

COSE 215: Theory of Computation

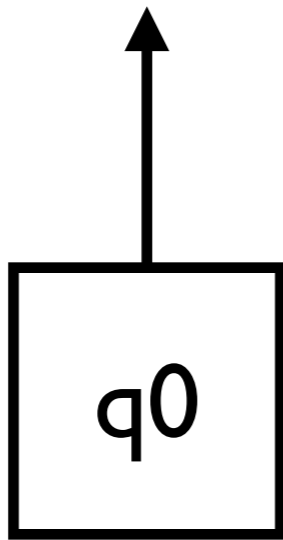
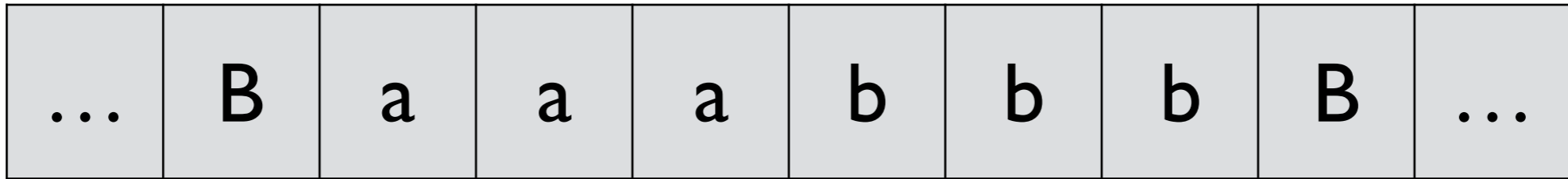
# Examples of Turing Machines

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2017 Spring

**Example 1.** Design a Turing machine that accepts  $L = \{a^n b^n \mid n \geq 1\}$ .

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$$

$$\begin{array}{lll} \delta(q_0, a) = (q_1, x, R) & \delta(q_2, y) = (q_2, y, L) & \delta(q_0, y) = (q_3, y, R) \\ \delta(q_1, a) = (q_1, a, R) & \delta(q_2, a) = (q_2, a, L) & \delta(q_3, y) = (q_3, y, R) \\ \delta(q_1, y) = (q_1, y, R) & \delta(q_2, x) = (q_0, x, R) & \delta(q_3, B) = (q_4, B, R) \\ \delta(q_1, b) = (q_2, y, L) & & \end{array}$$



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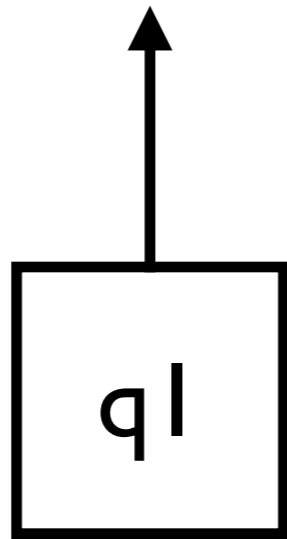
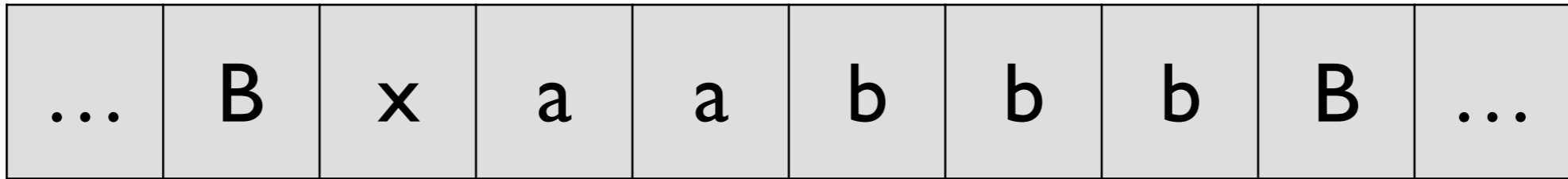
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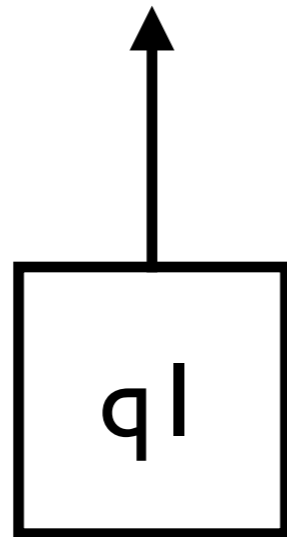
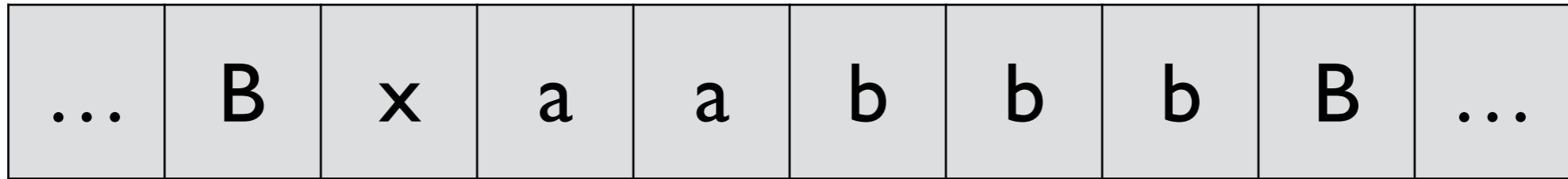
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(In  $q_1$ , move right to search for the leftmost 'b')

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

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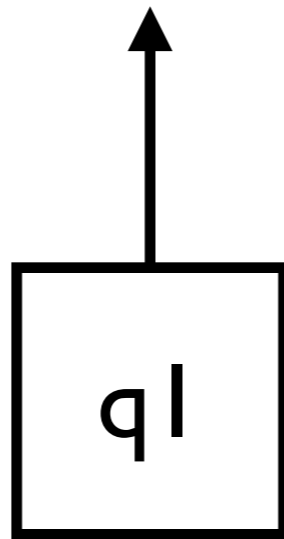
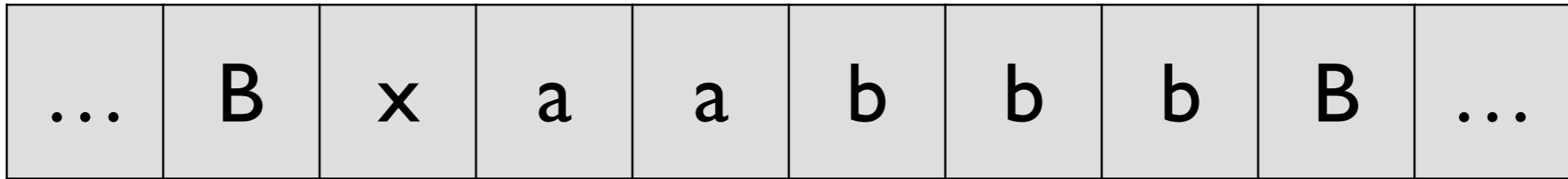
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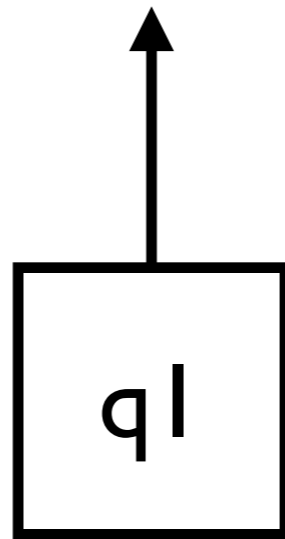
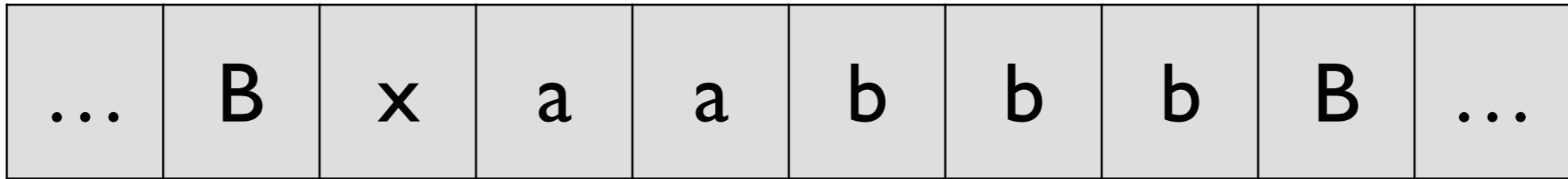
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

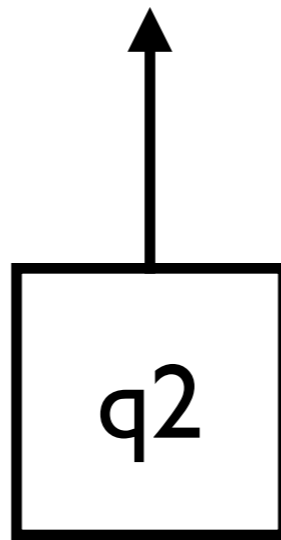
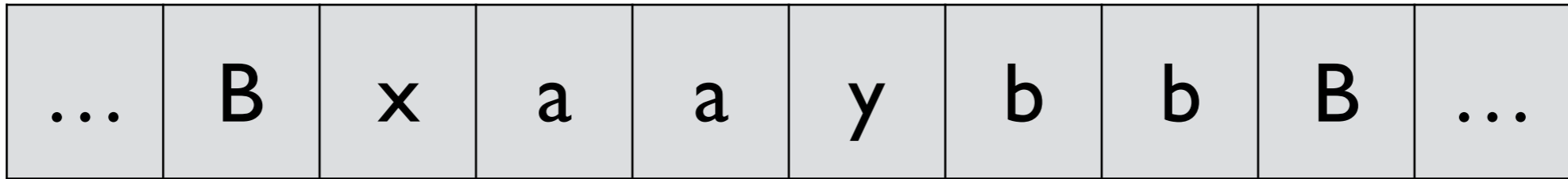
$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



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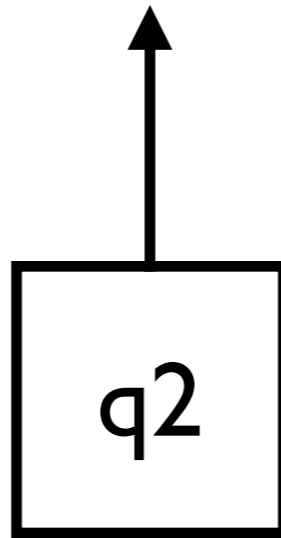
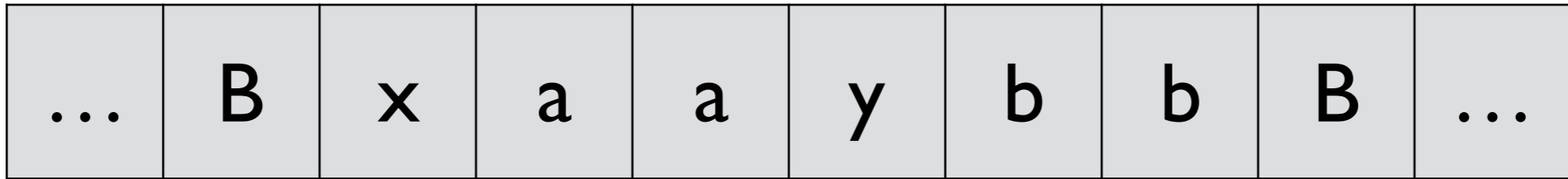


$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

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(In  $q_2$ , move left to search for the leftmost 'a')

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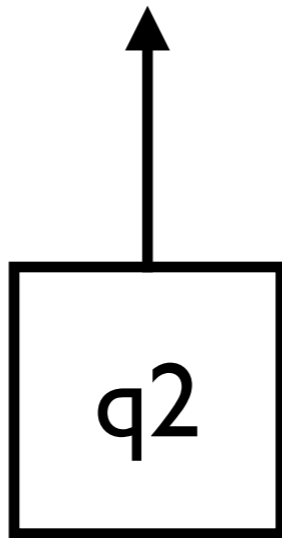
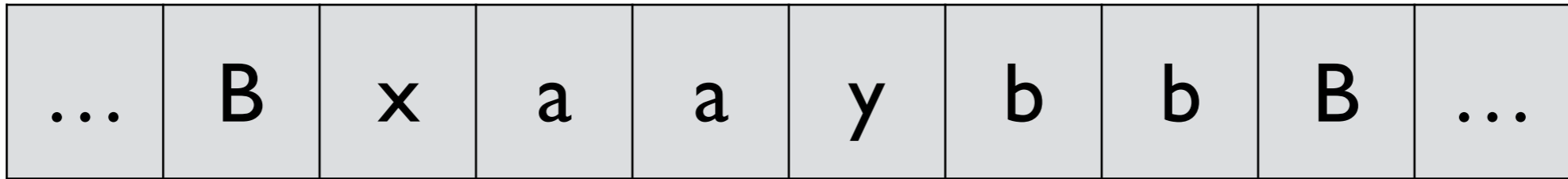
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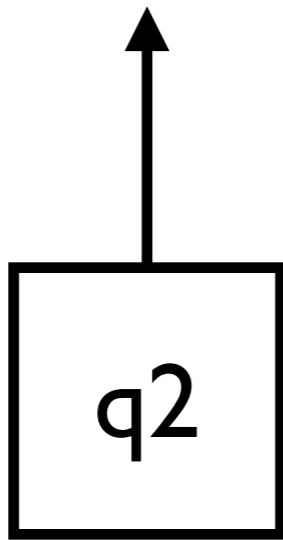
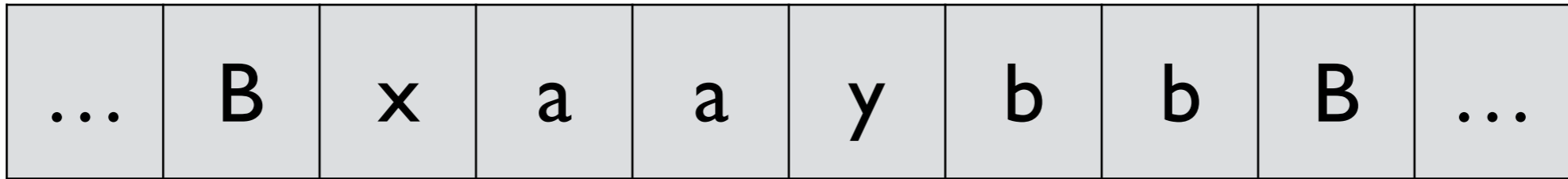
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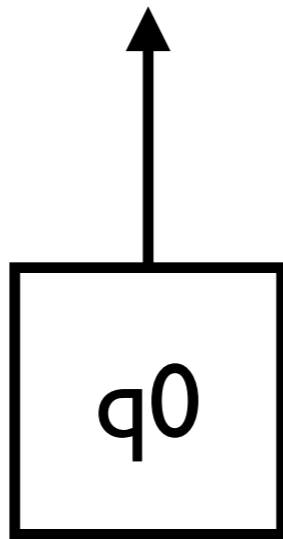
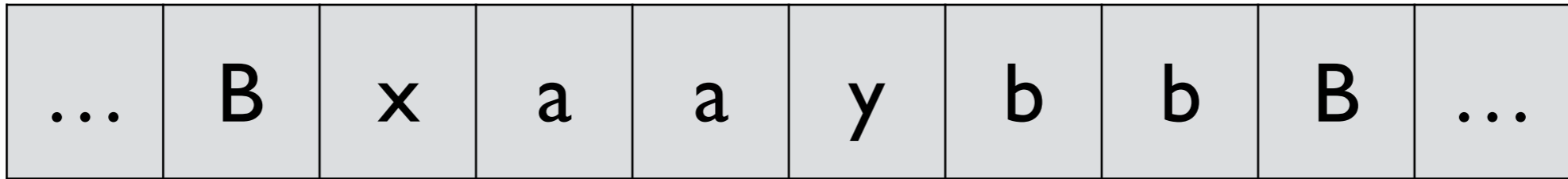
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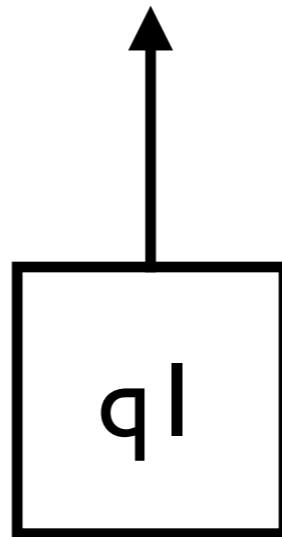
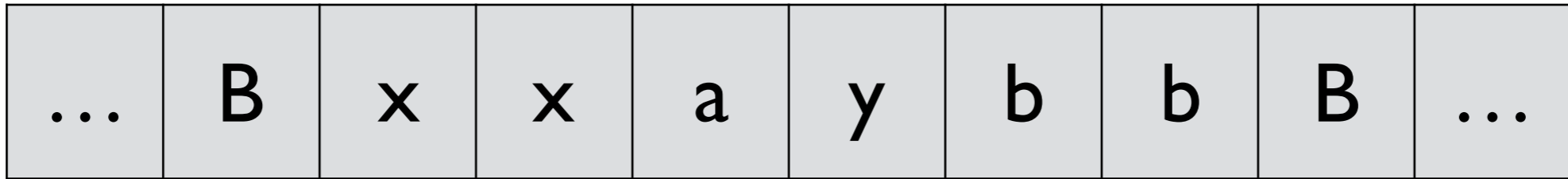
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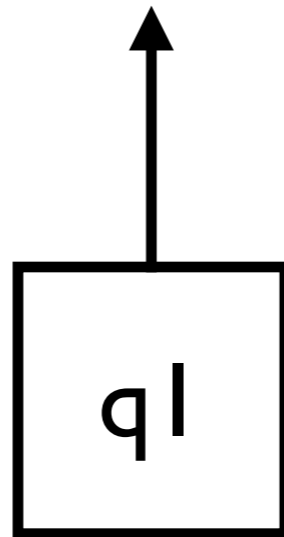
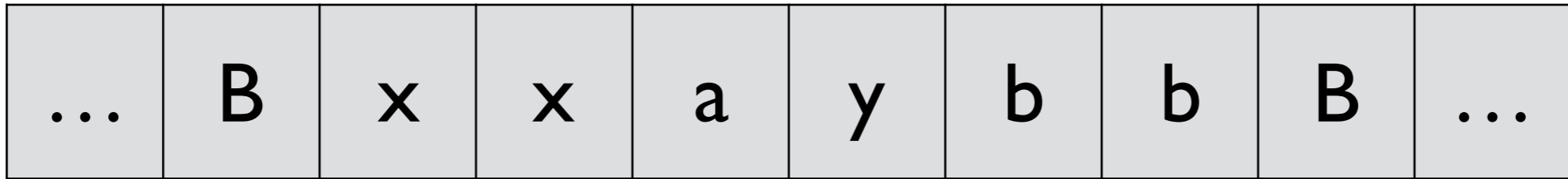
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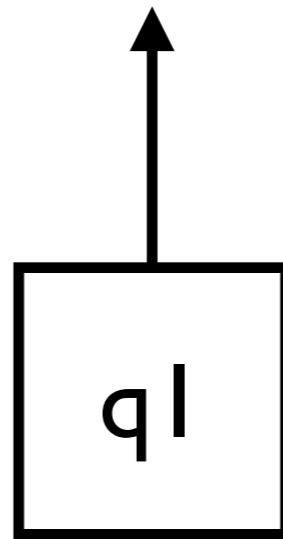
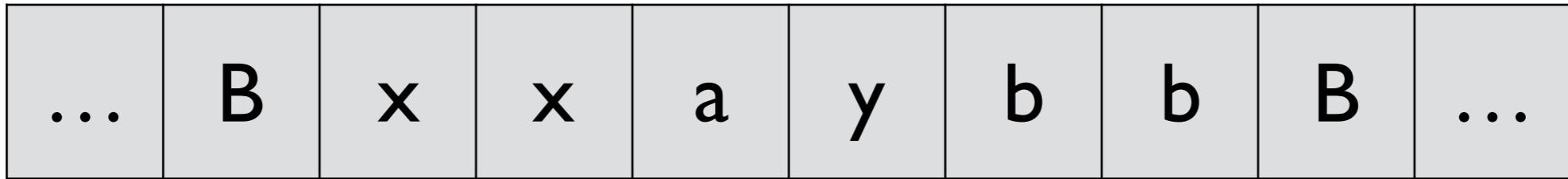
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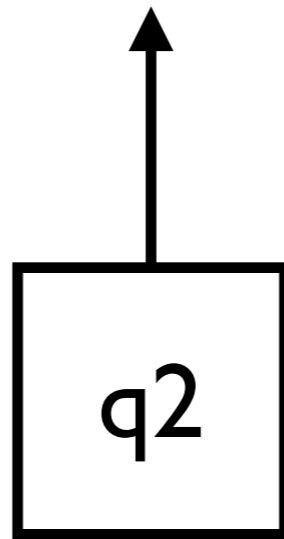
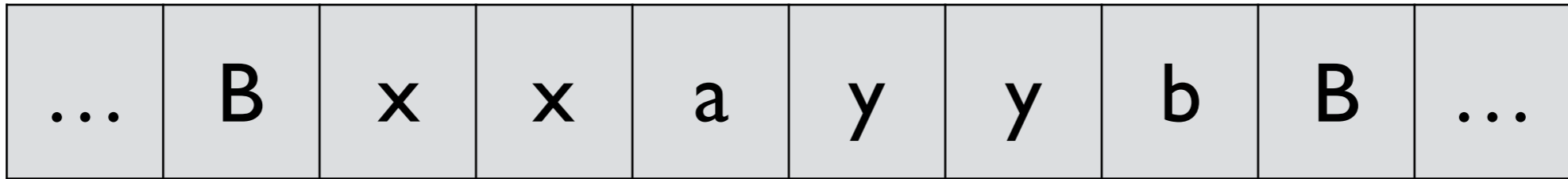
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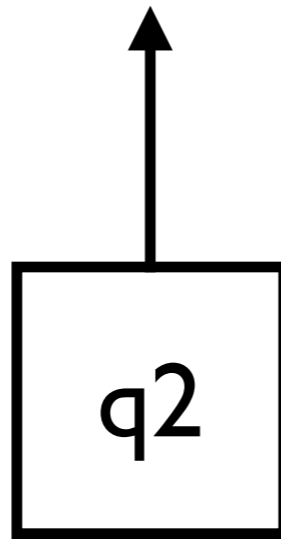
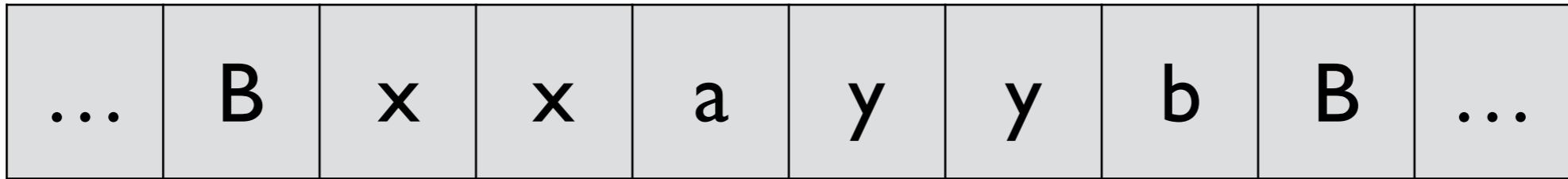


