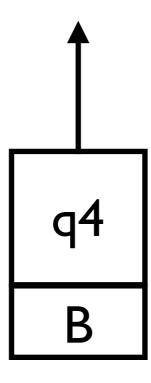
$$\begin{split} \delta((q_1, B), (B, a)) &= ((q_2, a), (*, a), R) \\ \delta((q_2, a), (B, b)) &= ((q_2, a), (B, b), R) \\ \delta((q_2, a), (B, c)) &= ((q_3, a), (B, c), R) \\ \delta((q_3, a), (*, b)) &= ((q_3, a), (*, b), R) \\ \delta((q_3, a), (B, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (*, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (B, c)) &= ((q_5, B), (B, c), L) \end{split}$$

•••	В	*	*	В	*	*	В	В	•••
•••	В	0		С	0		В	В	•••



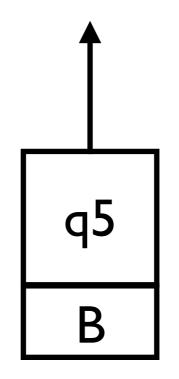
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•••	В	*	*	В	*	*	В	В	• • •
• • •	В	0		С	0		В	В	•••



 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \qquad \delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

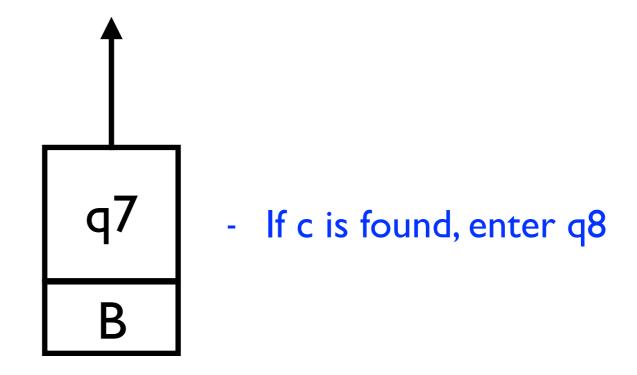
•••	В	*	*	В	*	*	В	В	•••
•••	В	0		С	0		В	В	•••



- If every symbols are checked, move right and enter q7

- $$\begin{split} \delta((q_1, B), (B, a)) &= ((q_2, a), (*, a), R) \\ \delta((q_2, a), (B, b)) &= ((q_2, a), (B, b), R) \\ \delta((q_2, a), (B, c)) &= ((q_3, a), (B, c), R) \\ \delta((q_3, a), (*, b)) &= ((q_3, a), (*, b), R) \\ \delta((q_3, a), (B, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (*, a)) &= ((q_4, B), (*, a), L) \\ \delta((q_4, B), (B, c)) &= ((q_5, B), (B, c), L) \end{split}$$
- $\delta((q_5, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (*, a)) = ((q_1, B), (*, a), R)$ $\delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_7, B), (B, c)) = ((q_8, B), (B, c), R)$ $\delta((q_8, B), (*, a)) = ((q_8, B), (*, a), R)$ $\delta((q_8, B), (B, B)) = ((q_9, B), (B, B), R)$

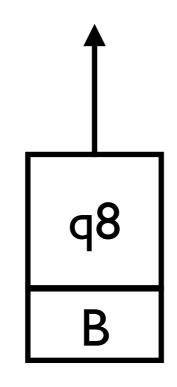
•••	В	*	*	В	*	*	В	В	• • •
• • •	В	0		С	0		В	В	•••



 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \qquad \delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L) \qquad \delta((q_8, B), (*, a)) = ((q_8, B), (*, a), R)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

 $\delta((q_5, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (*, a)) = ((q_1, B), (*, a), R)$ $\delta((q_3, a), (B, a)) = ((q_4, B), (*, a), L) \qquad \delta((q_7, B), (B, c)) = ((q_8, B), (B, c), R)$ $\delta((q_8, B), (B, B)) = ((q_9, B), (B, B), R)$

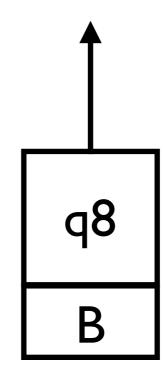
•••	В	*	*	В	*	*	В	В	•••
•••	В	0	I	С	0		В	В	•••



- move right until it finds B

 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

•••	В	*	*	В	*	*	В	В	• • •
•••	В	0		С	0		В	В	• • •



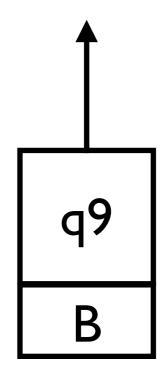
 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \qquad \delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

•••	В	*	*	В	*	*	В	В	• • •
• • •	В	0		С	0		В	В	•••

 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

 $\delta((q_5, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (*, a)) = ((q_1, B), (*, a), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \qquad \delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_3, a), (B, a)) = ((q_4, B), (*, a), L) \qquad \delta((q_7, B), (B, c)) = ((q_8, B), (B, c), R)$ $\delta((q_8, B), (*, a)) = ((q_8, B), (*, a), R)$ $\delta((q_8, B), (B, B)) = ((q_9, B), (B, B), R)$

•••	В	*	*	В	*	*	В	В	•••
•••	В	0		С	0		В	В	•••



 $\delta((q_1, B), (B, a)) = ((q_2, a), (*, a), R)$ $\delta((q_2, a), (B, b)) = ((q_2, a), (B, b), R)$ $\delta((q_2, a), (B, c)) = ((q_3, a), (B, c), R)$ $\delta((q_4, B), (*, a)) = ((q_4, B), (*, a), L)$ $\delta((q_4, B), (B, c)) = ((q_5, B), (B, c), L)$

 $\delta((q_5, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (B, a)) = ((q_6, B), (B, a), L)$ $\delta((q_6, B), (*, a)) = ((q_1, B), (*, a), R)$ $\delta((q_3, a), (*, b)) = ((q_3, a), (*, b), R) \qquad \delta((q_5, B), (*, a)) = ((q_7, B), (*, a), R)$ $\delta((q_3, a), (B, a)) = ((q_4, B), (*, a), L) \qquad \delta((q_7, B), (B, c)) = ((q_8, B), (B, c), R)$ $\delta((q_8, B), (*, a)) = ((q_8, B), (*, a), R)$ $\delta((q_8, B), (B, B)) = ((q_9, B), (B, B), R)$