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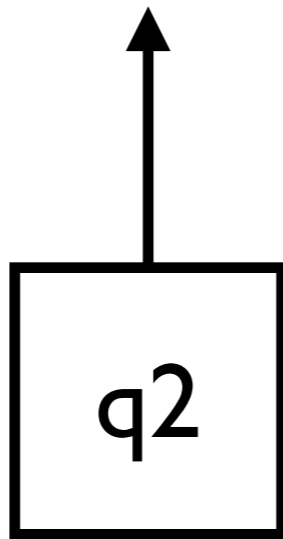
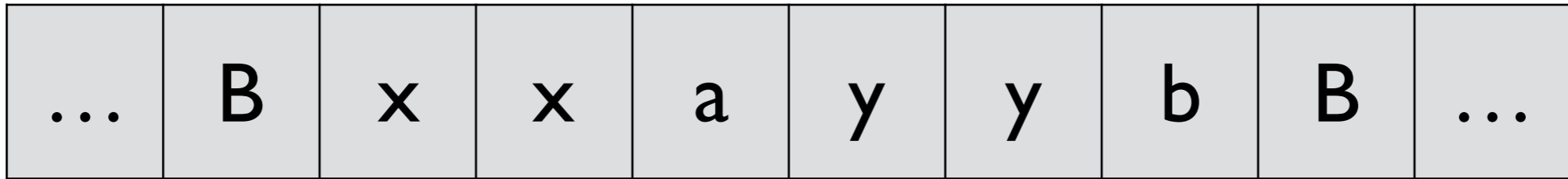
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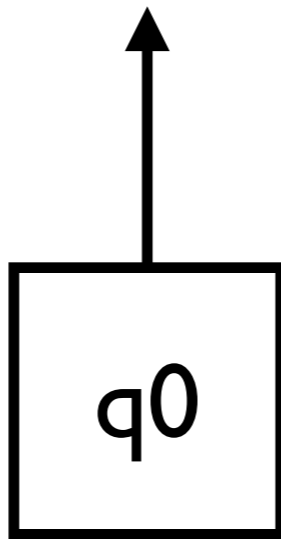
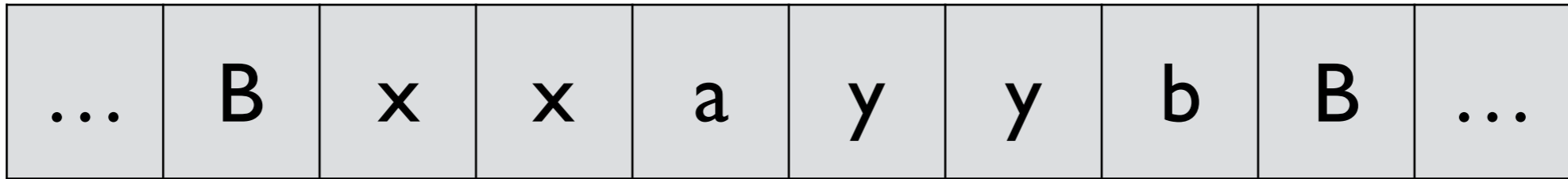
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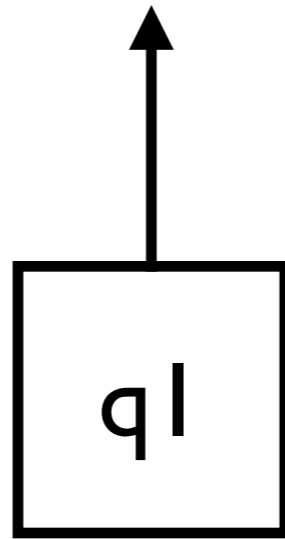
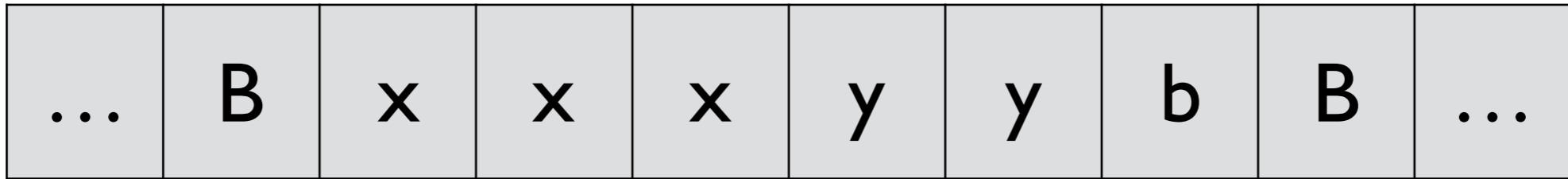
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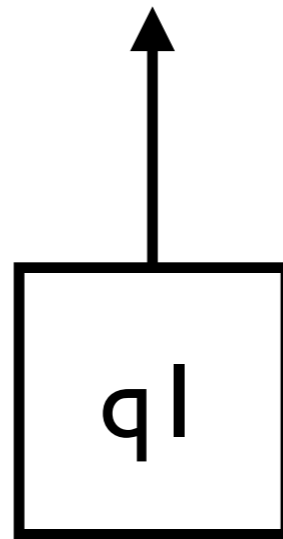
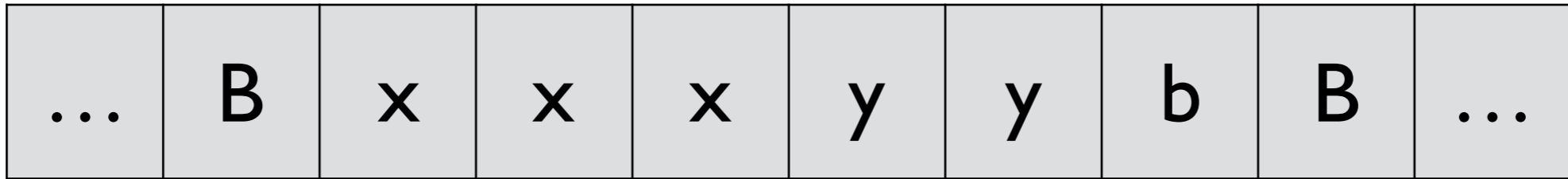
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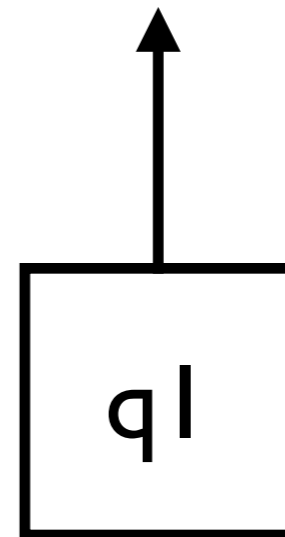
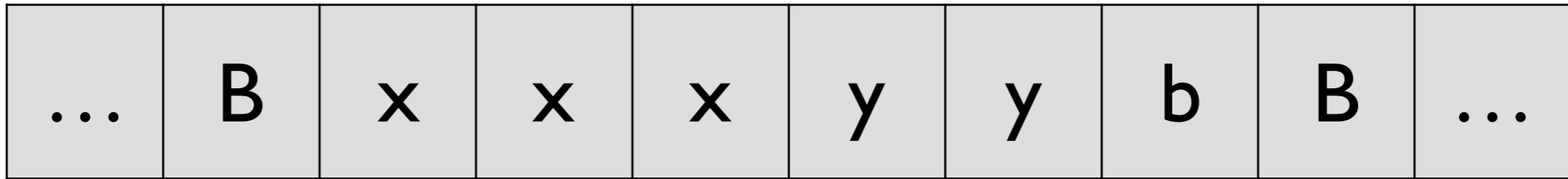
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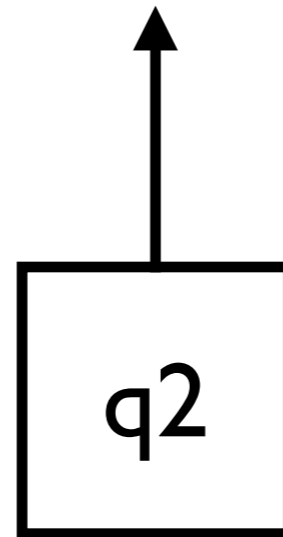
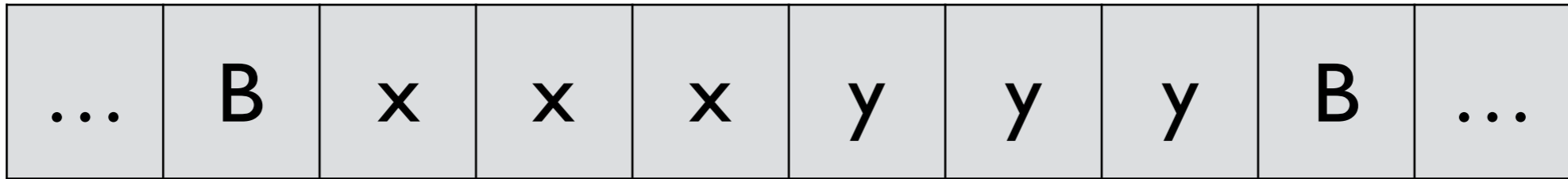
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

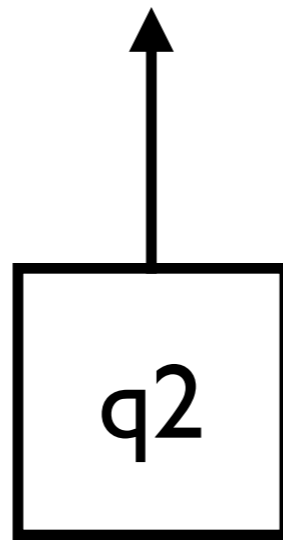
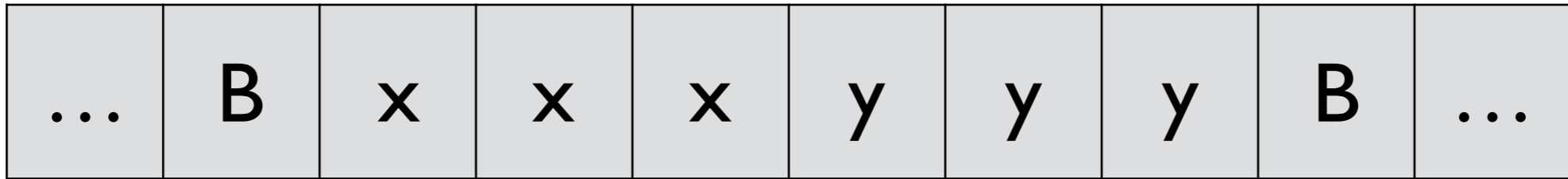
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

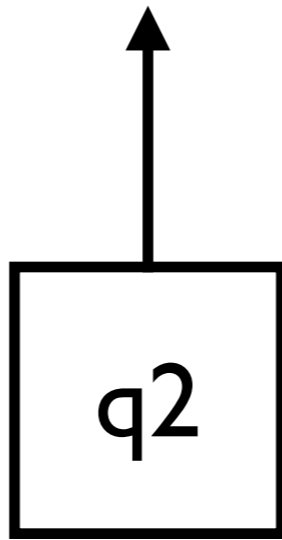
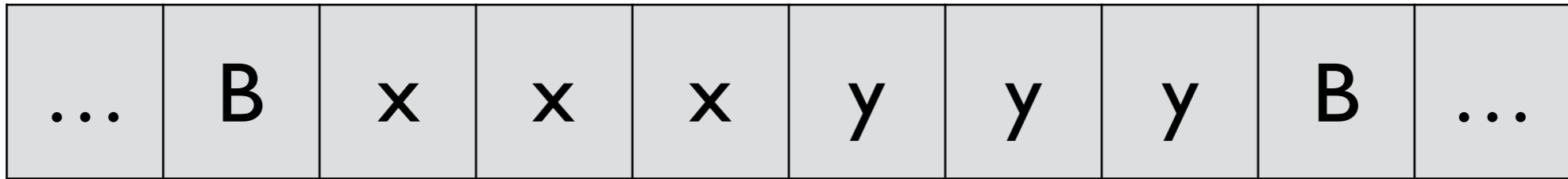
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$





$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

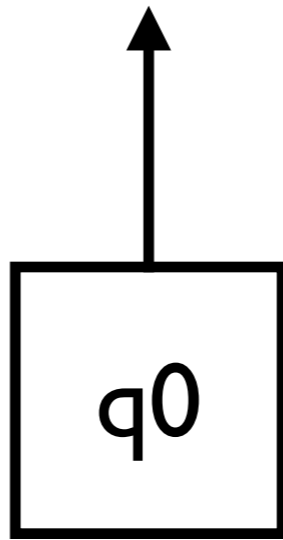
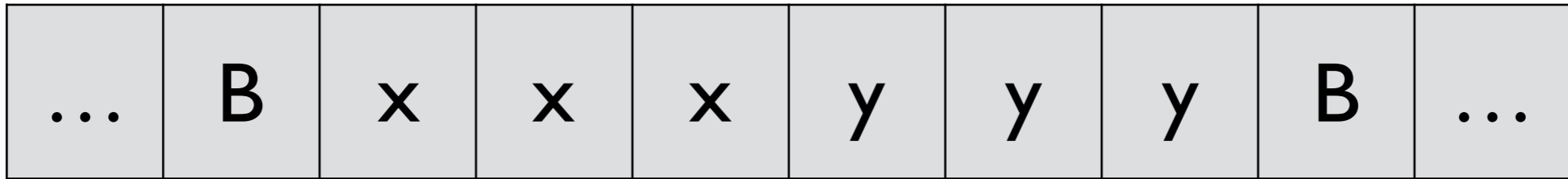
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

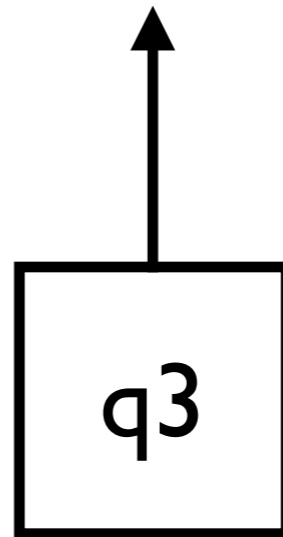
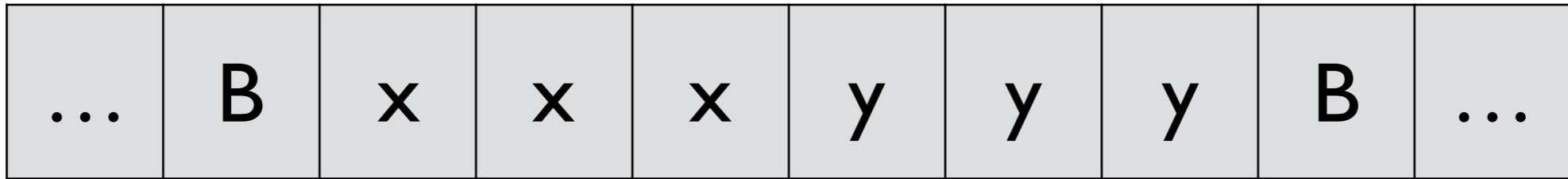
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

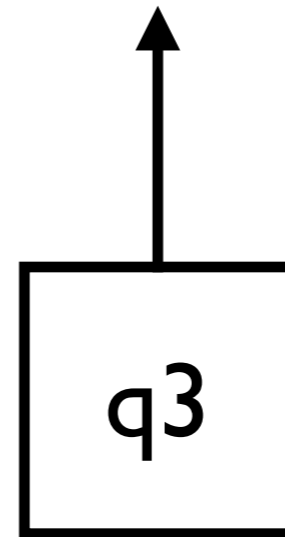
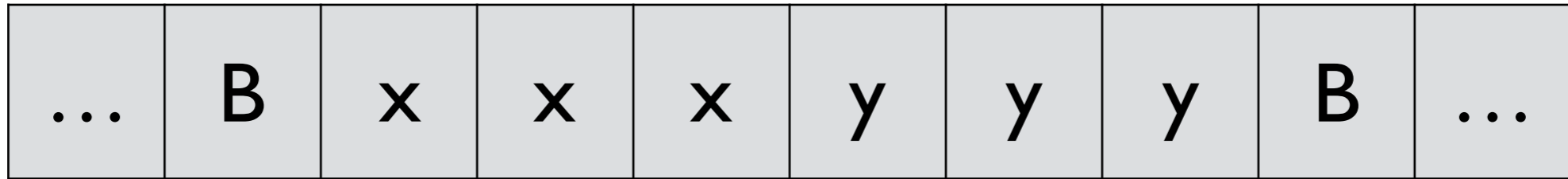
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



(In  $q_3$ , move right to check that there are no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

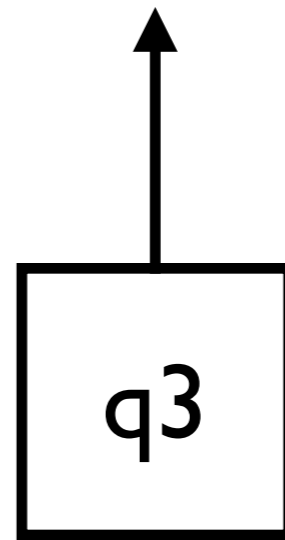
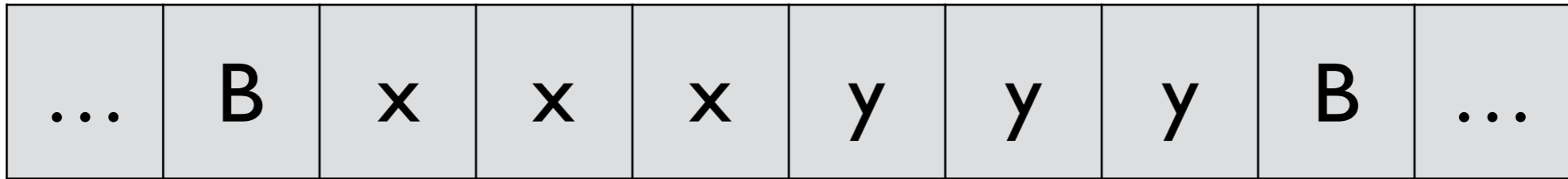
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

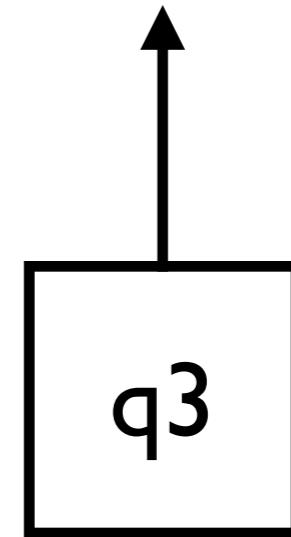
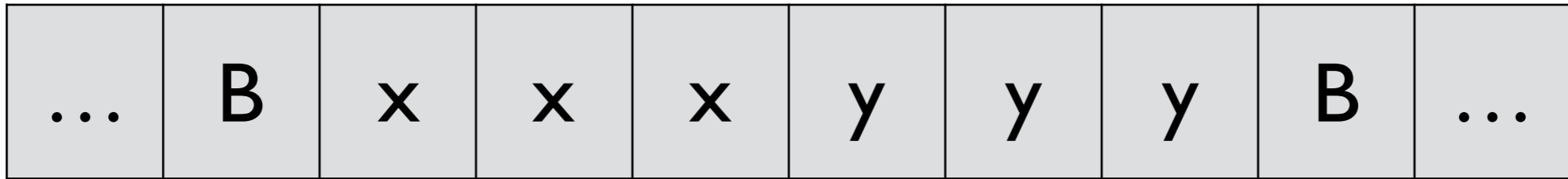
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$



(no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

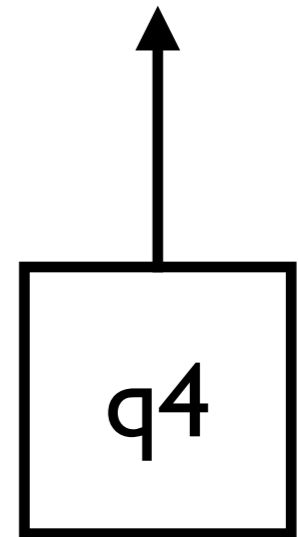
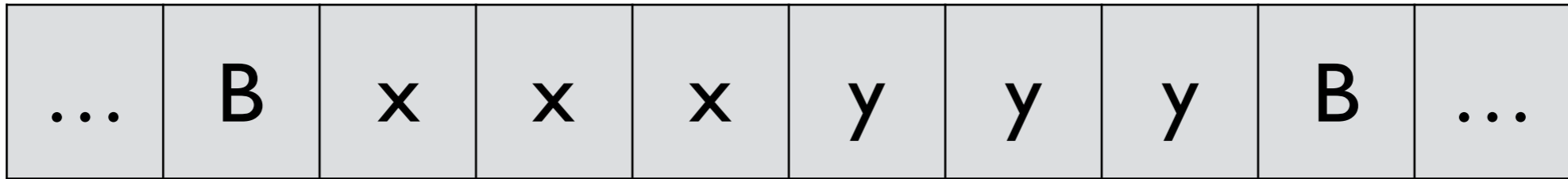
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



“final state”

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

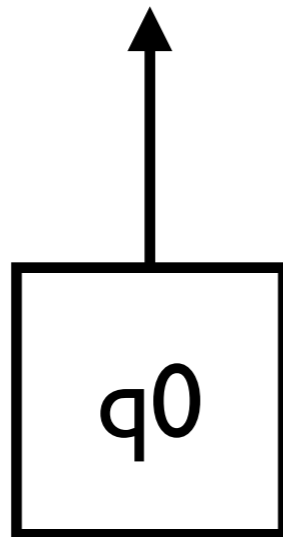
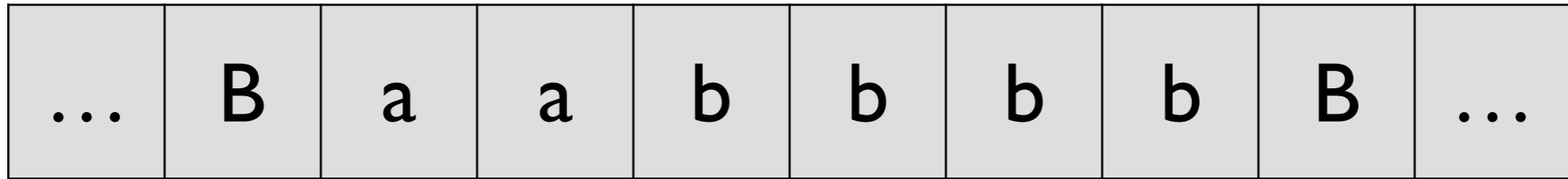
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

When the input string is not in the language:



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

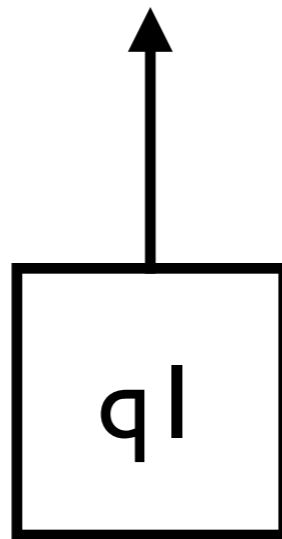
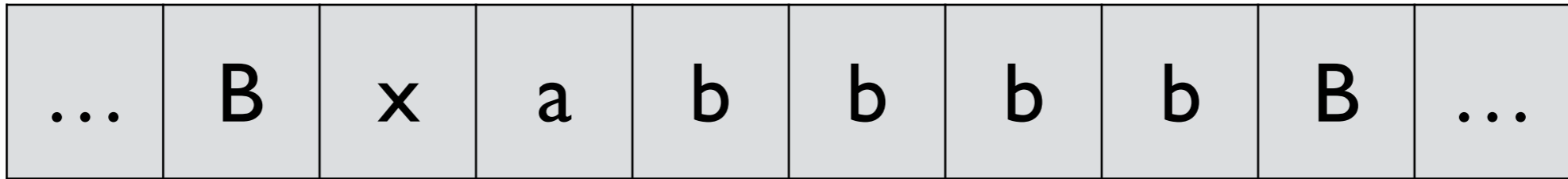
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$





$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

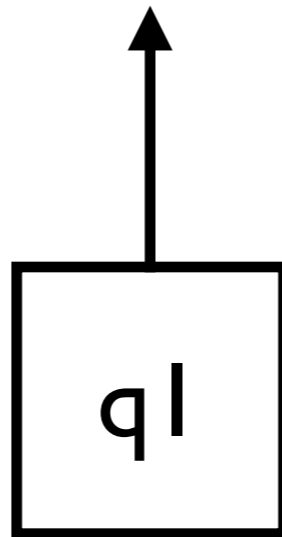
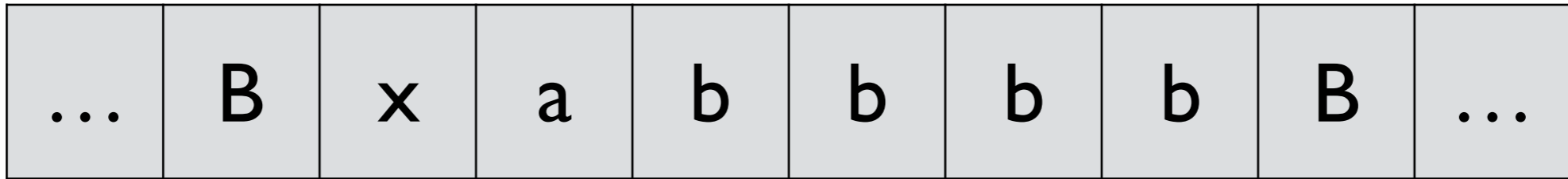
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

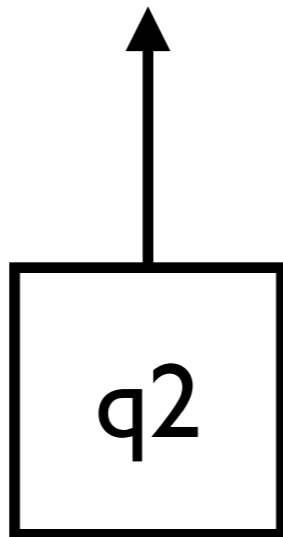
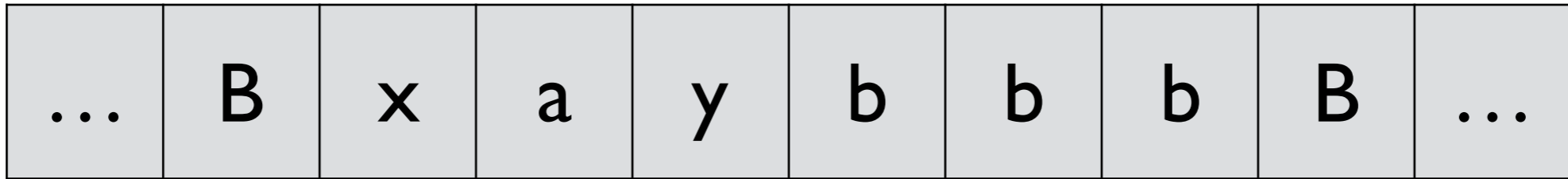
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

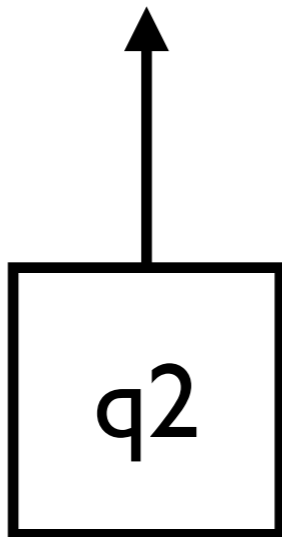
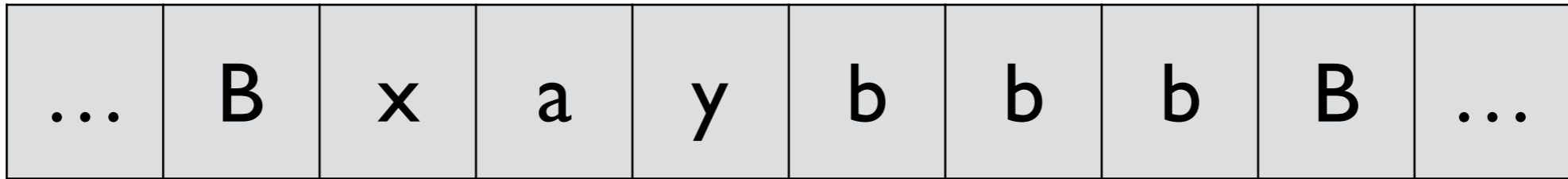
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

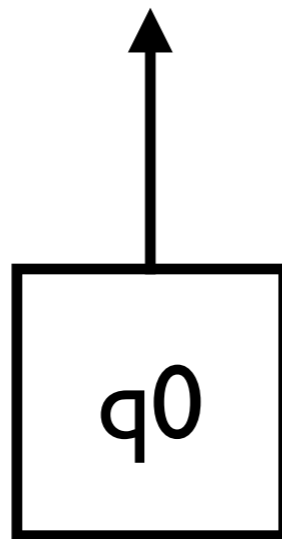
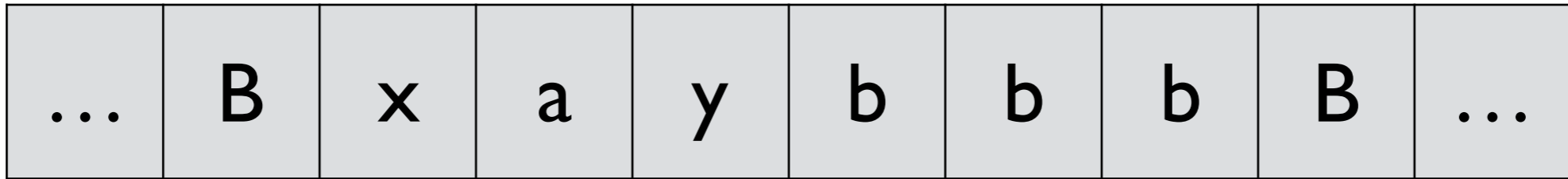
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

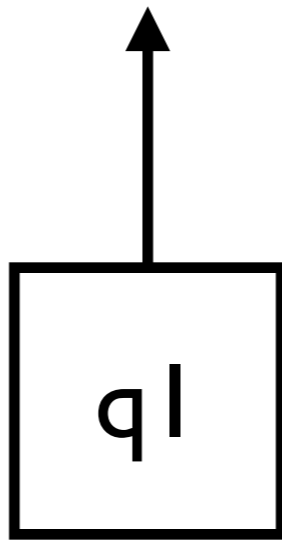
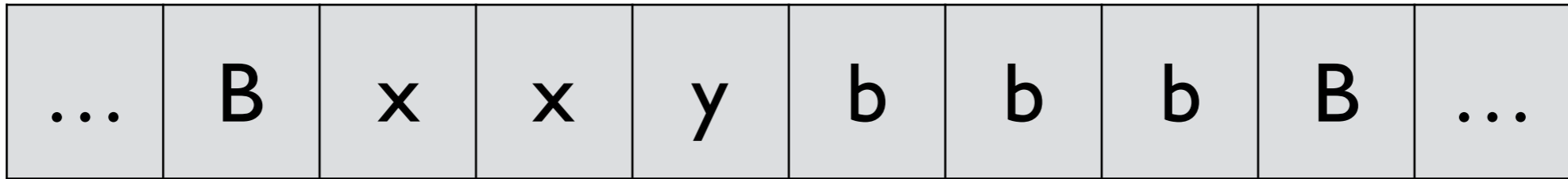
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

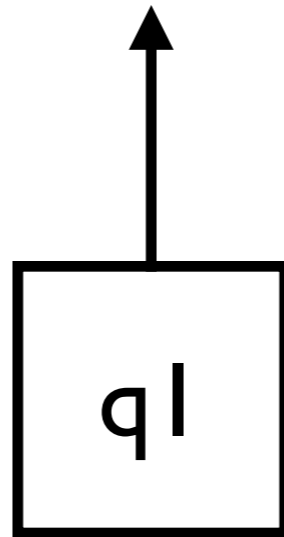
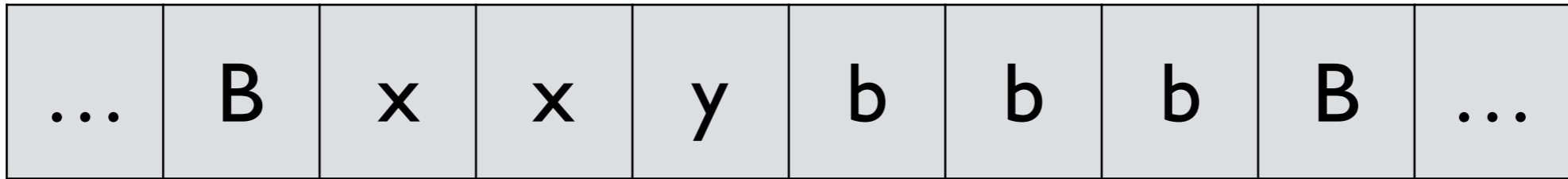
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

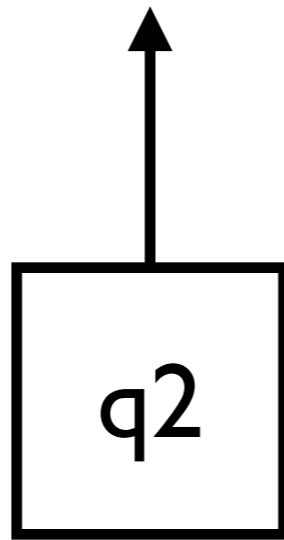
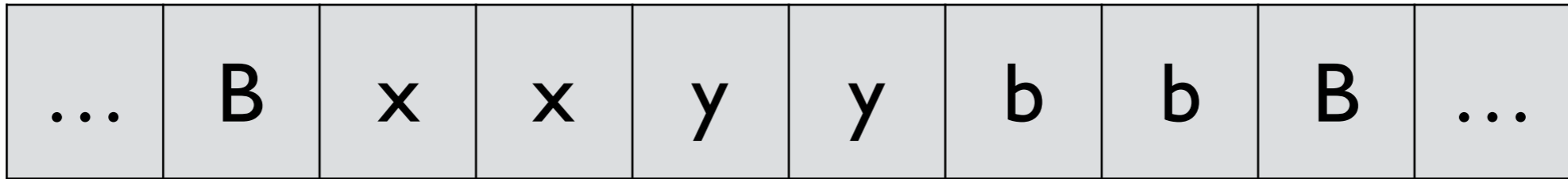
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

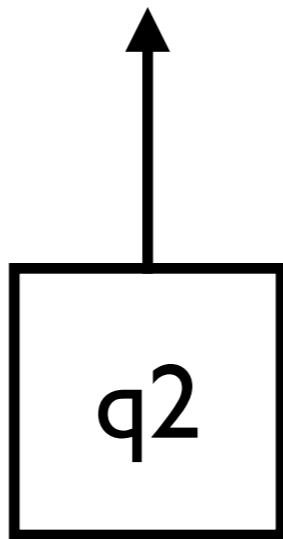
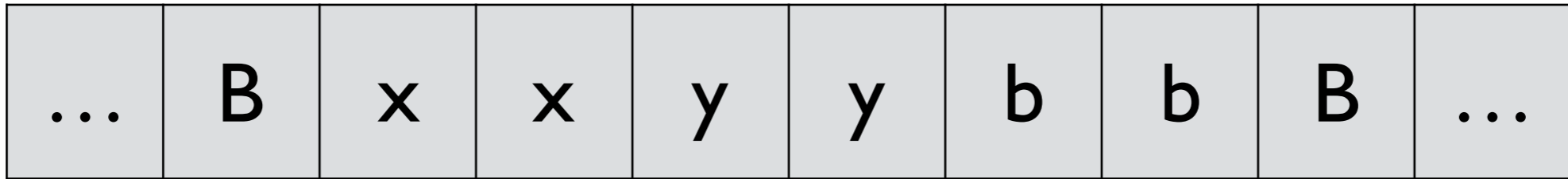


$$\begin{aligned} \delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L) \end{aligned}$$

$$\begin{aligned} \delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R) \end{aligned}$$

$$\begin{aligned} \delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R) \end{aligned}$$





$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

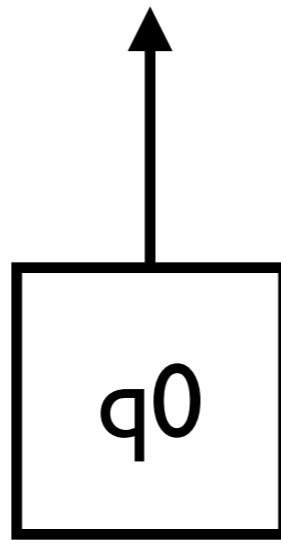
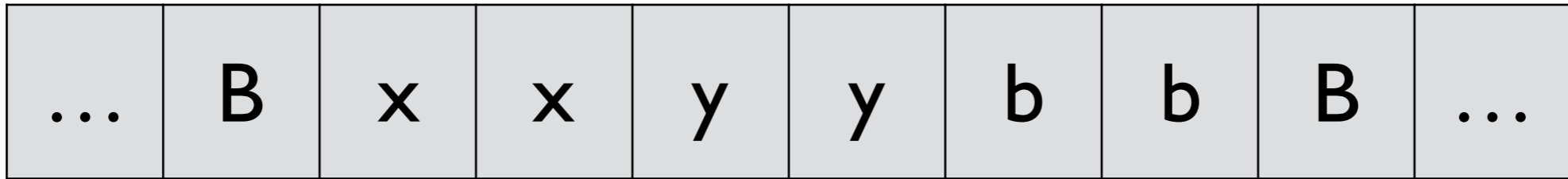
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

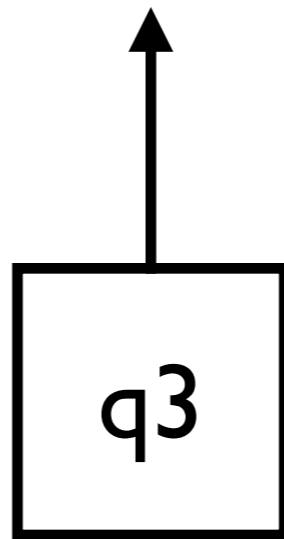
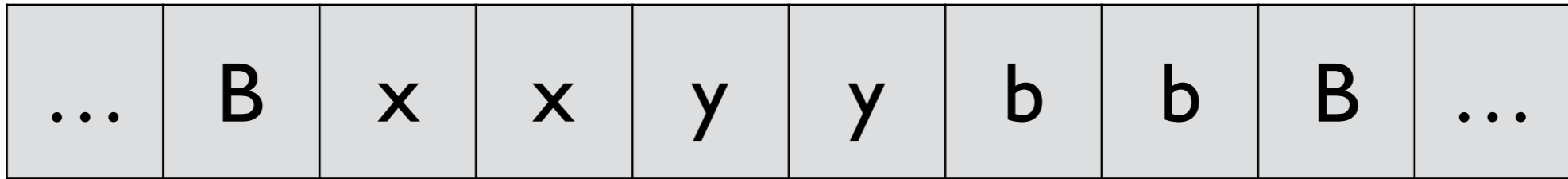
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

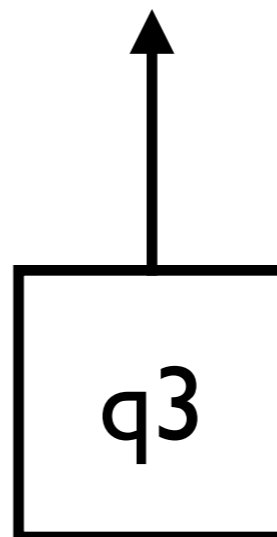
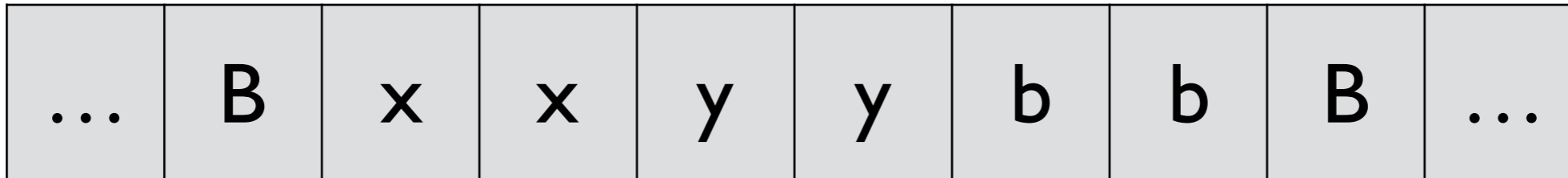
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$



(undefined, halt)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

**Example 2.** Given  $x$  and  $y$ , design a Turing machine that computes  $x + y$ .

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_4\})$$

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

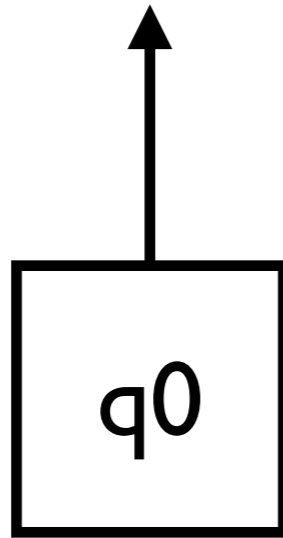
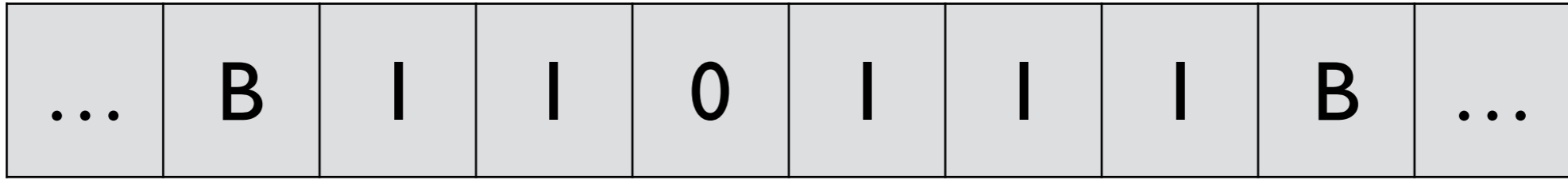
$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

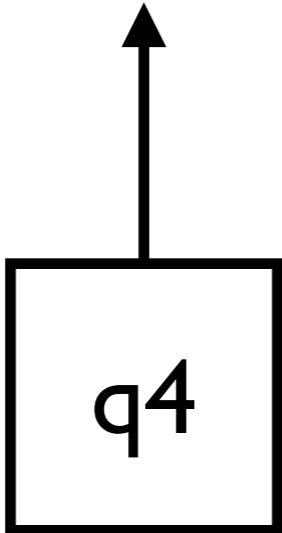
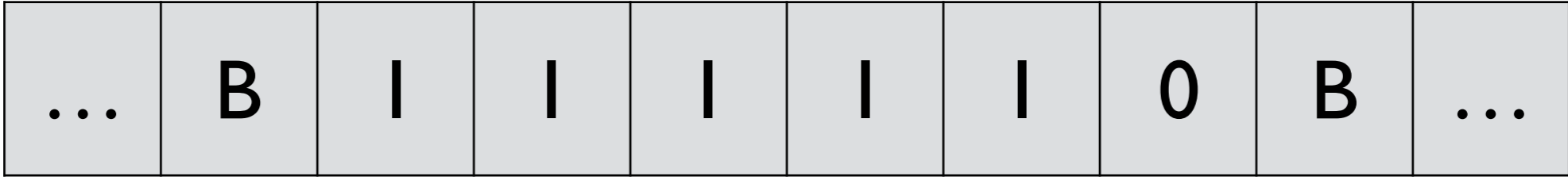
$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$

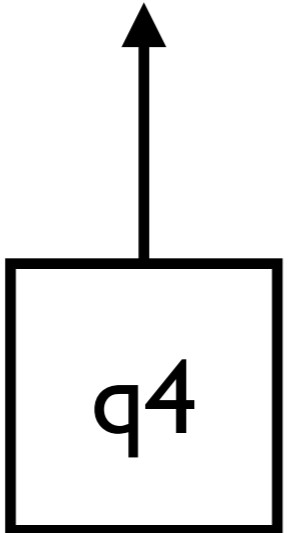
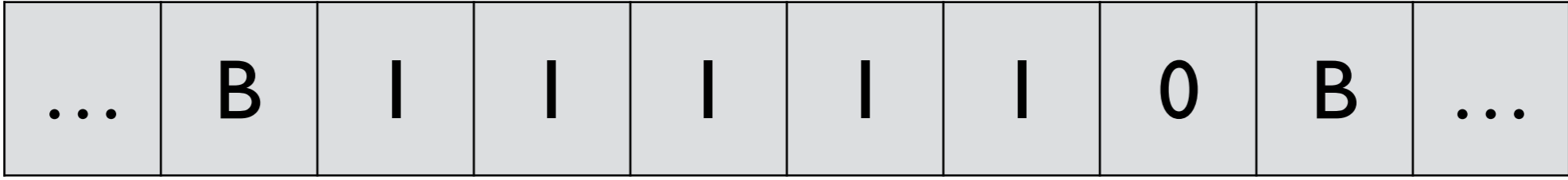
Initial machine configuration:



Final machine configuration:

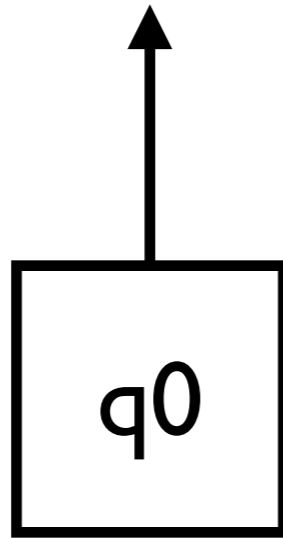
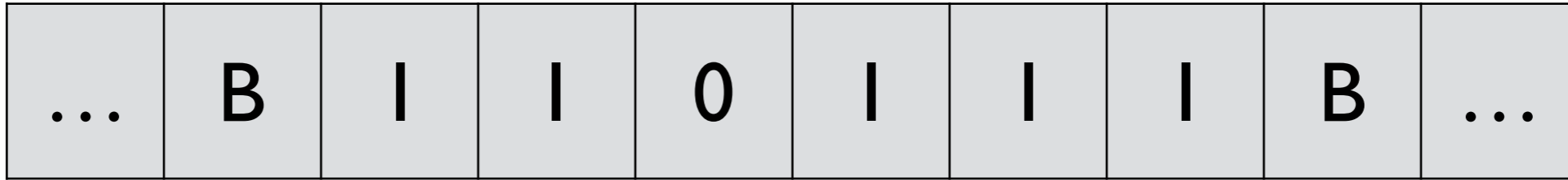


Final machine configuration:

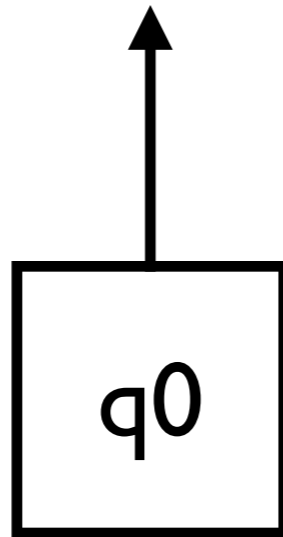
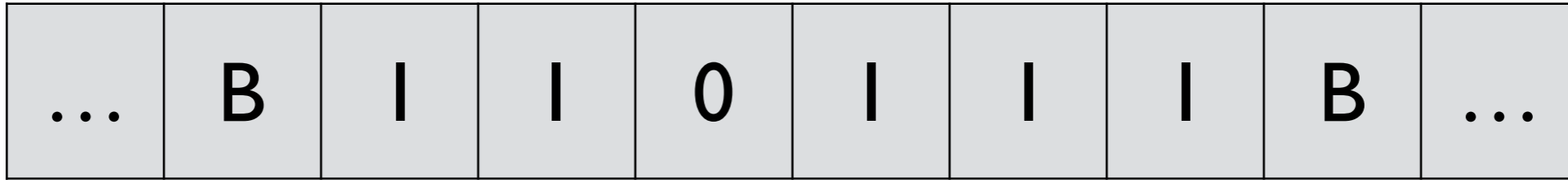


Addition in math is to move 0 to the right end





$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

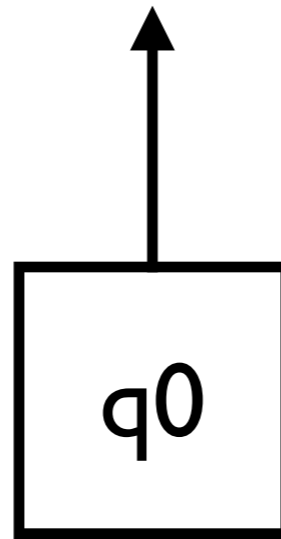
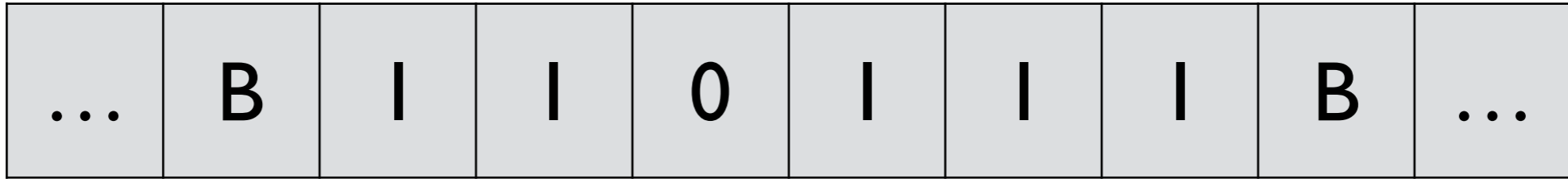
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

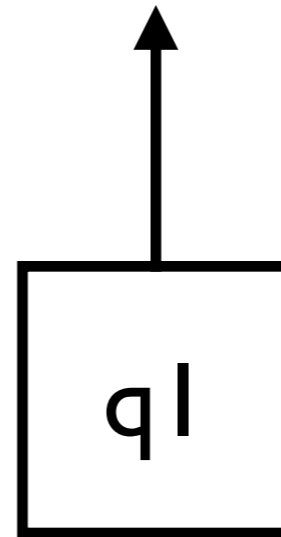
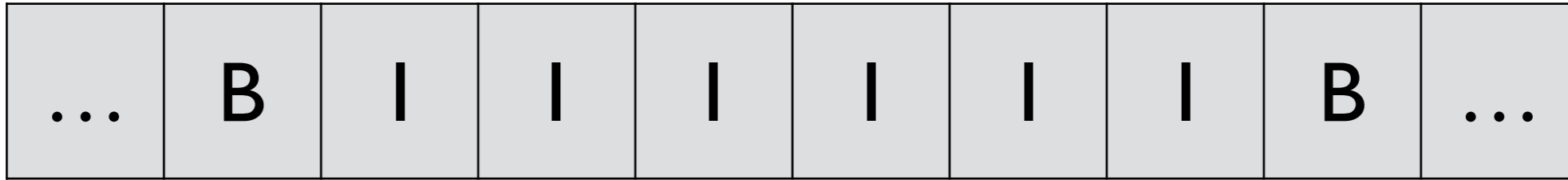
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



(In  $q_1$ , search for the right end of  $y$ )

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

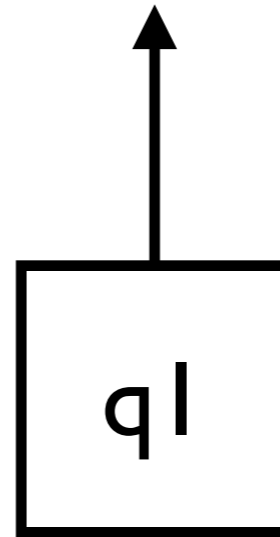
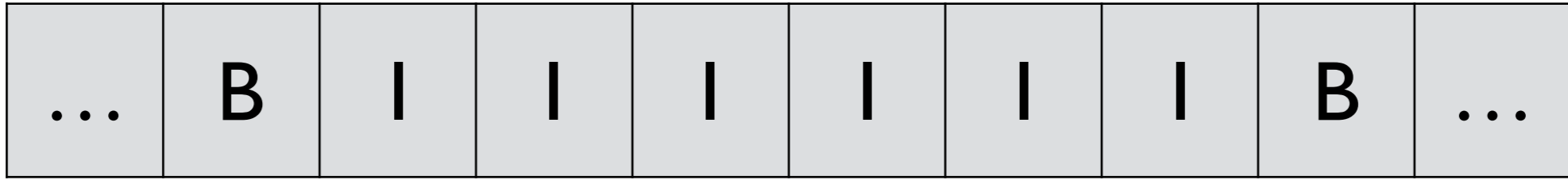
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

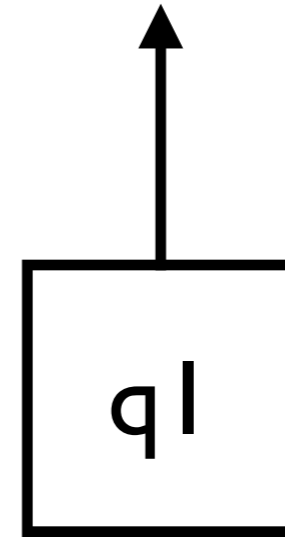
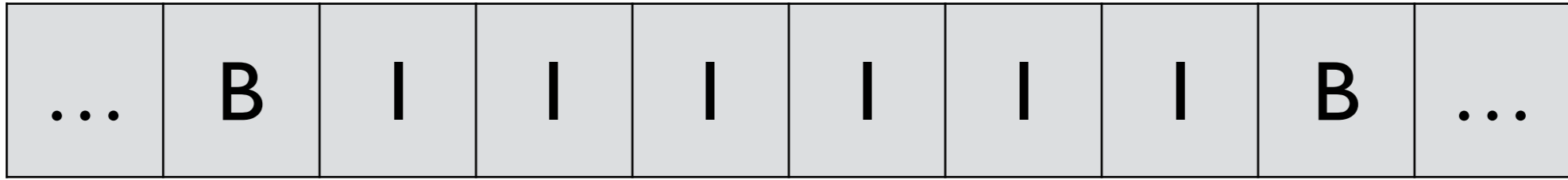
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

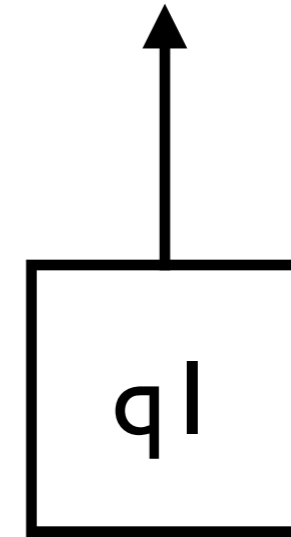
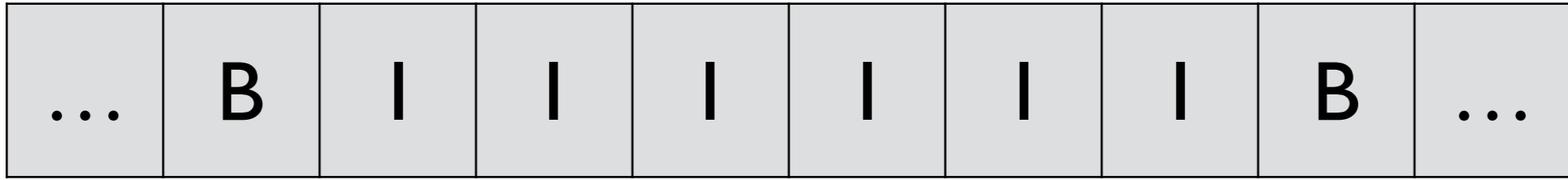
$$\delta(q_3, B) = (q_4, B, R)$$



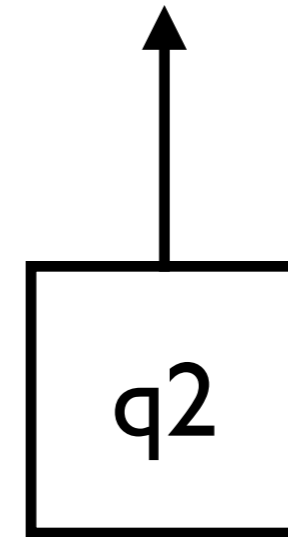
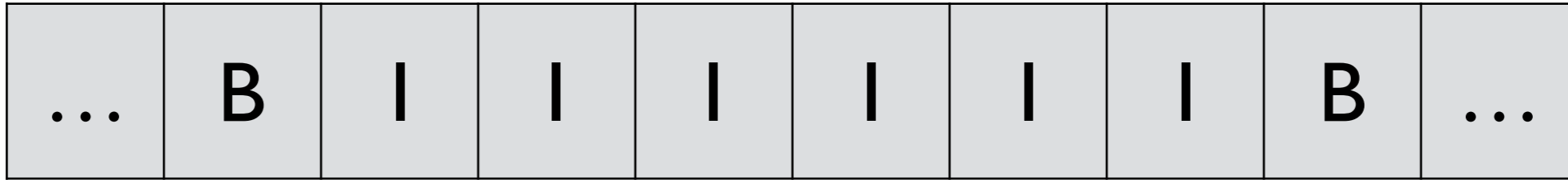
$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



(In  $q_2$ , replace the rightmost  $I$  by  $0$ )

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

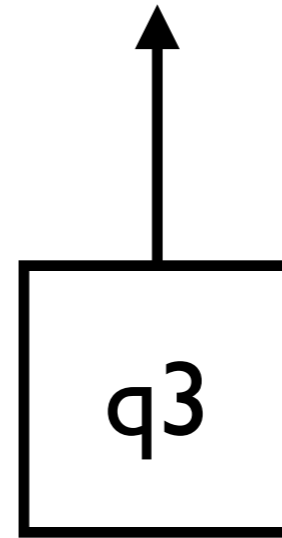
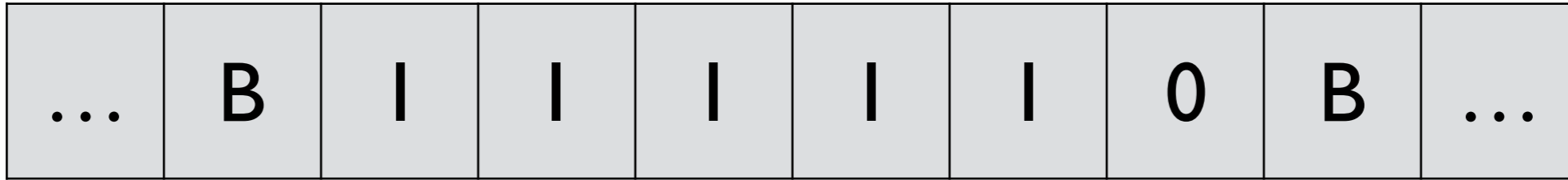
$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$





(In  $q_3$ , look for the leftmost  $I$ )

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

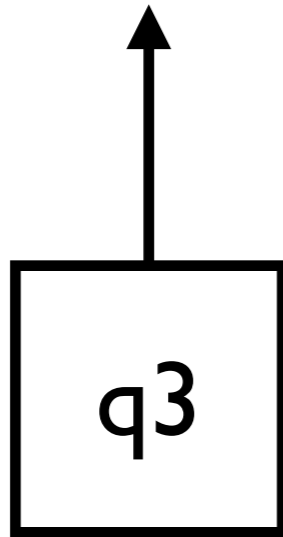
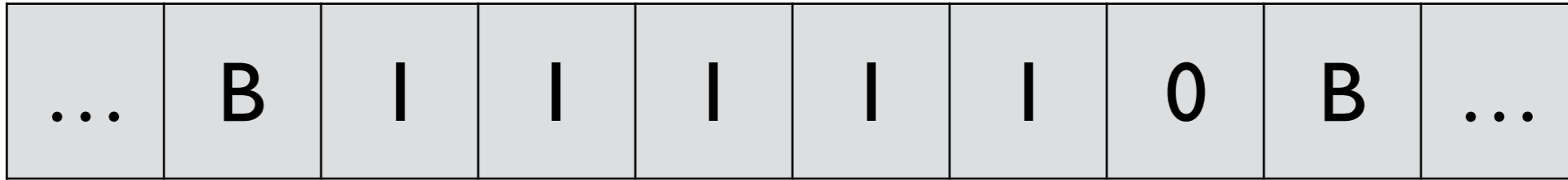
$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

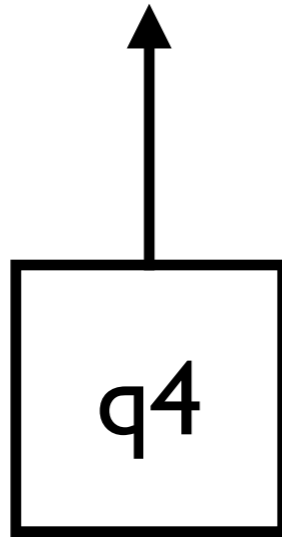
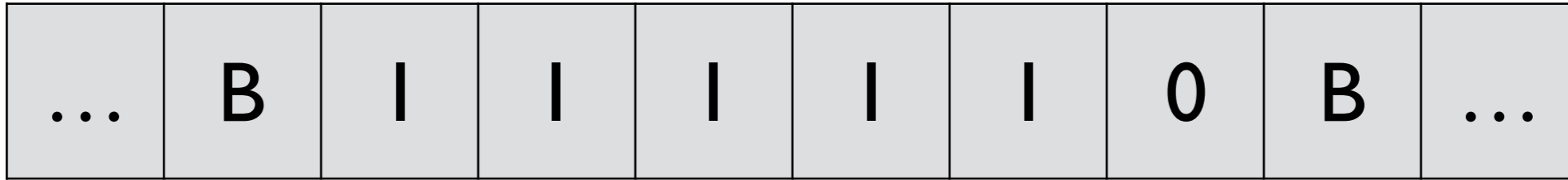
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



“final state”

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$

**Example 3.** Design a Turing machine that transforms  $w$  into  $ww$ .

$$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$$

$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

$$\delta(q_1, 1) = (q_1, 1, L)$$

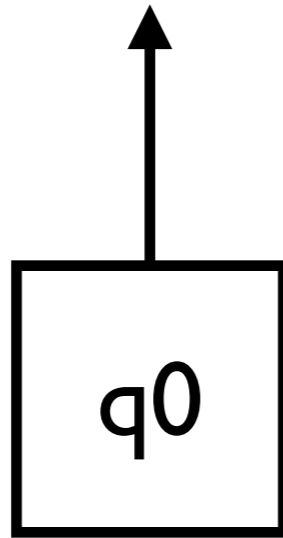
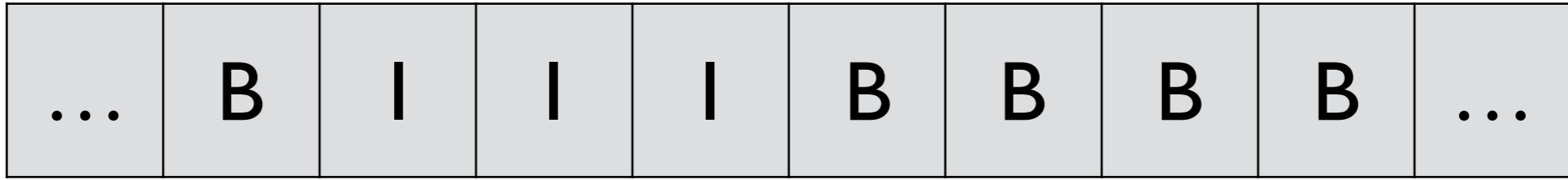
$$\delta(q_1, x) = (q_2, 1, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

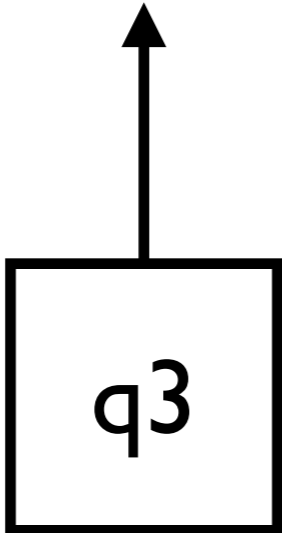
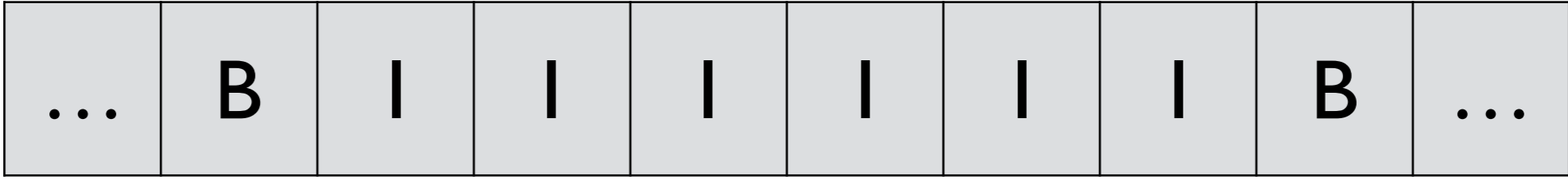
$$\delta(q_2, B) = (q_1, 1, L)$$

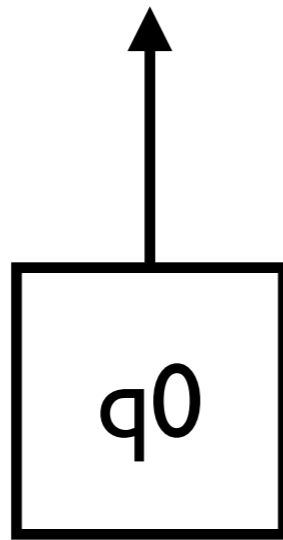
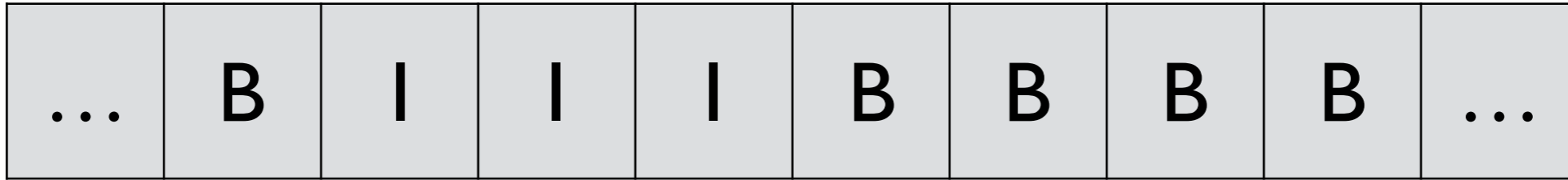
$$\delta(q_1, B) = (q_3, B, R)$$

Initial machine configuration:

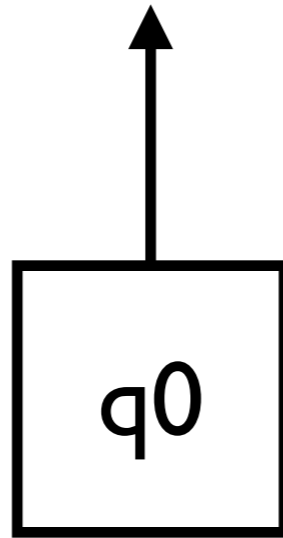
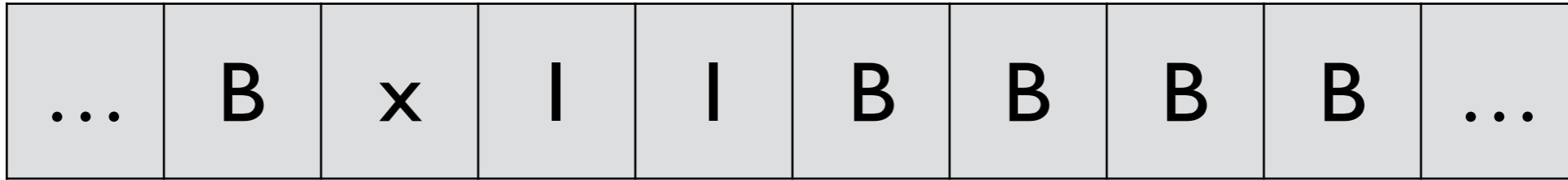


Final machine configuration:



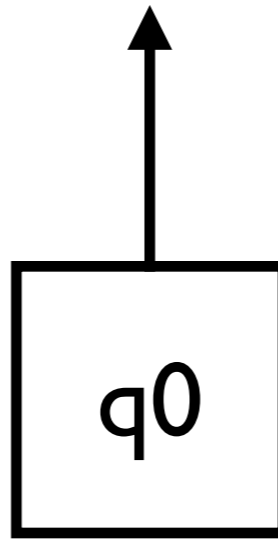
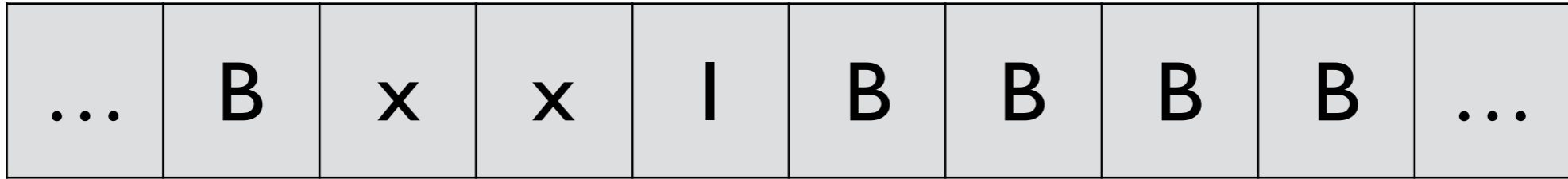


- $\delta(q_0, 1) = (q_0, x, R)$
- $\delta(q_0, B) = (q_1, B, L)$
- $\delta(q_1, 1) = (q_1, 1, L)$
- $\delta(q_1, x) = (q_2, 1, R)$
- $\delta(q_2, 1) = (q_2, 1, R)$
- $\delta(q_2, B) = (q_1, 1, L)$
- $\delta(q_1, B) = (q_3, B, R)$

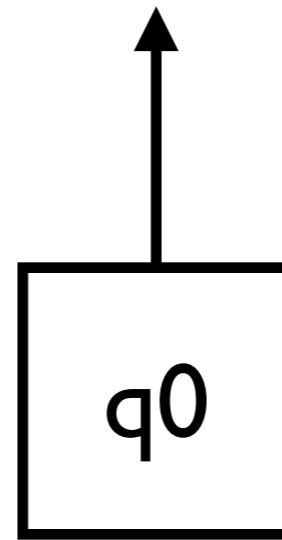
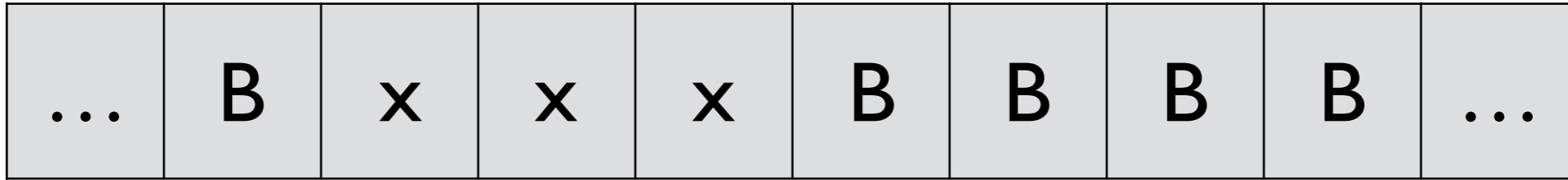


$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



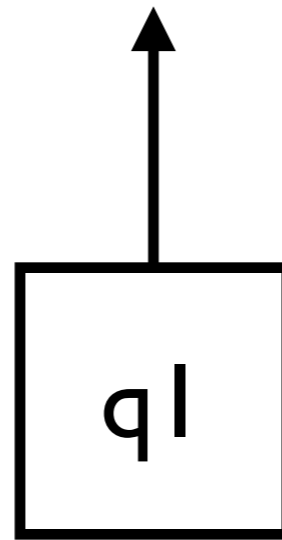


$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



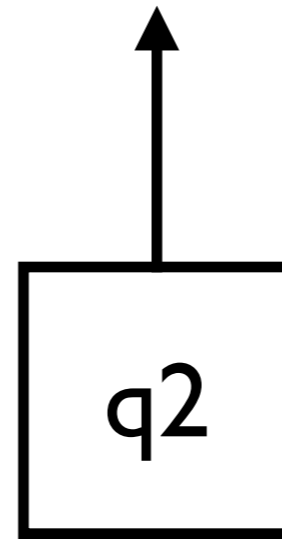
Initially, replace every  $1$  by  $x$

$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$



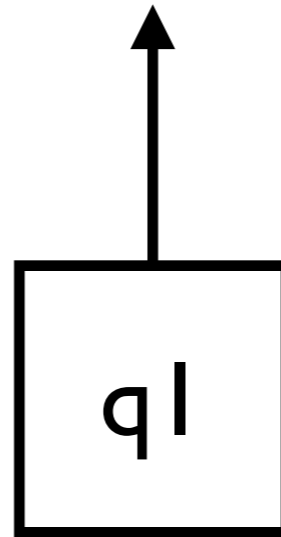
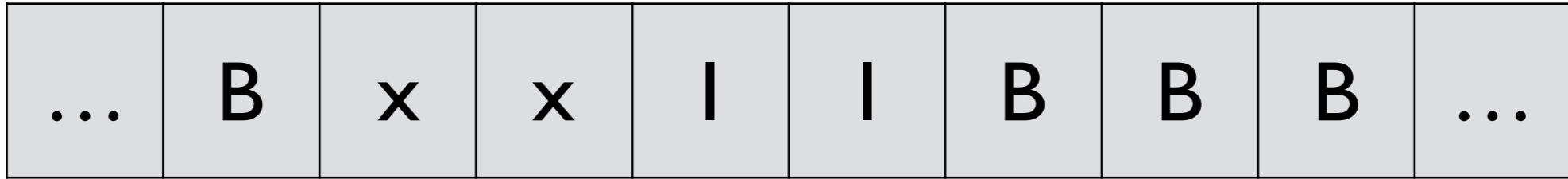
In  $q_1$ , look for the rightmost  $x$

$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$



In  $q_2$ , look for the first blank and write I

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

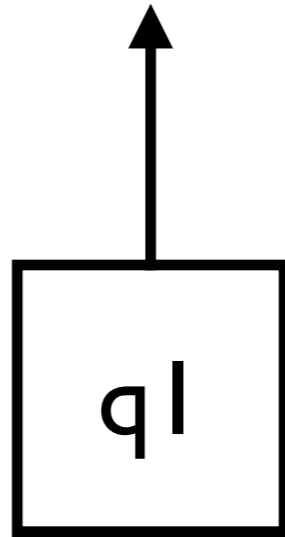
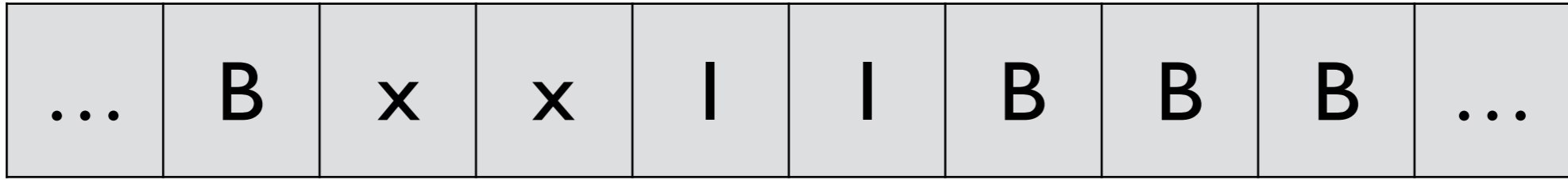
$$\delta(q_1, 1) = (q_1, 1, L)$$

$$\delta(q_1, x) = (q_2, 1, R)$$

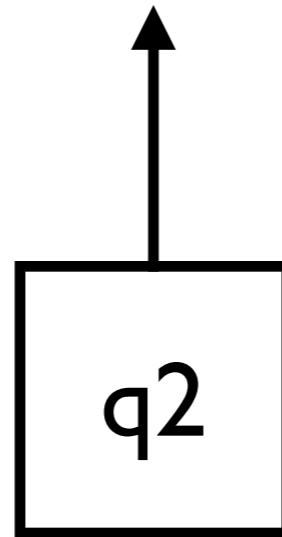
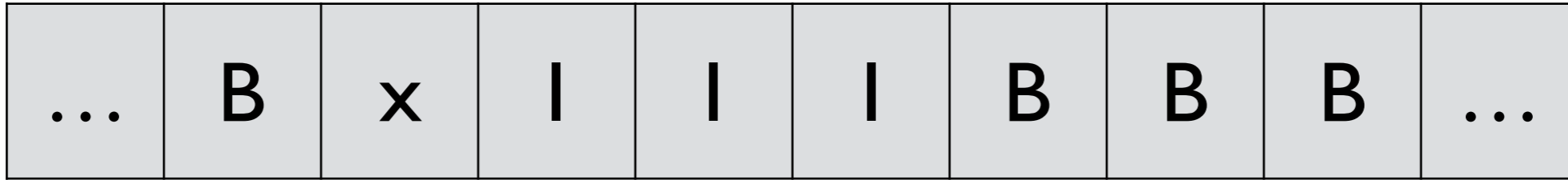
$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

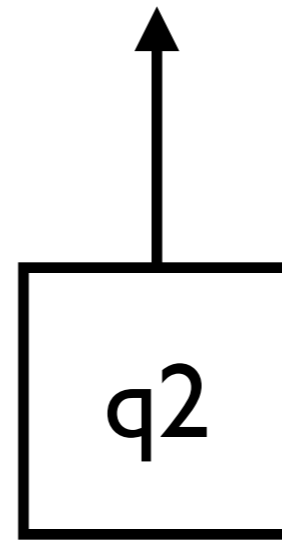
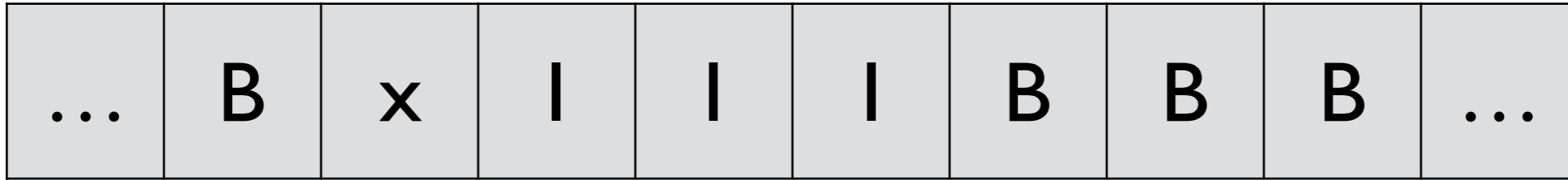
$$\delta(q_1, B) = (q_3, B, R)$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

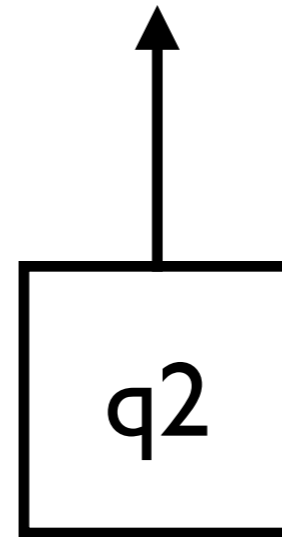
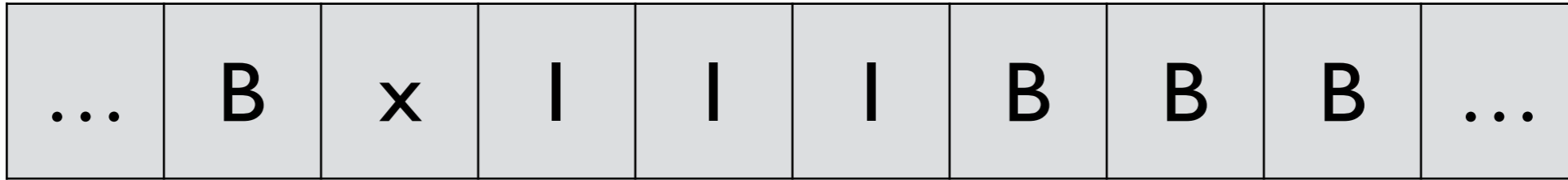


$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

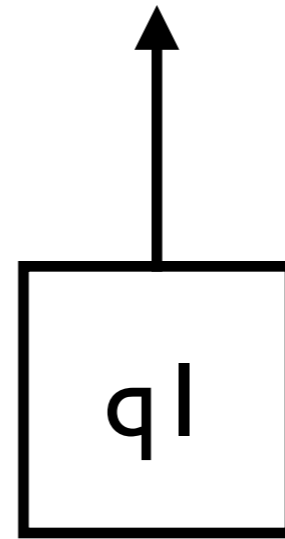
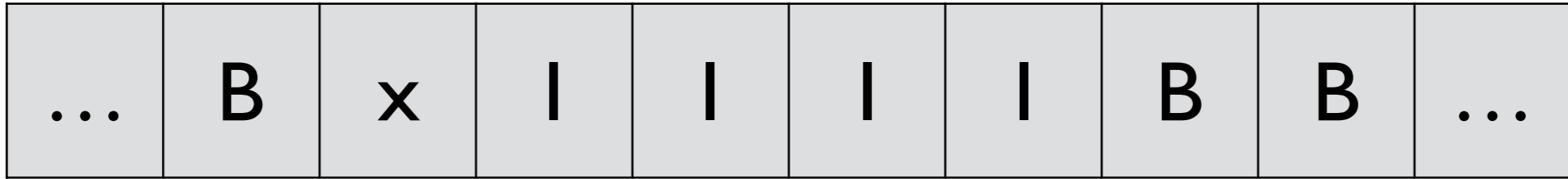


- $\delta(q_0, 1) = (q_0, x, R)$
- $\delta(q_0, B) = (q_1, B, L)$
- $\delta(q_1, 1) = (q_1, 1, L)$
- $\delta(q_1, x) = (q_2, 1, R)$
- $\delta(q_2, 1) = (q_2, 1, R)$
- $\delta(q_2, B) = (q_1, 1, L)$
- $\delta(q_1, B) = (q_3, B, R)$

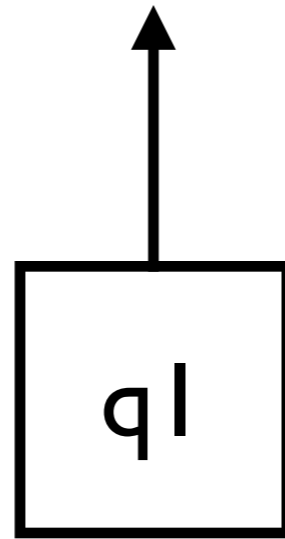
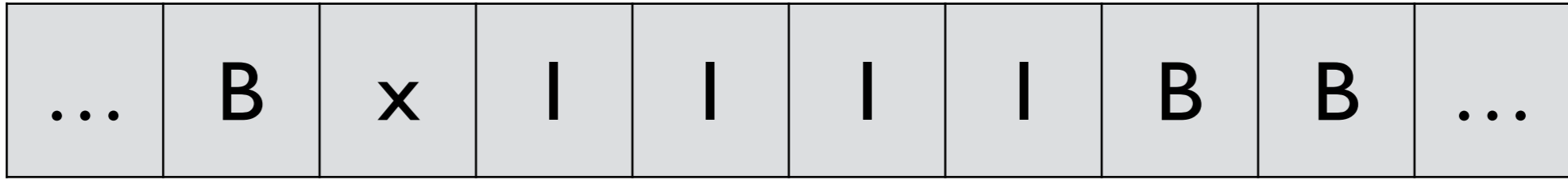




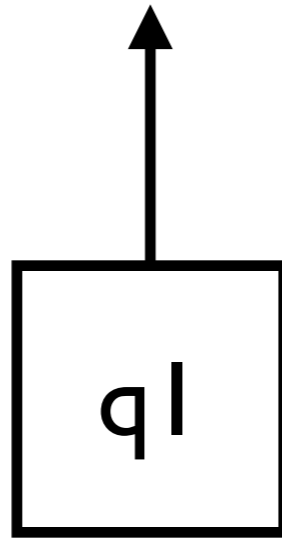
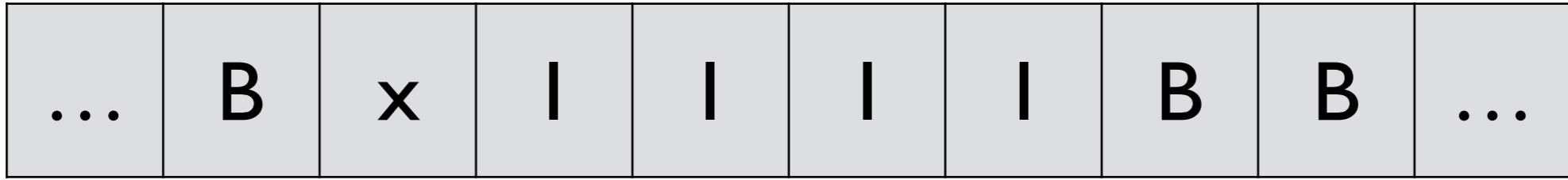
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



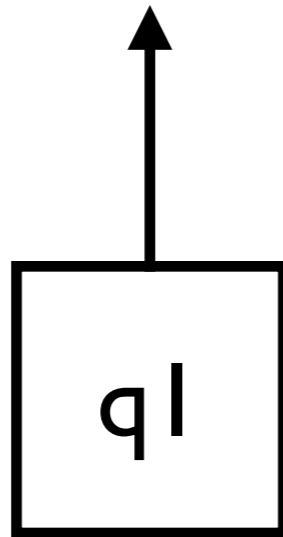
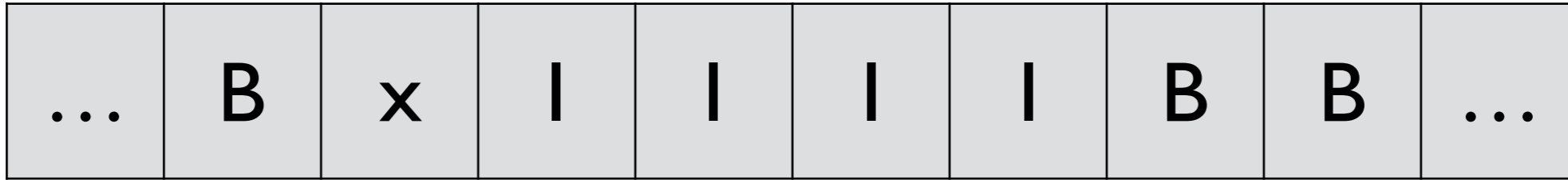
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



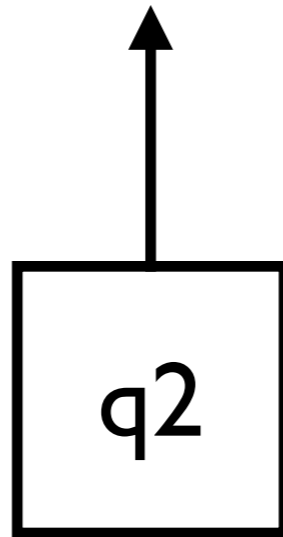
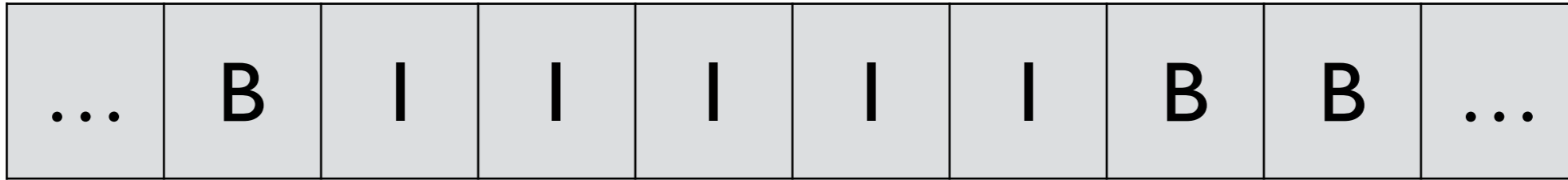
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



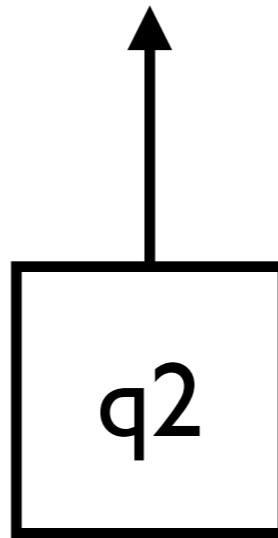
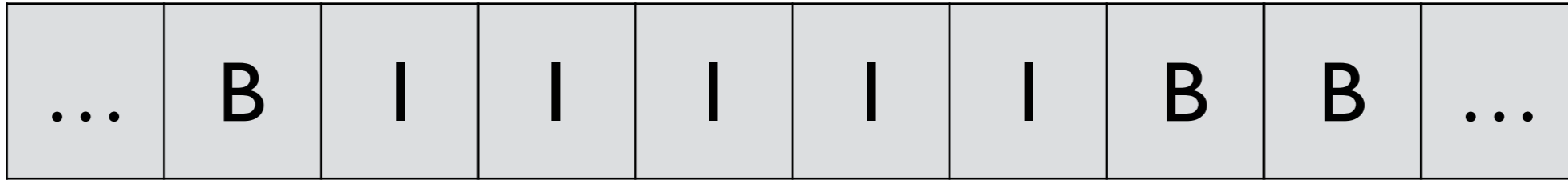
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



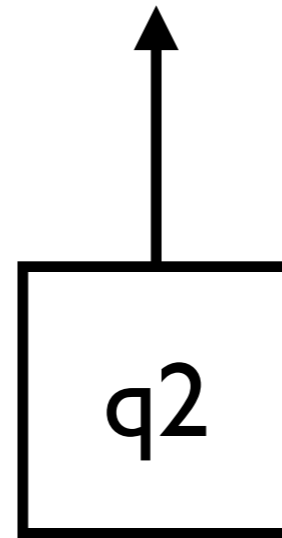
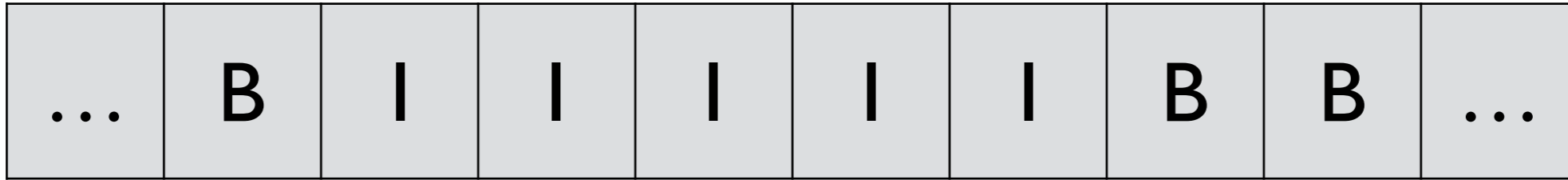
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



- $\delta(q_0, 1) = (q_0, x, R)$
- $\delta(q_0, B) = (q_1, B, L)$
- $\delta(q_1, 1) = (q_1, 1, L)$
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- $\delta(q_2, 1) = (q_2, 1, R)$
- $\delta(q_2, B) = (q_1, 1, L)$
- $\delta(q_1, B) = (q_3, B, R)$

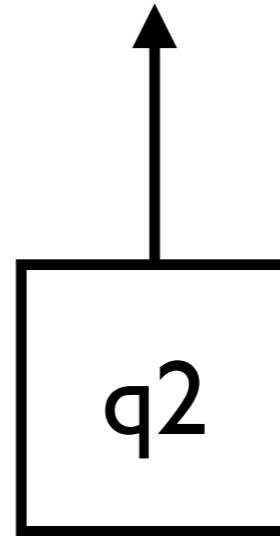
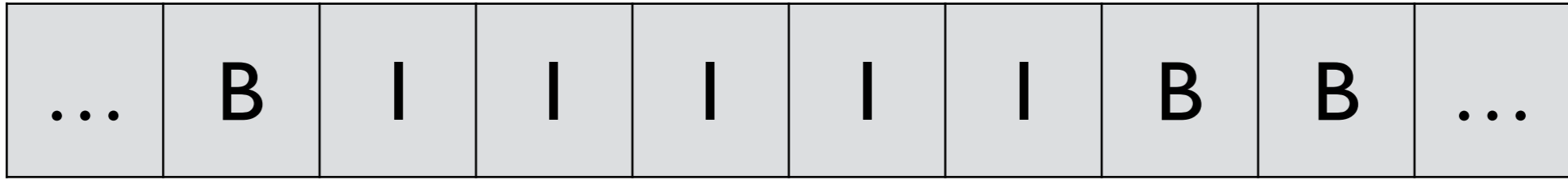


$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

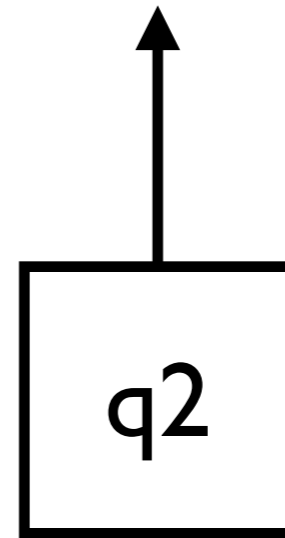
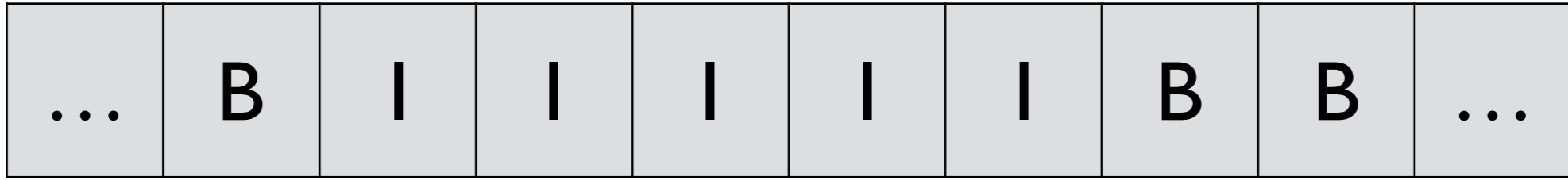


$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

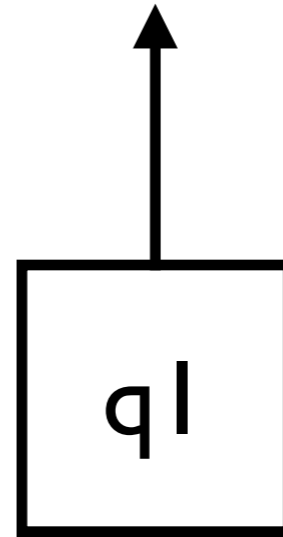
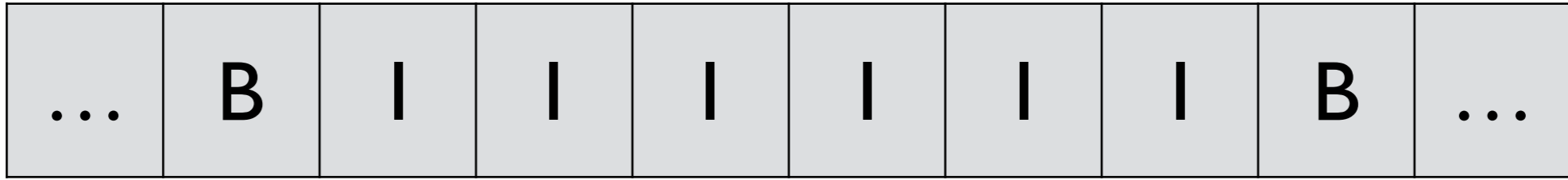




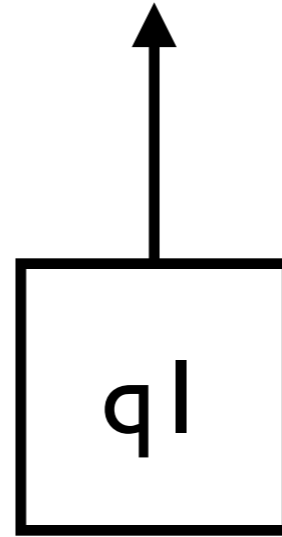
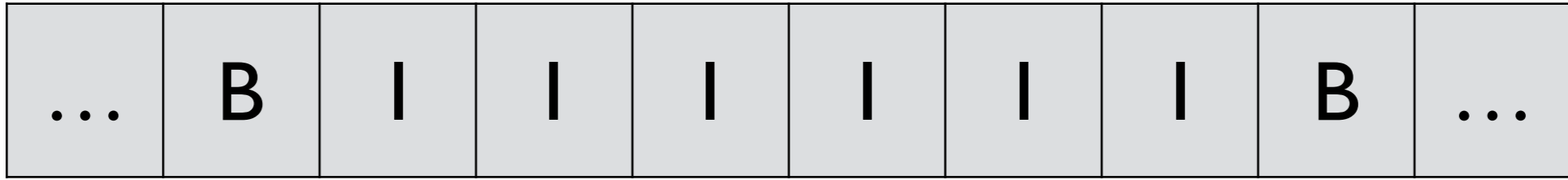
- $\delta(q_0, 1) = (q_0, x, R)$
- $\delta(q_0, B) = (q_1, B, L)$
- $\delta(q_1, 1) = (q_1, 1, L)$
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- $\delta(q_2, 1) = (q_2, 1, R)$
- $\delta(q_2, B) = (q_1, 1, L)$
- $\delta(q_1, B) = (q_3, B, R)$



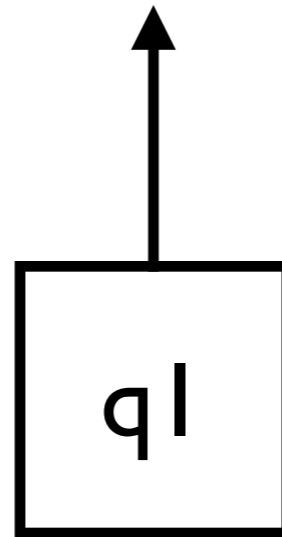
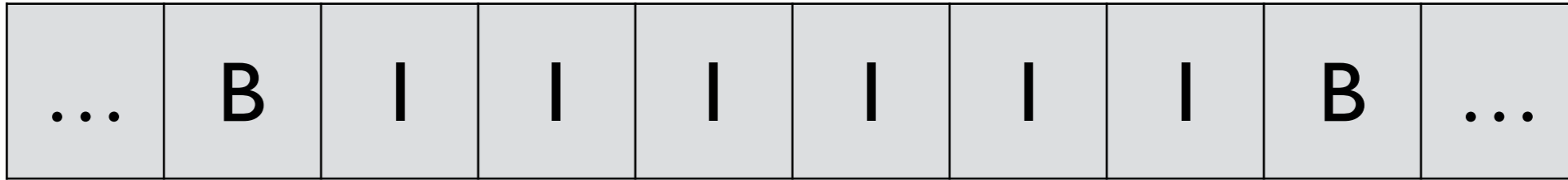
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



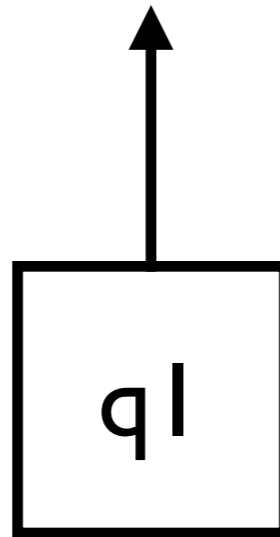
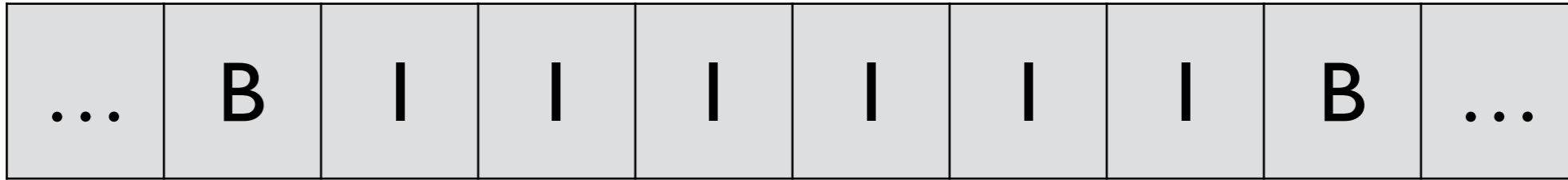
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



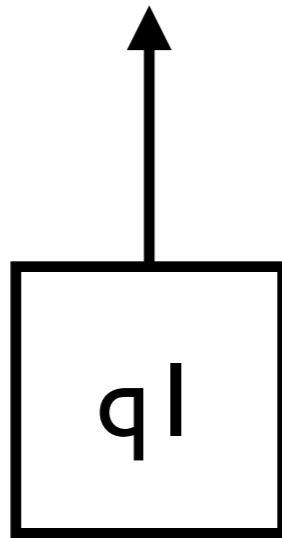
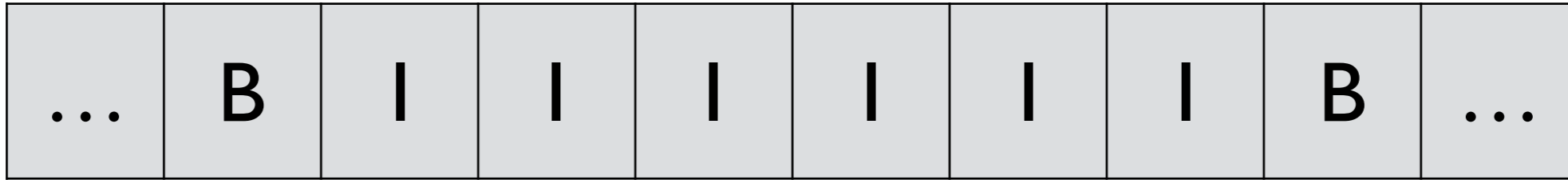
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



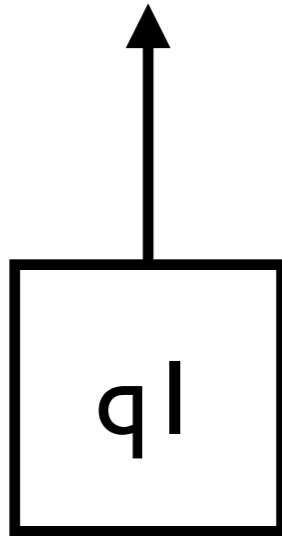
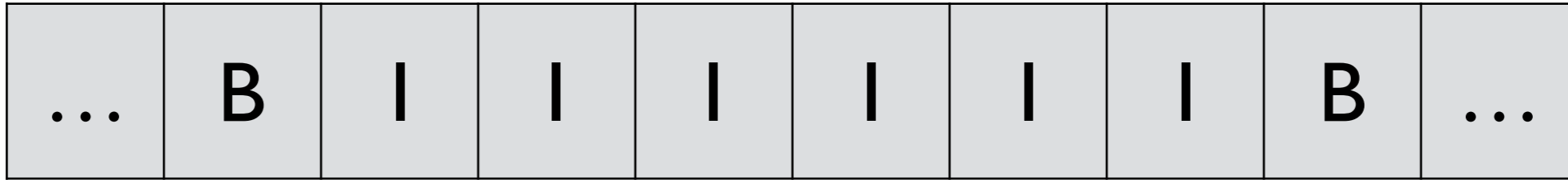
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$



$$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$



- $\delta(q_0, 1) = (q_0, x, R)$
- $\delta(q_0, B) = (q_1, B, L)$
- $\delta(q_1, 1) = (q_1, 1, L)$
- $\delta(q_1, x) = (q_2, 1, R)$
- $\delta(q_2, 1) = (q_2, 1, R)$
- $\delta(q_2, B) = (q_1, 1, L)$
- $\delta(q_1, B) = (q_3, B, R)$



$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

$$\delta(q_1, 1) = (q_1, 1, L)$$

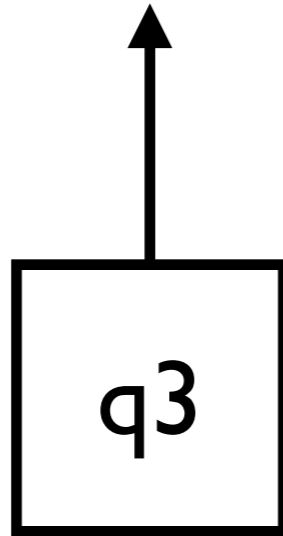
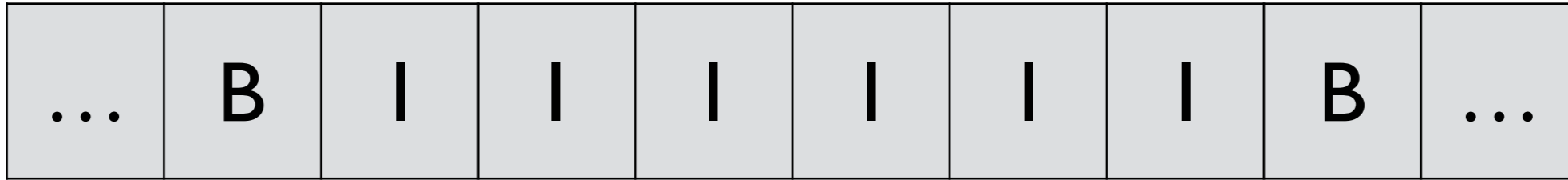
$$\delta(q_1, x) = (q_2, 1, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

$$\delta(q_2, B) = (q_1, 1, L)$$

$$\delta(q_1, B) = (q_3, B, R)$$





“final state”

$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

**Example 4.** Design a Turing machine that computes  $f(m, n)$ .

$$f(m, n) = \max(m - n, 0) = \text{if } m \geq n \text{ then } m - n \text{ else } 0$$

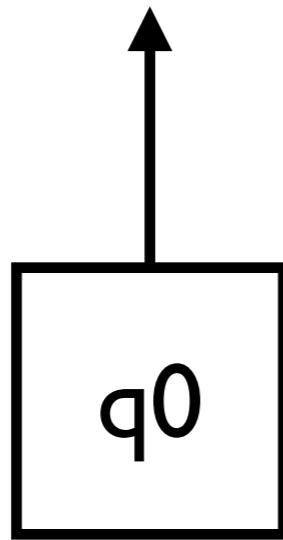
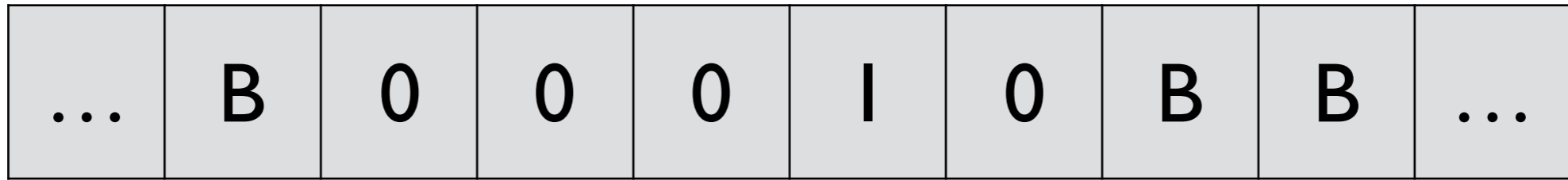
**Example 4.** Design a Turing machine that computes  $f(m, n)$ .

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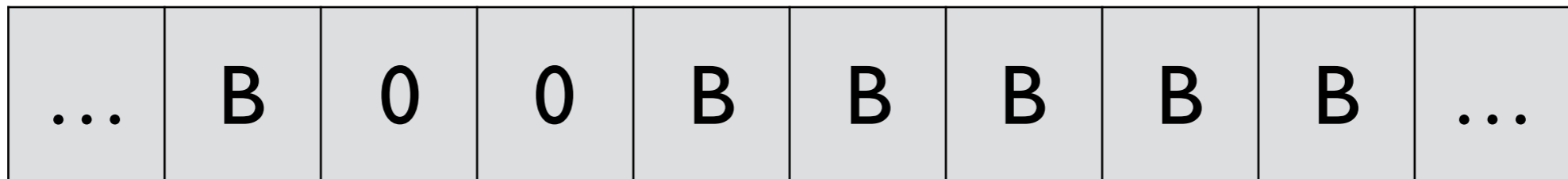
$$M = (\{q_0, q_1, \dots, q_6\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_6\})$$

	0	1	B
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

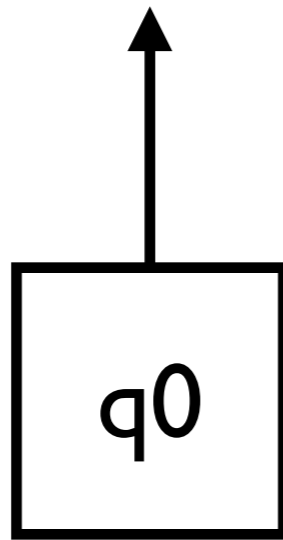
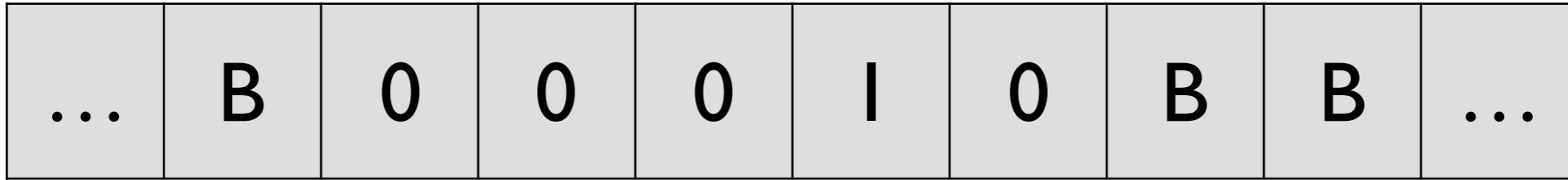
Initial machine configuration: e.g.,  $f(3, 1)$



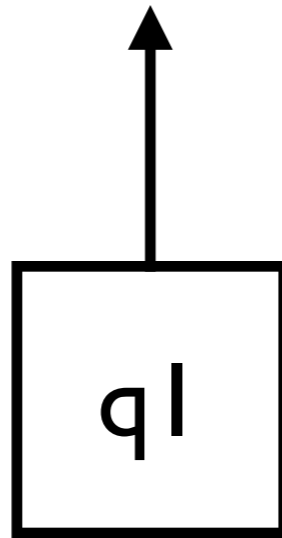
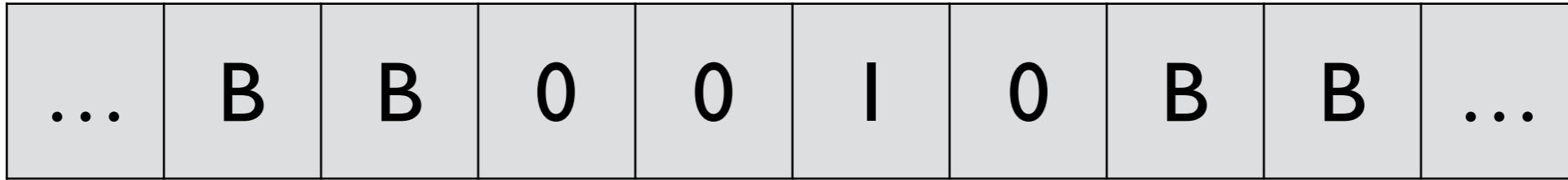
When the machine halts:



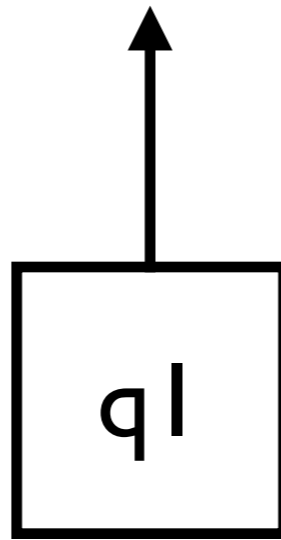
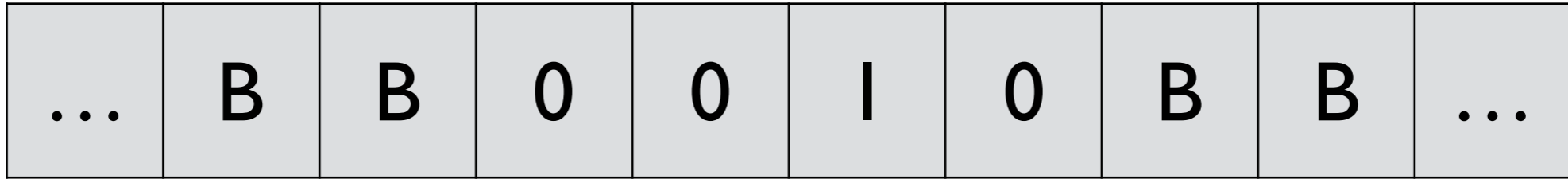
e.g.,  $f(3, 1)$



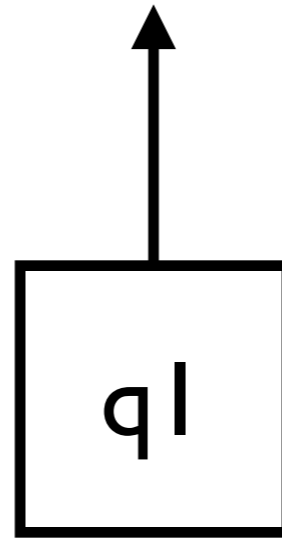
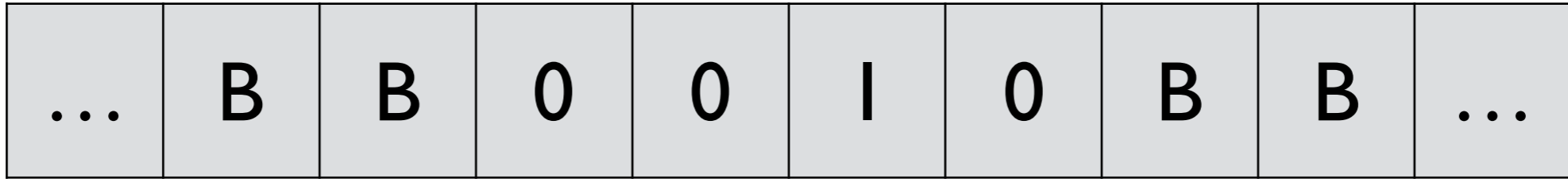
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

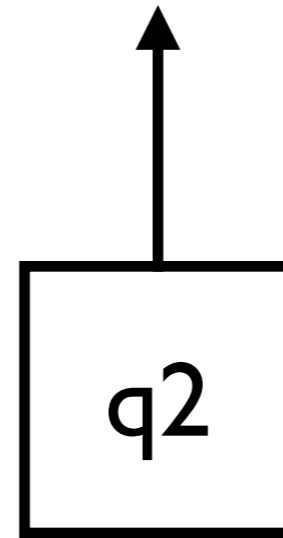
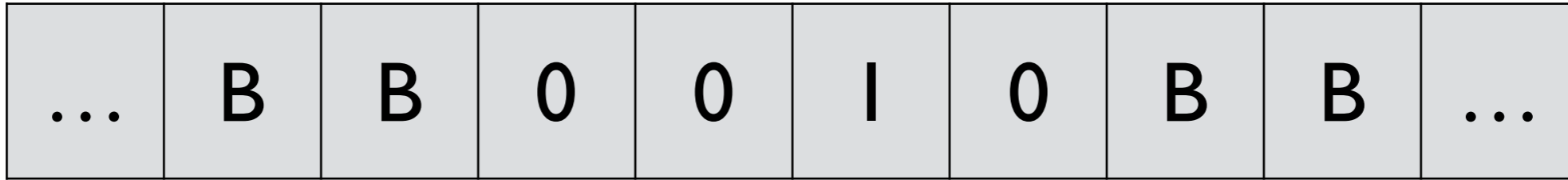


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

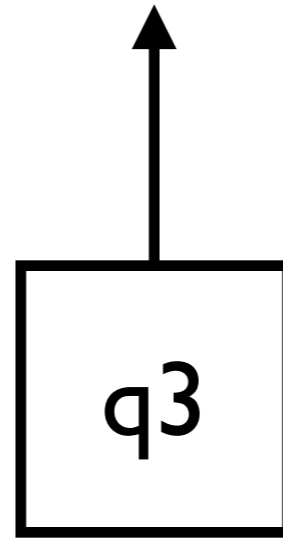
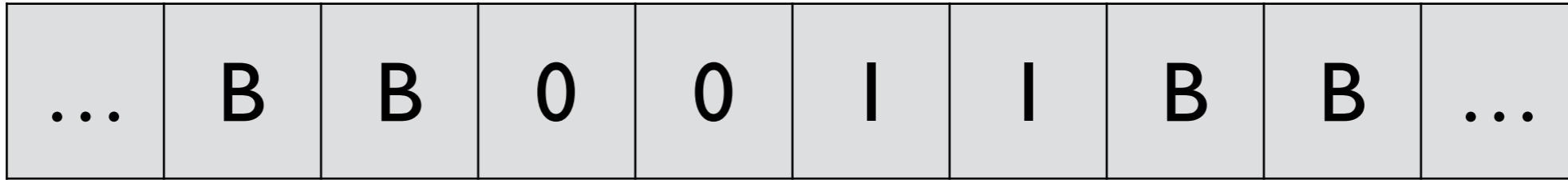


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

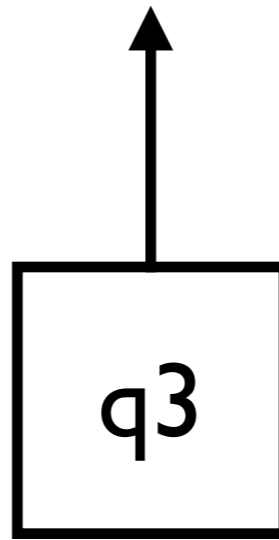
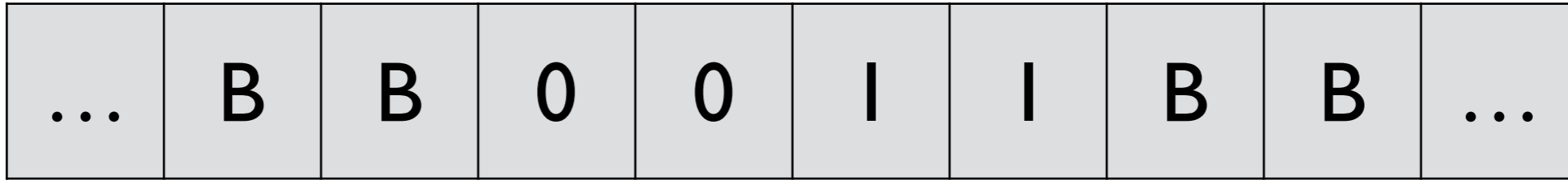




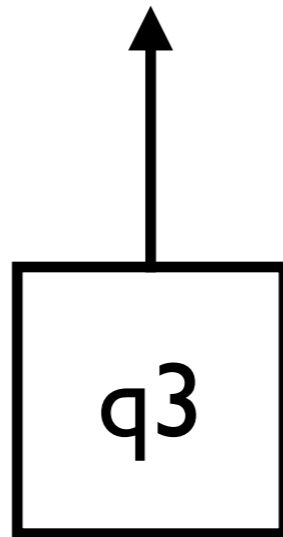
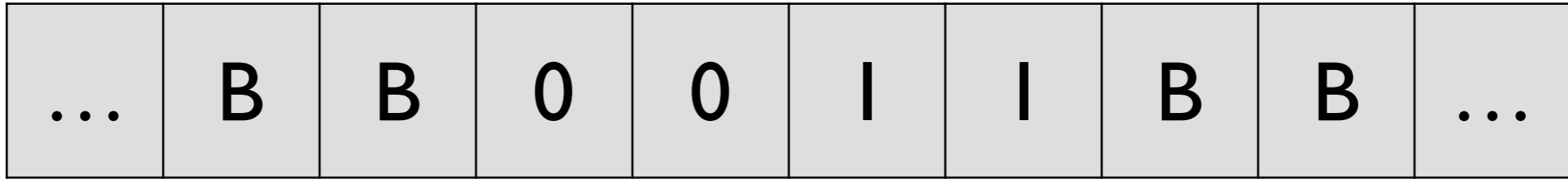
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



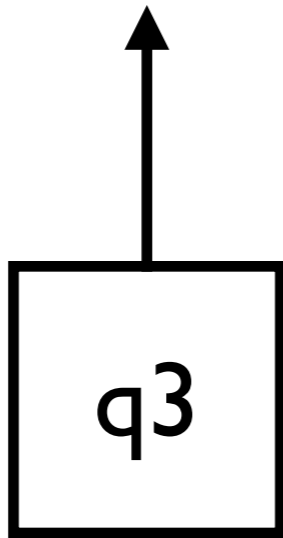
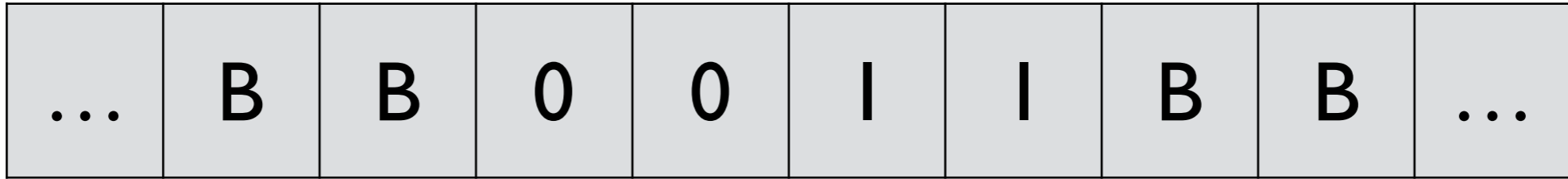
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



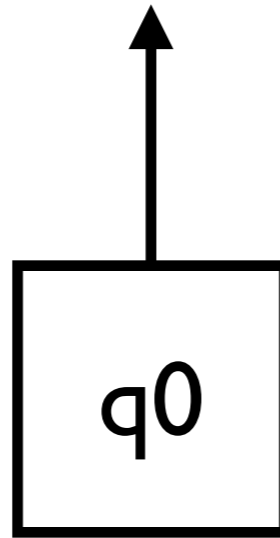
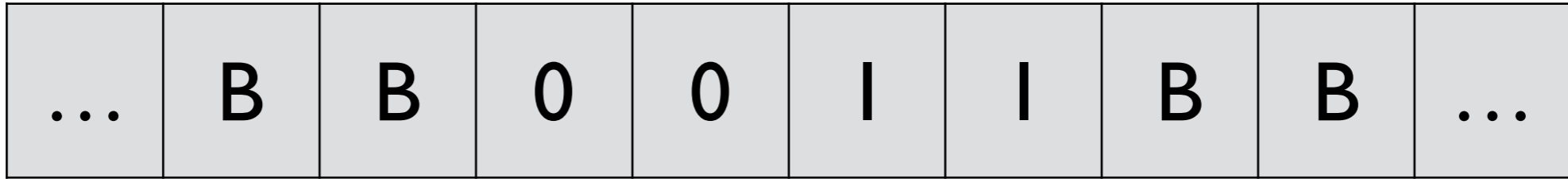
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



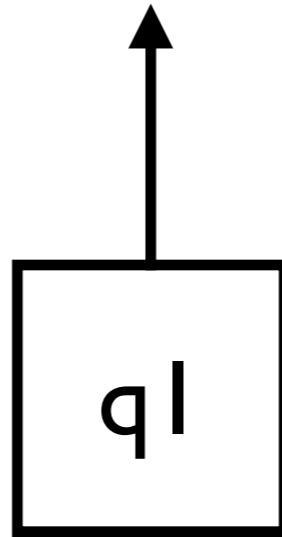
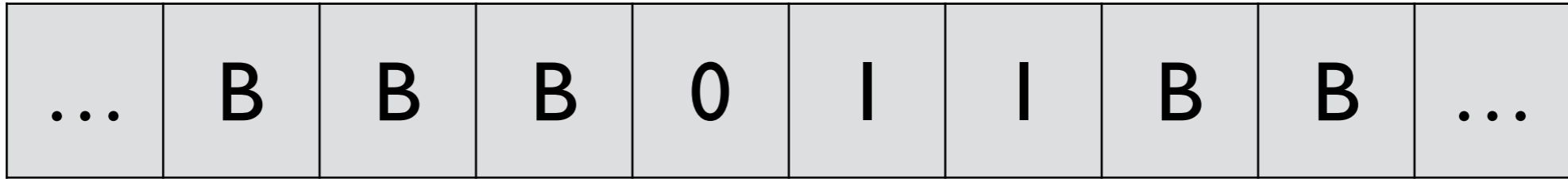
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



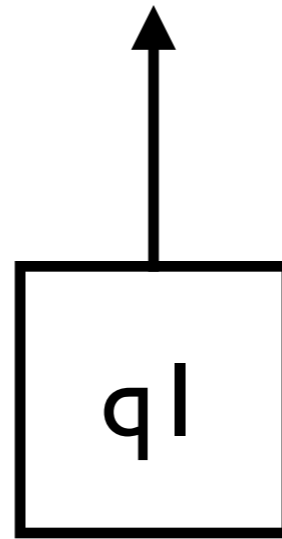
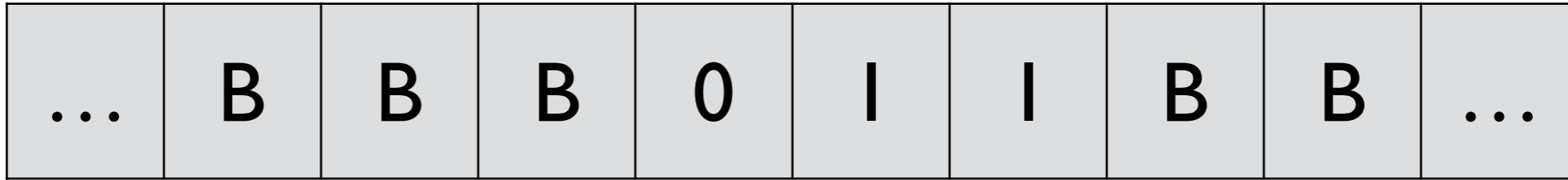
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

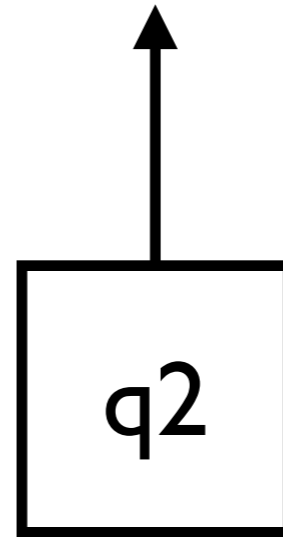
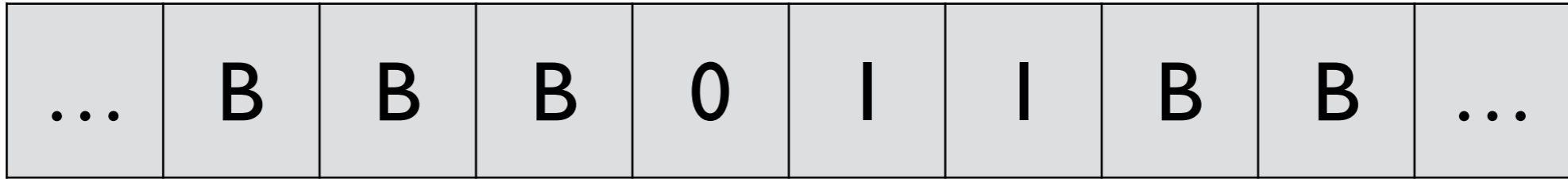


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

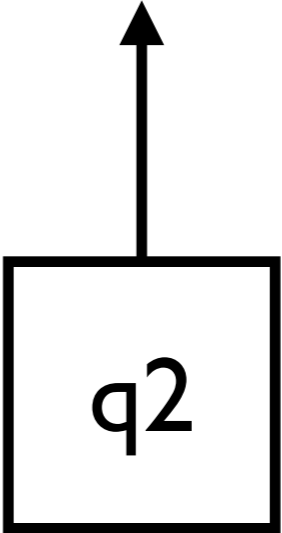
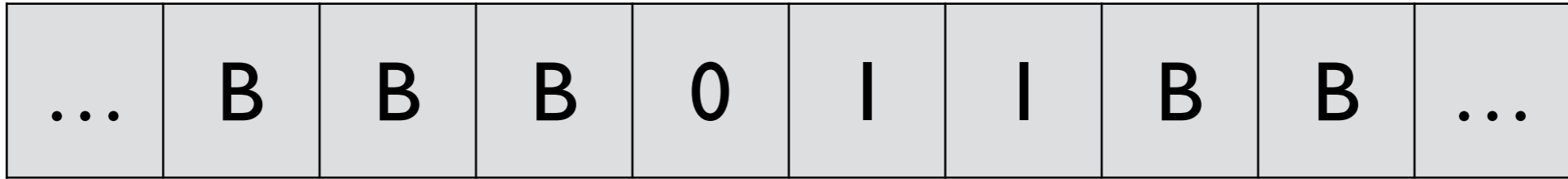


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

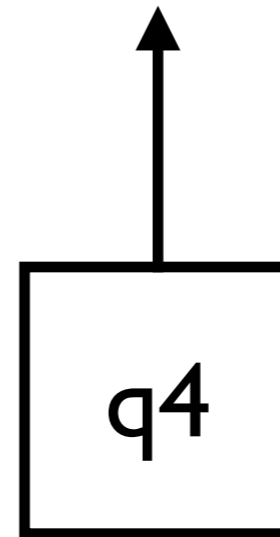
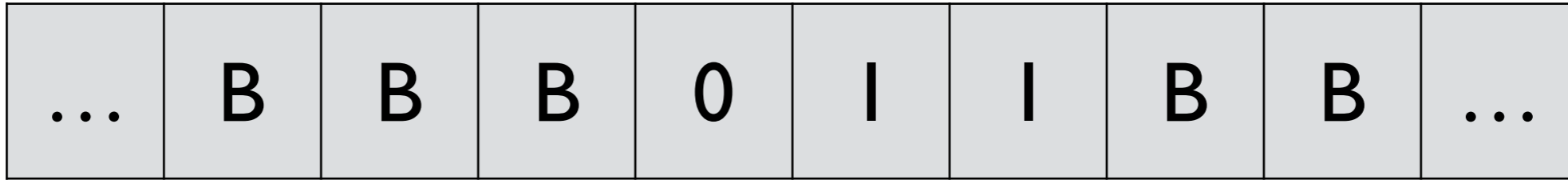




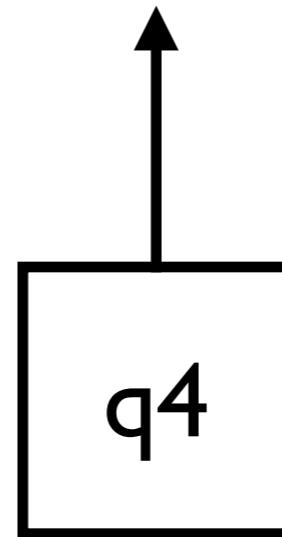
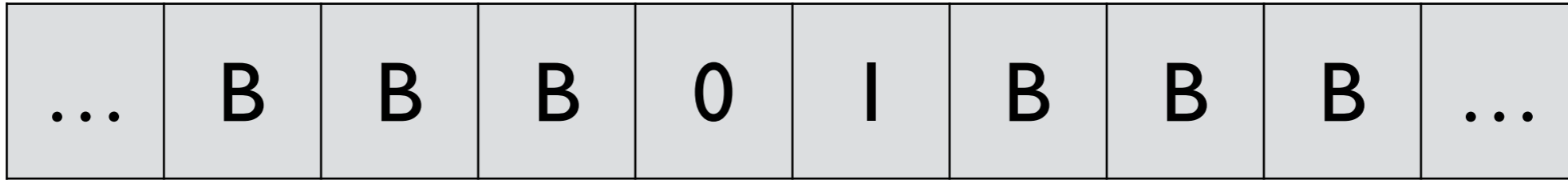
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



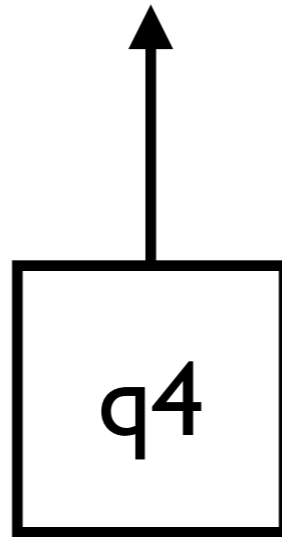
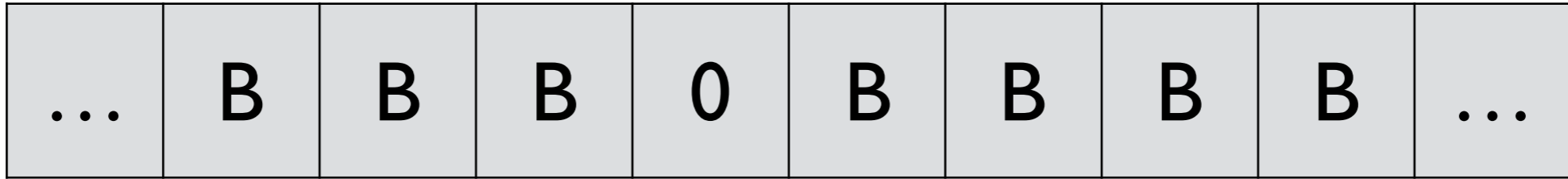
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



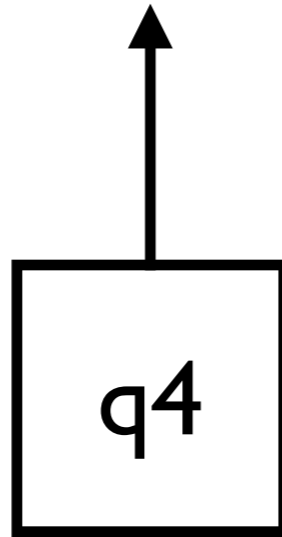
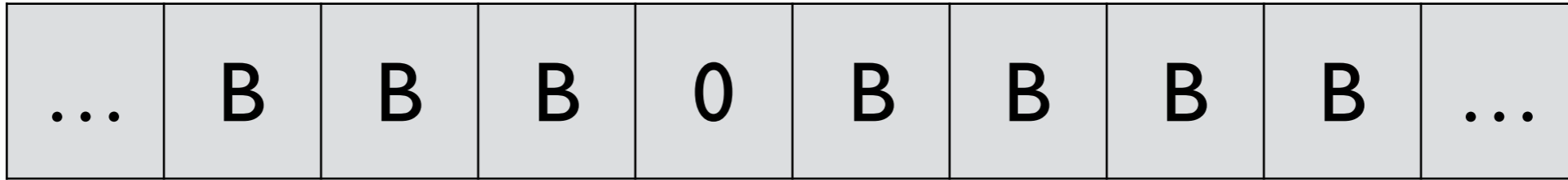
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



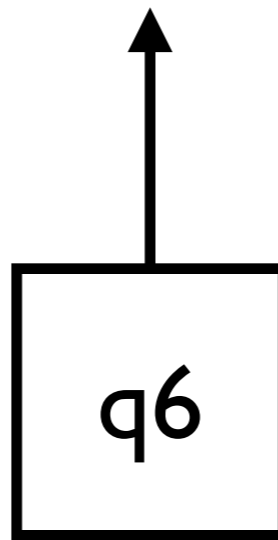
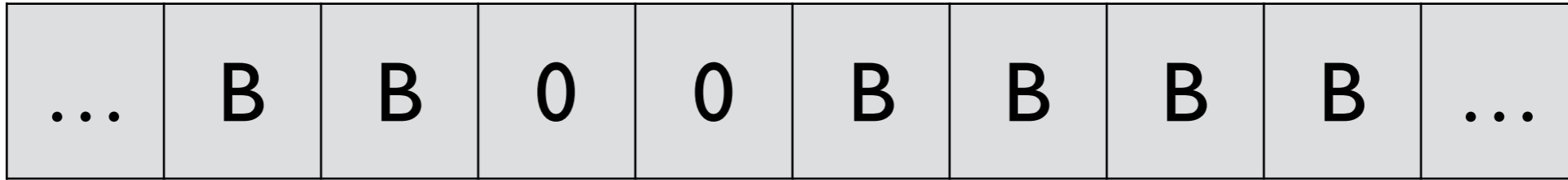
	$0$	$1$	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

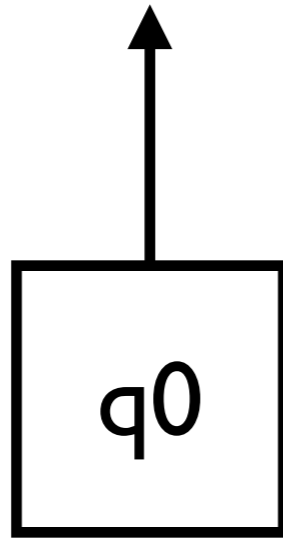
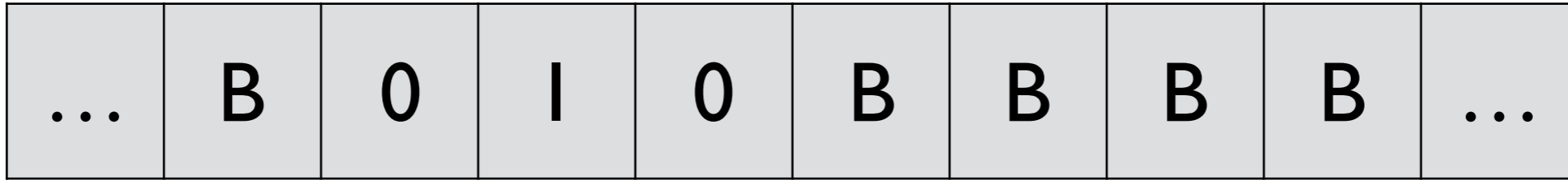


	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



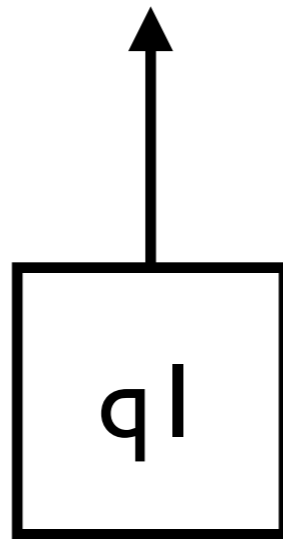
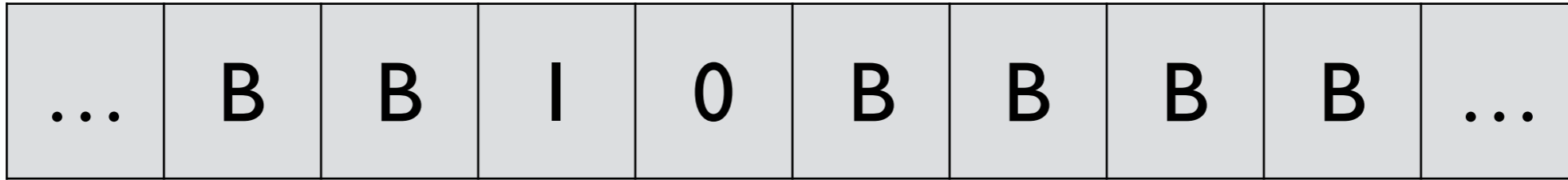
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

e.g.,  $f(1, 1)$

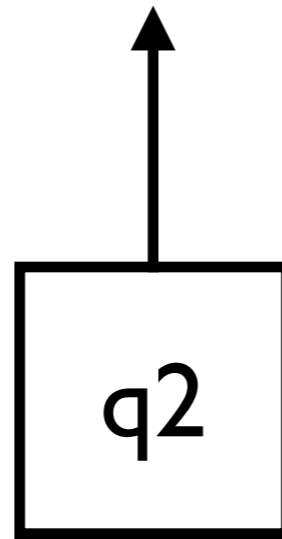
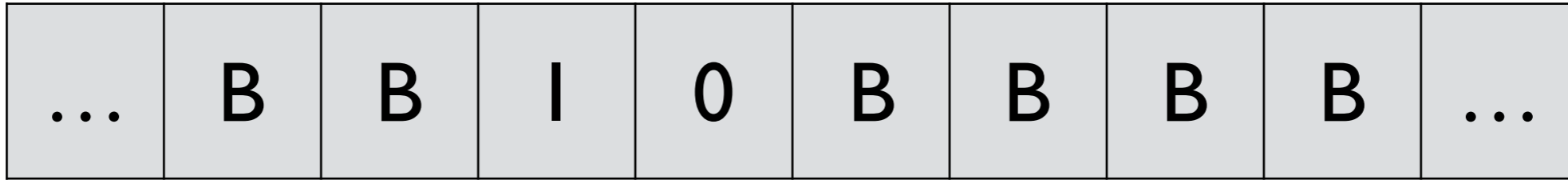


	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

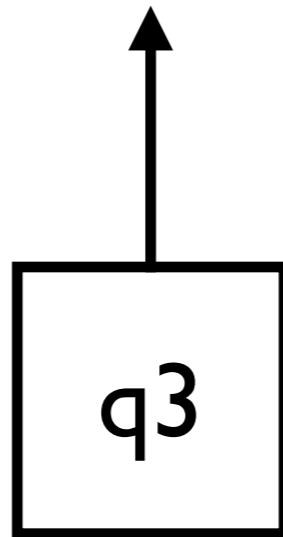
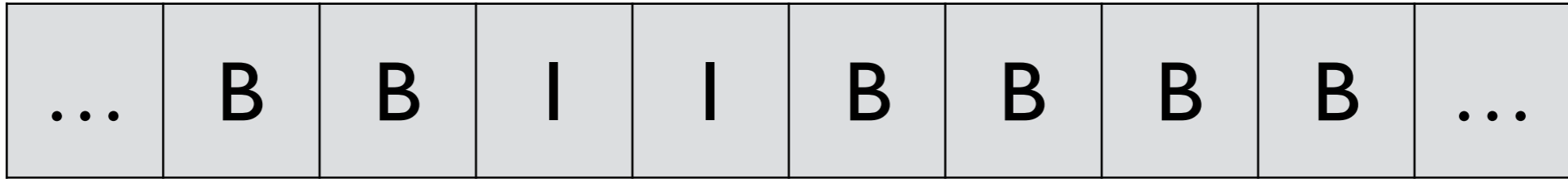




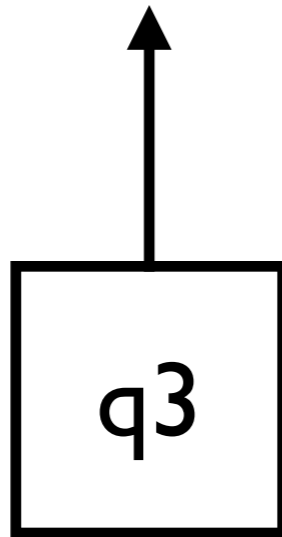
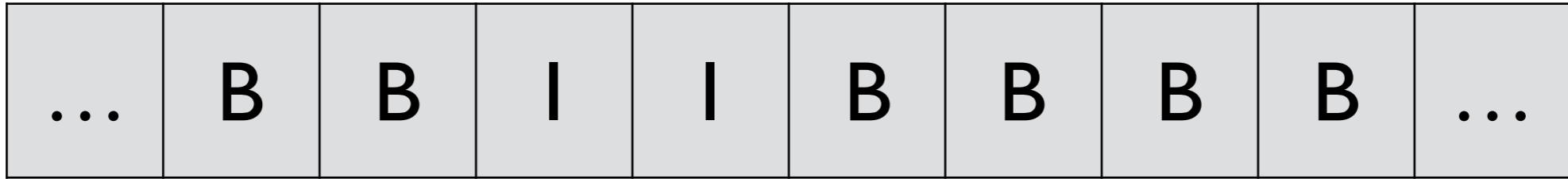
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



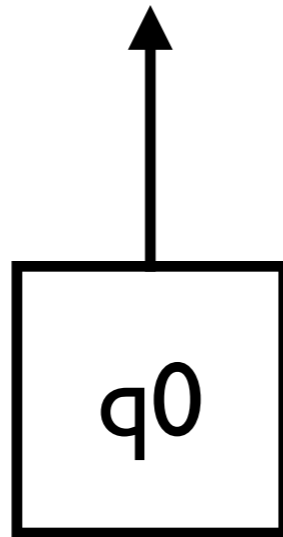
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



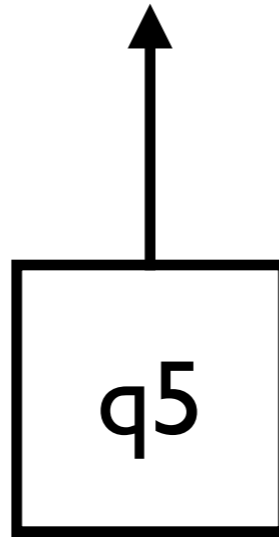
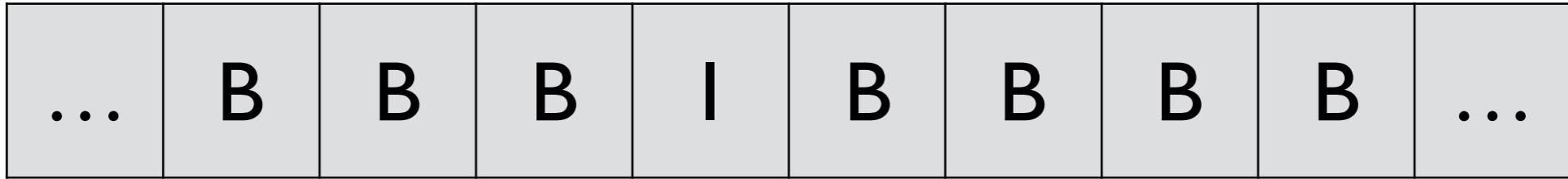
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



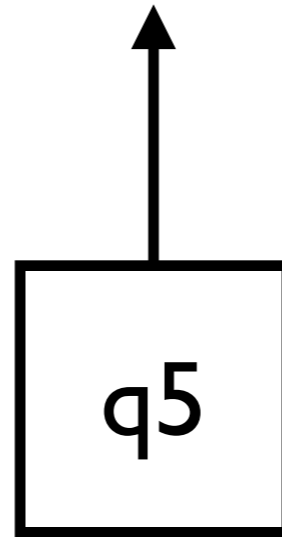
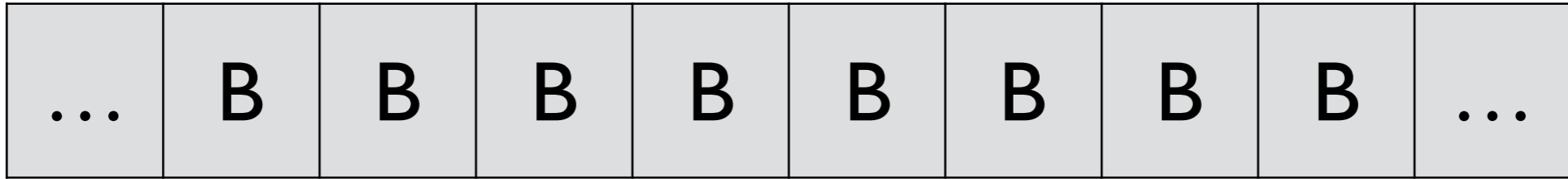
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



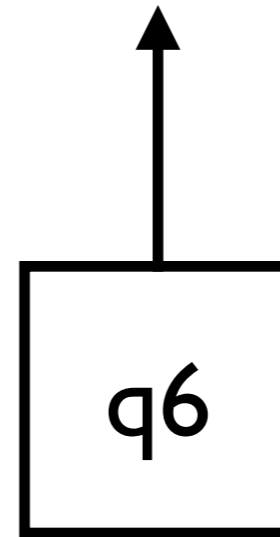
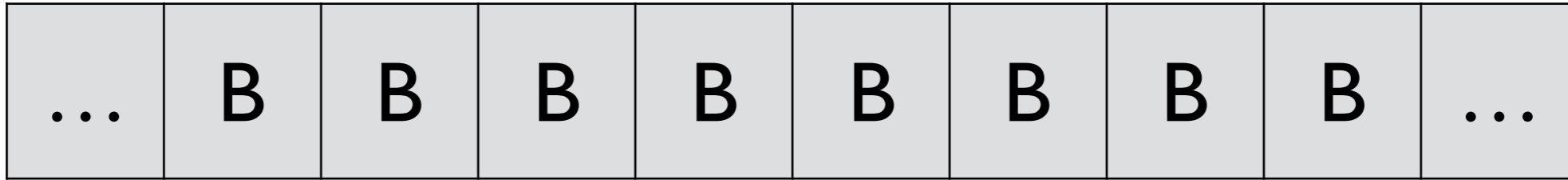
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



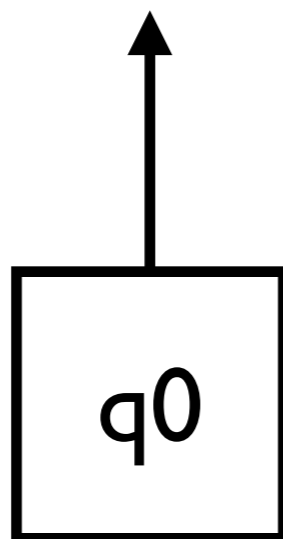
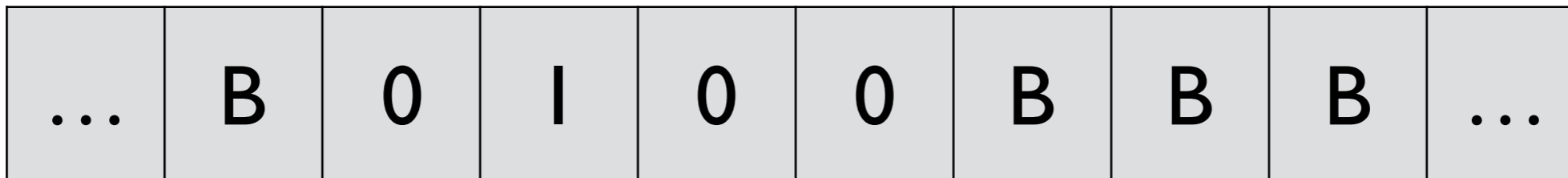
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



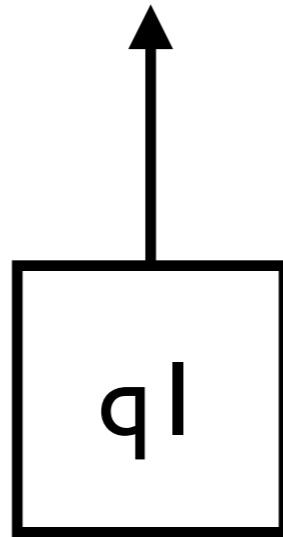
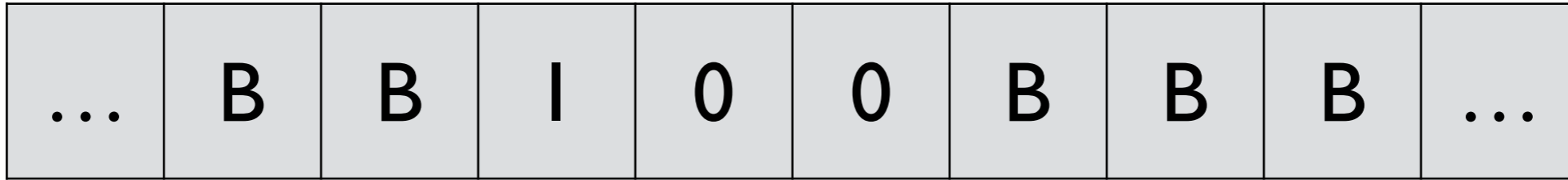
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



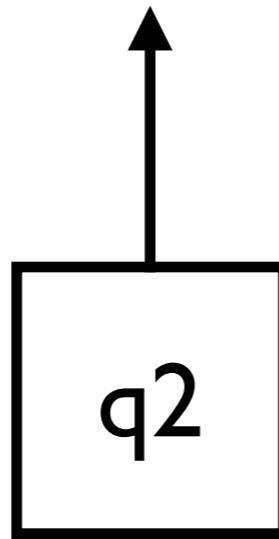
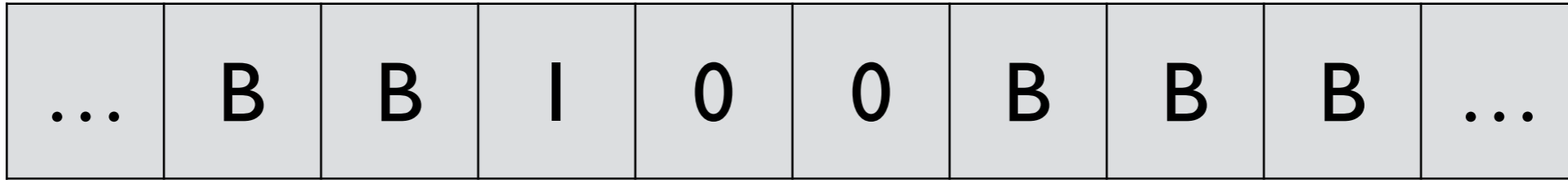
e.g.,  $f(1,2)$



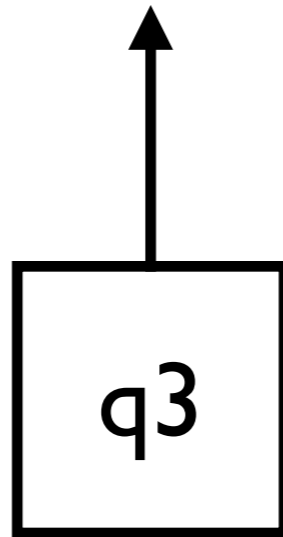
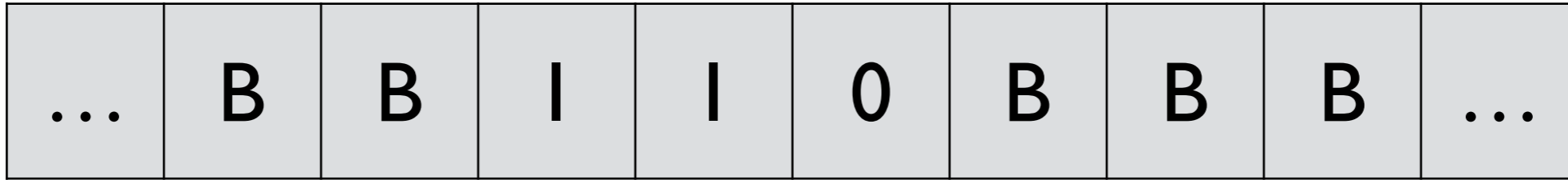
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



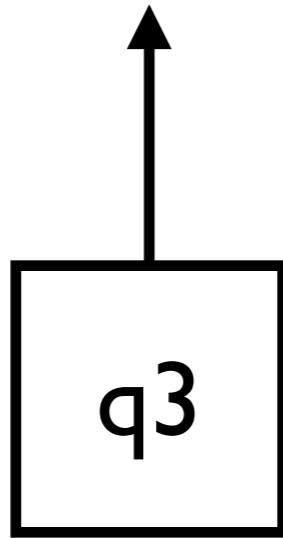
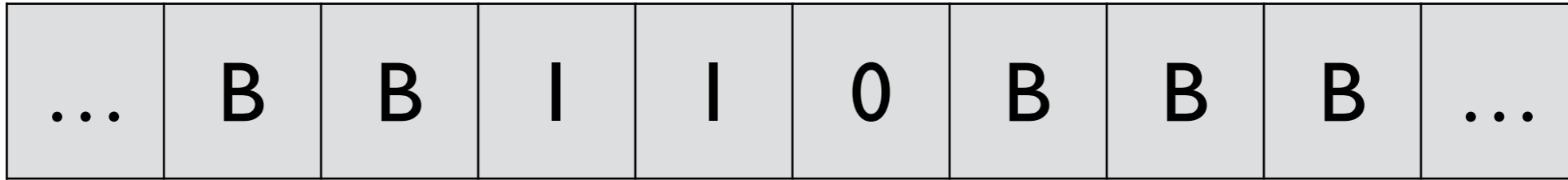
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



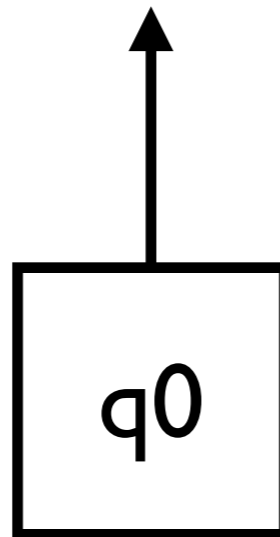
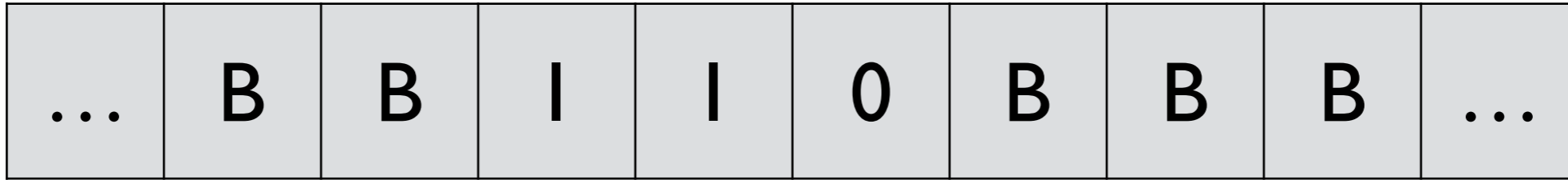
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



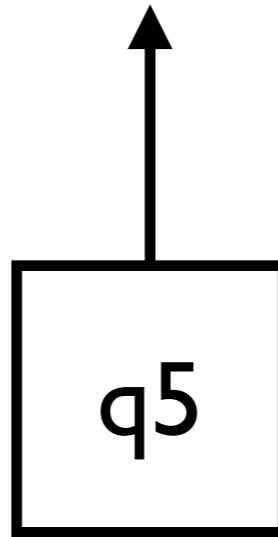
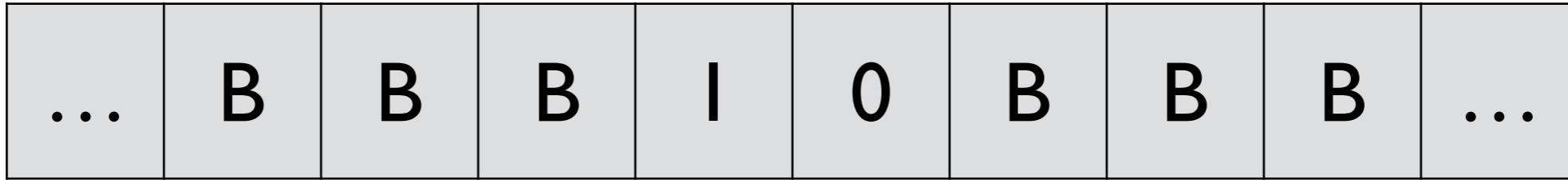
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



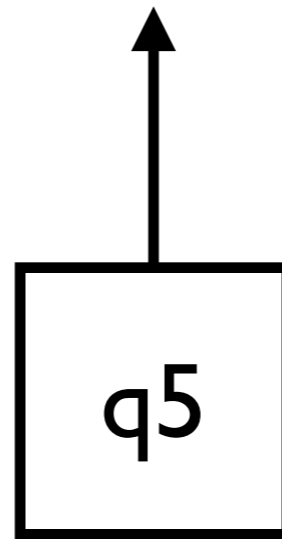
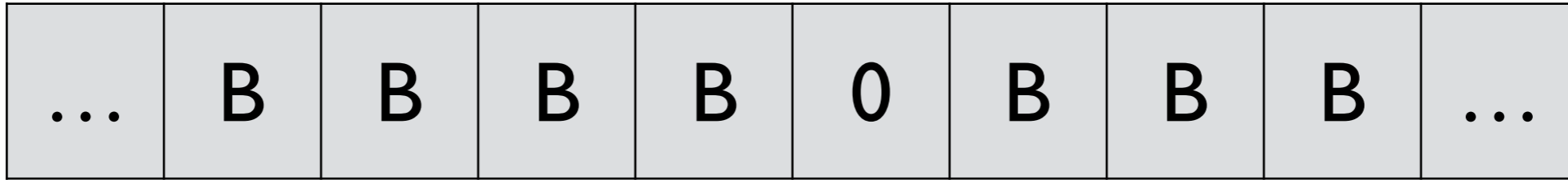
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

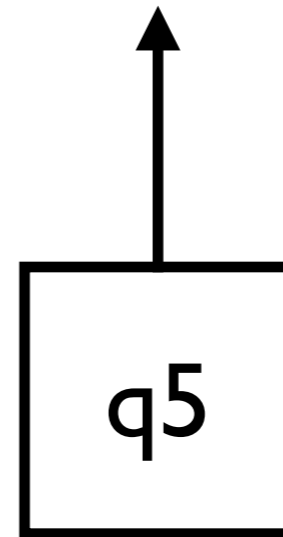
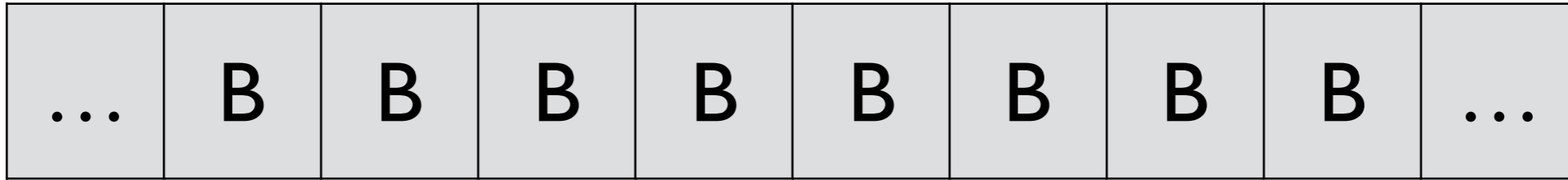


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

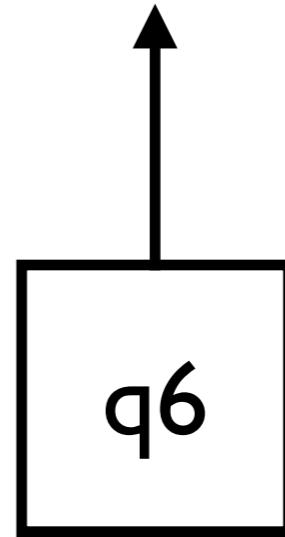
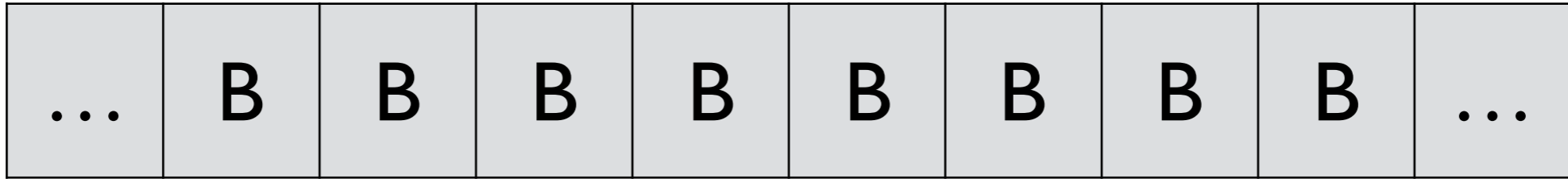


	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			





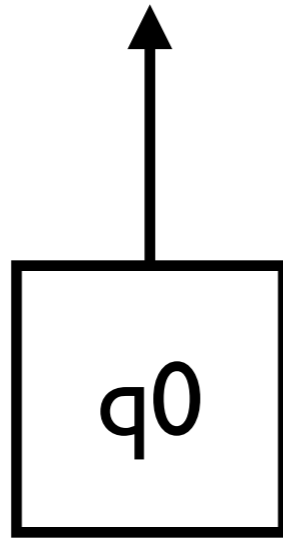
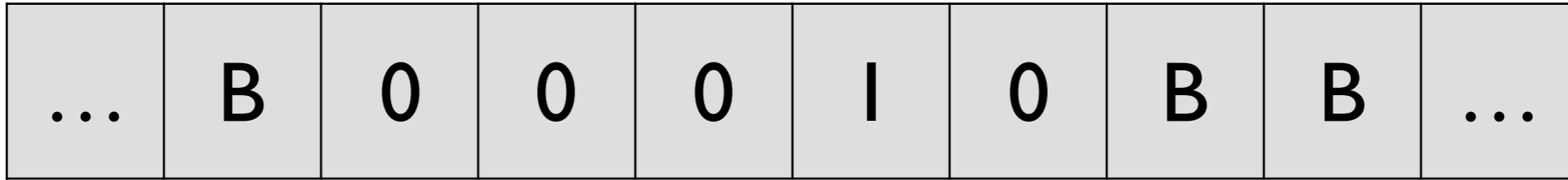
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

**Example, revisited**

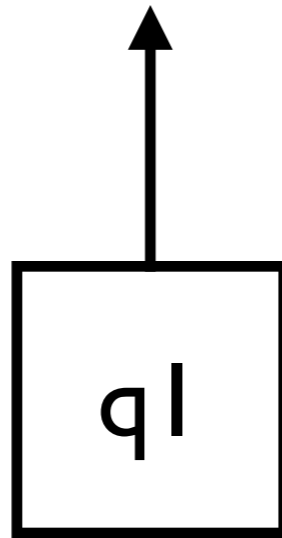
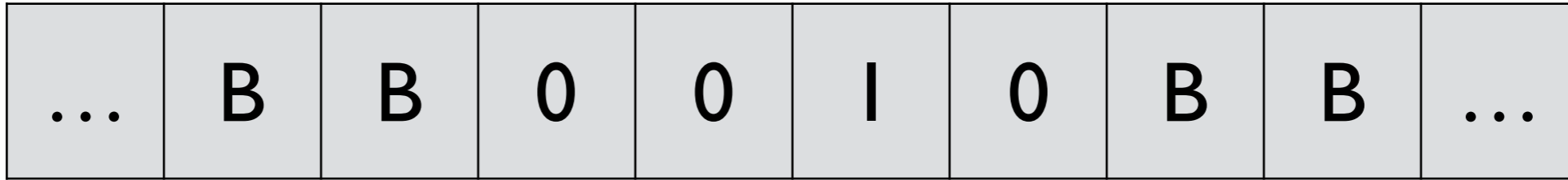
e.g.,  $f(3, 1)$



Scanned 0 in  $q_0$ , the cycle must repeat:

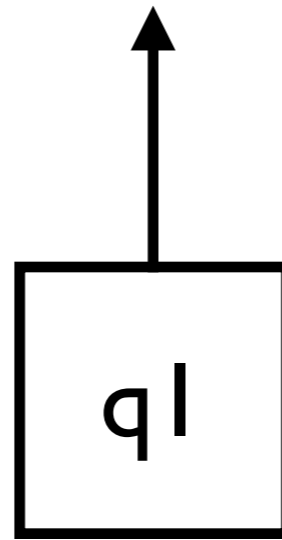
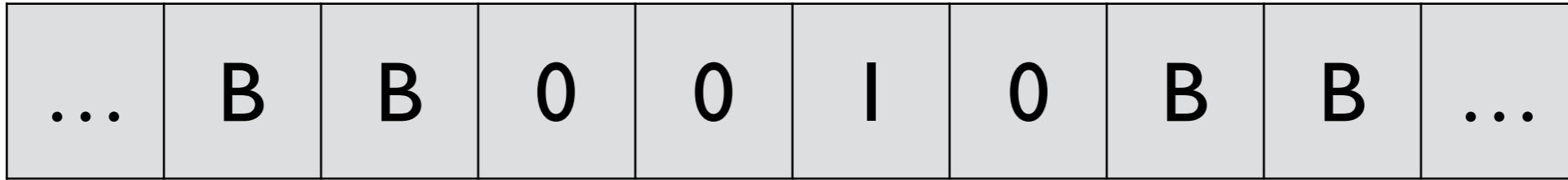
1. replace 0 by B
2. move right
3. enter  $q_1$

	0	1	B
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



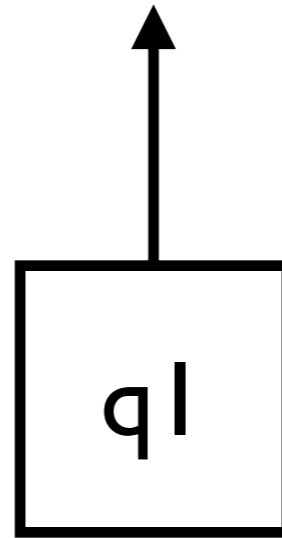
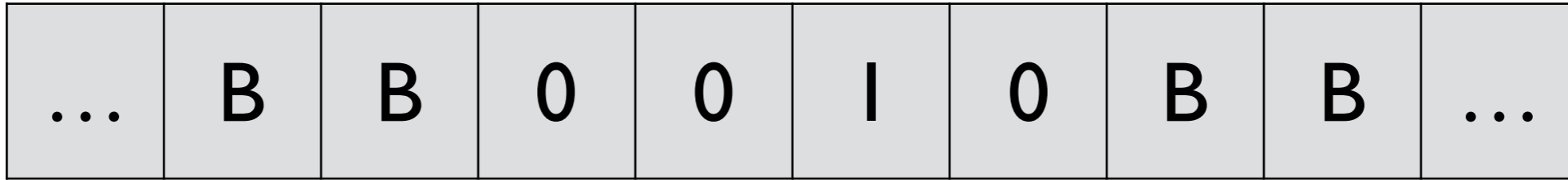
In  $q_1$ , search right,  
looking for leftmost 1

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



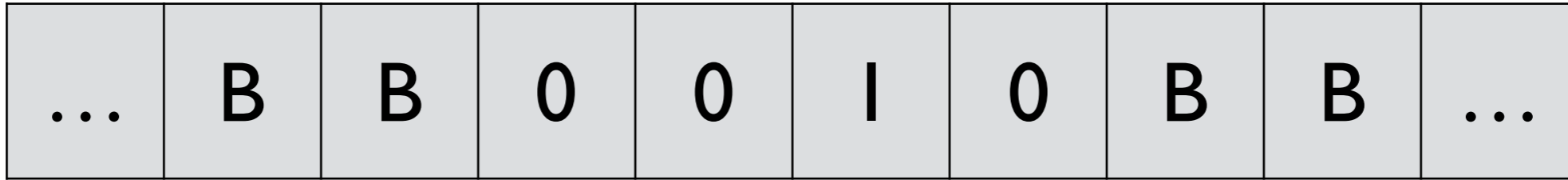
In  $q_1$ , search right,  
looking for leftmost 1

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

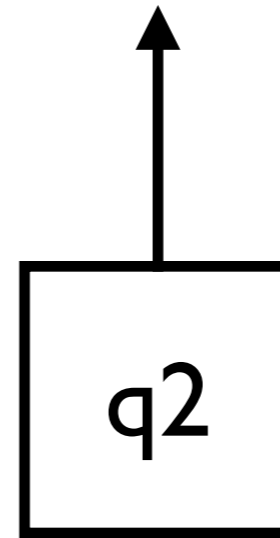


When found, enter  $q_2$

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

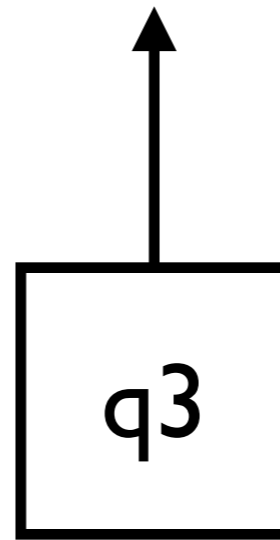
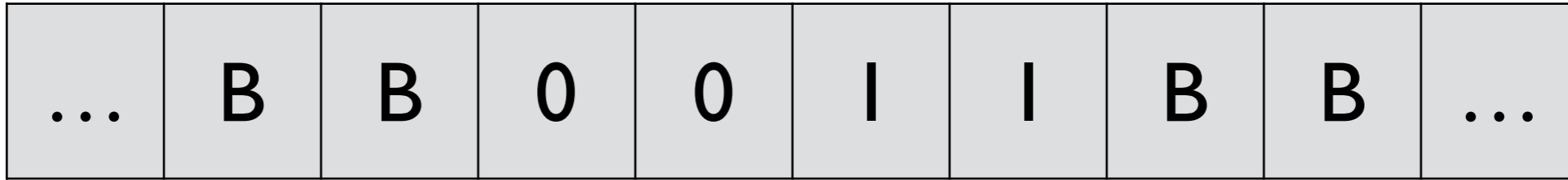


- In  $q_2$ , move right until it finds 0
- When found, change it by 1
- enter  $q_3$



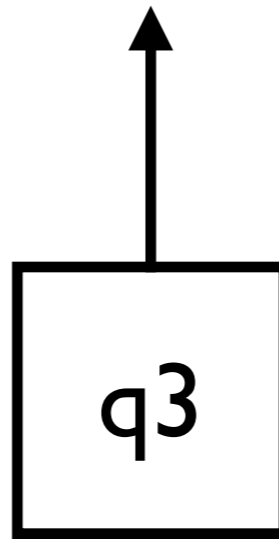
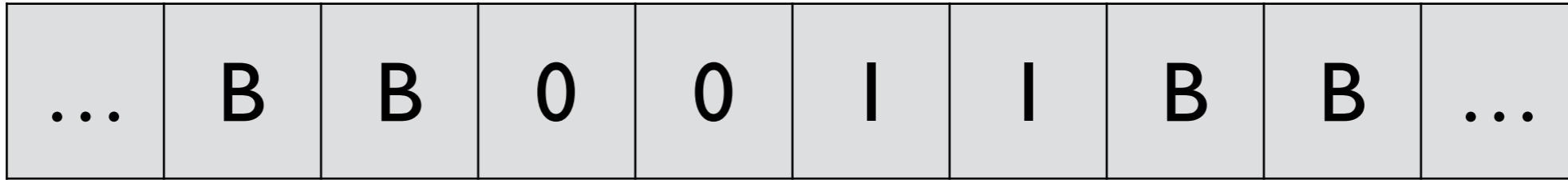
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



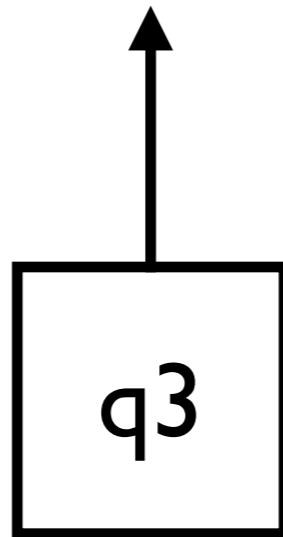
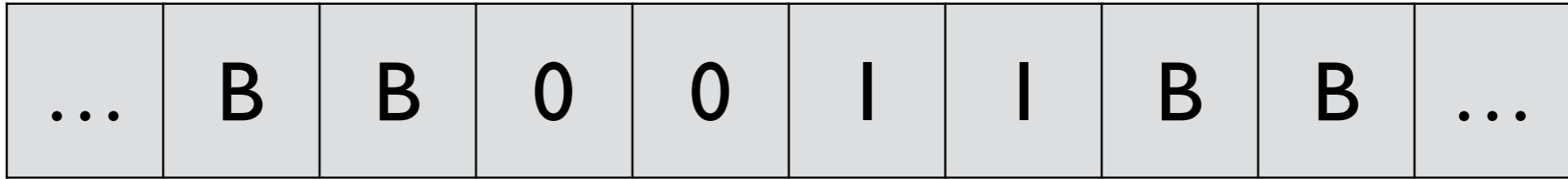


In  $q_3$ , move left until it find B

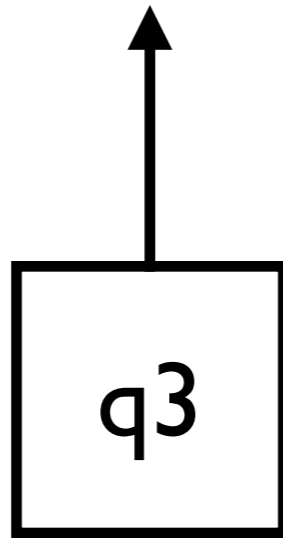
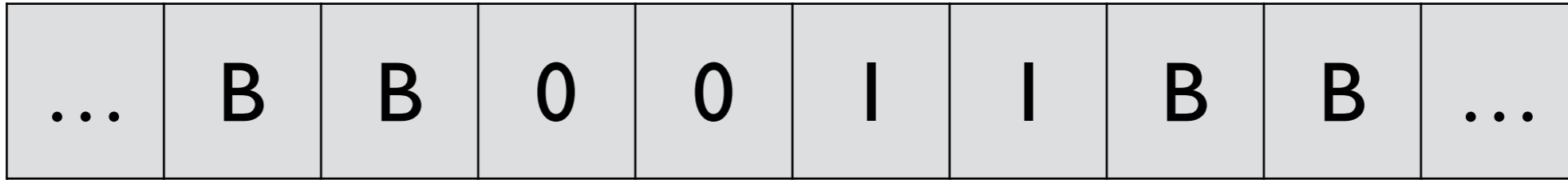
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

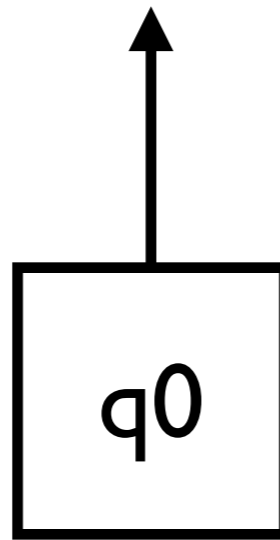
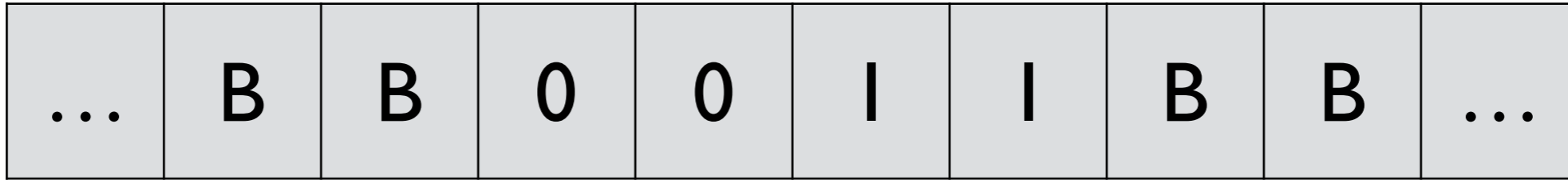


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



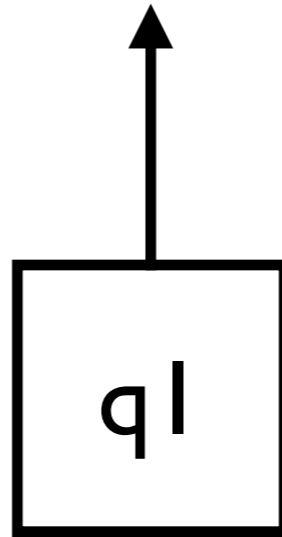
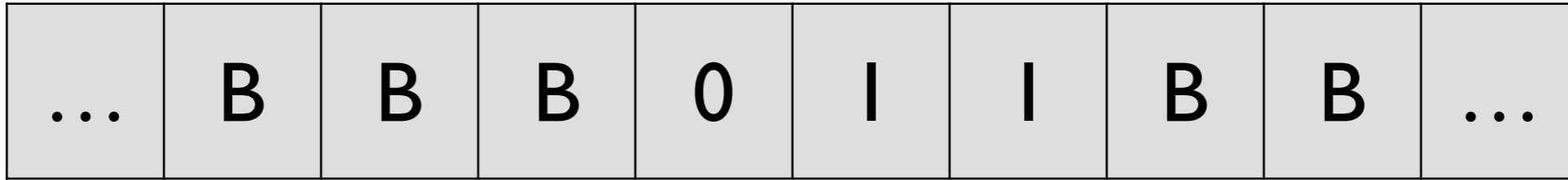
When B found, enter  $q_0$  and begin the cycle

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



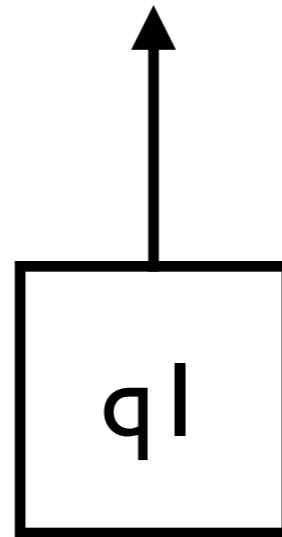
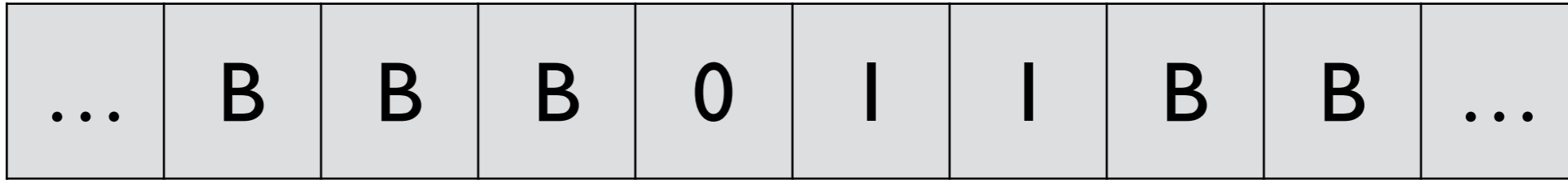
Begin a new cycle.

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



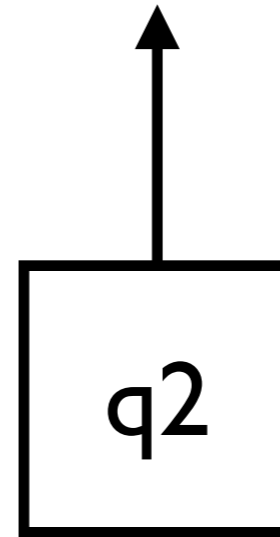
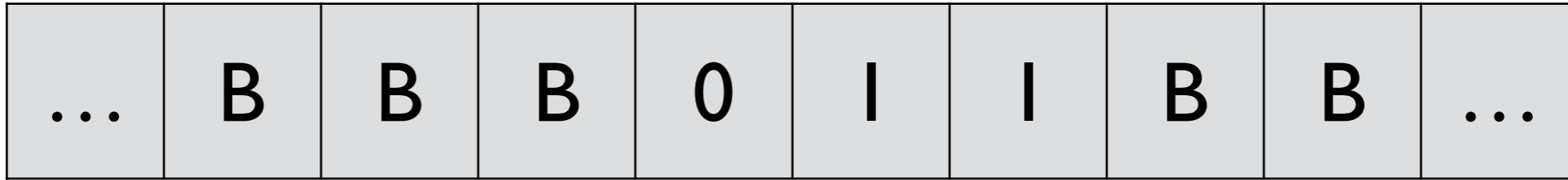
look for leftmost 1

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



When found, enter  $q_2$

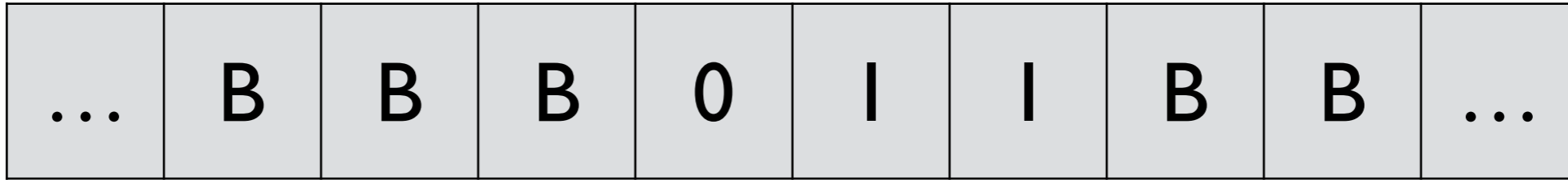
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



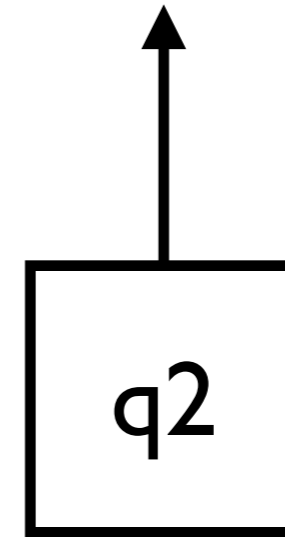
move right until it finds 0

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

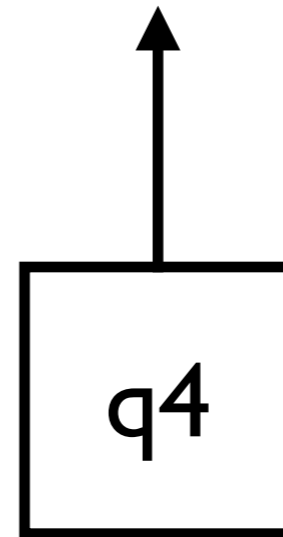
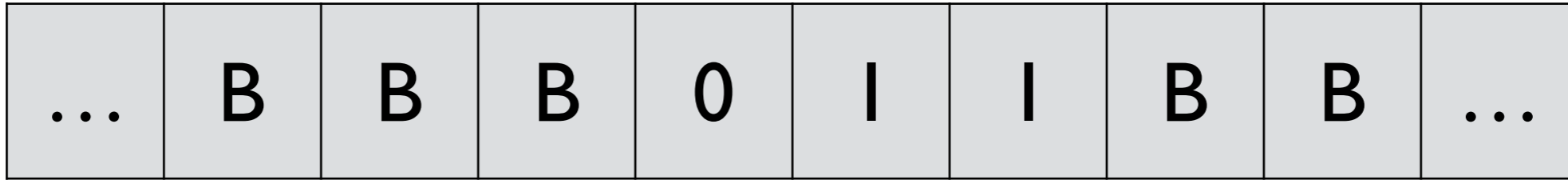




- No more 0's.
- All  $n$  0's changed to 1
- $n+1$  0's changed to B
- $m-(n+1)$  0's on the tape
- replace all 1's by B and put one 0

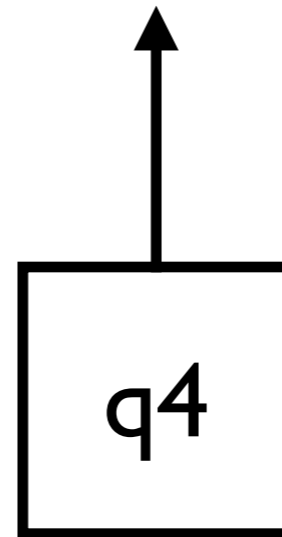
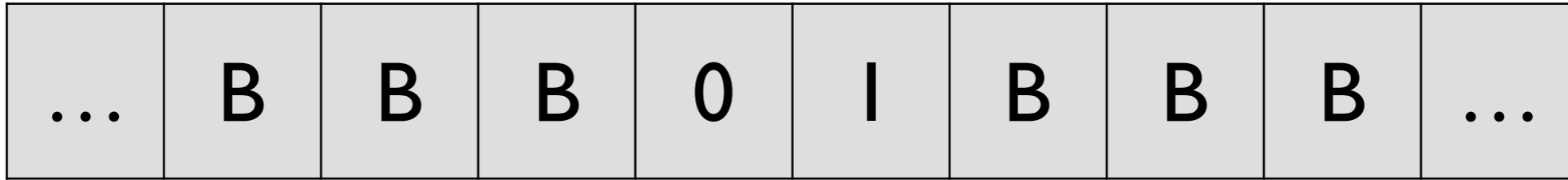


	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

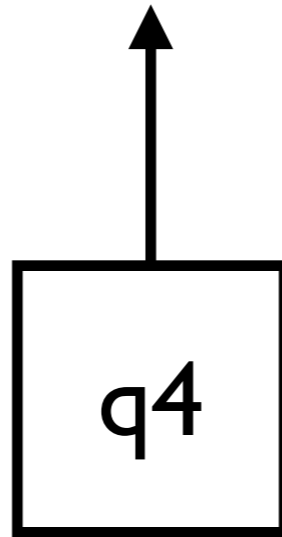
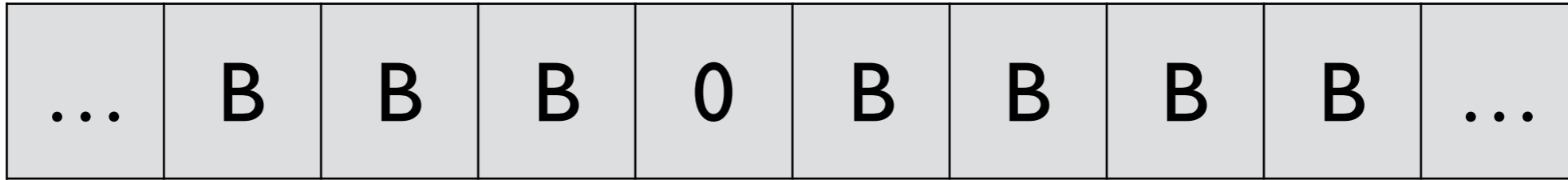


move left, changing I by B, until it finds B

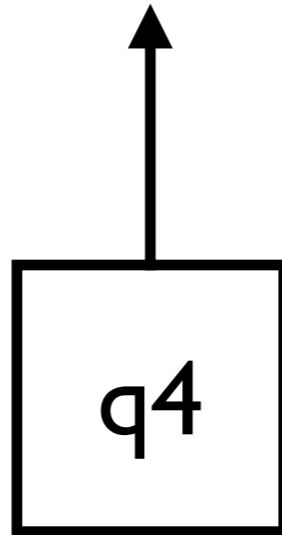
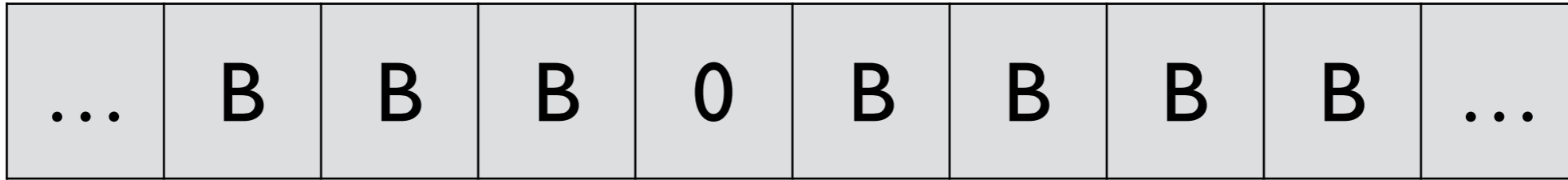
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	$0$	$1$	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

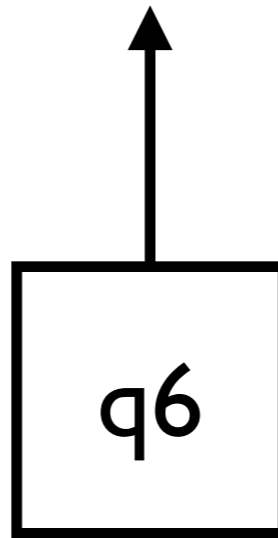
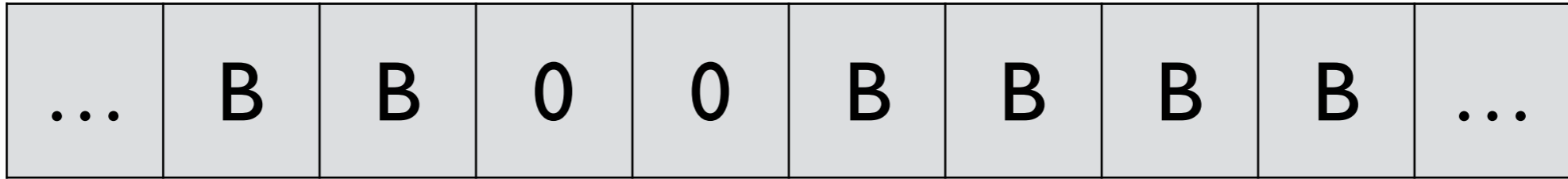


	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



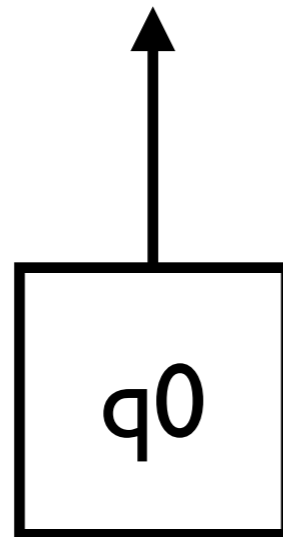
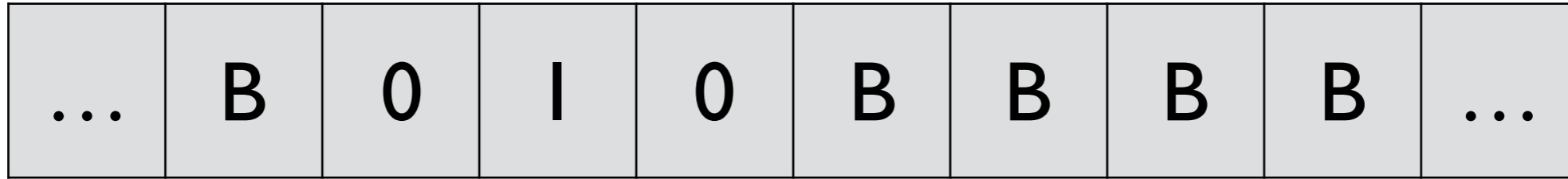
- change that B by 0
- enter the final state

	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



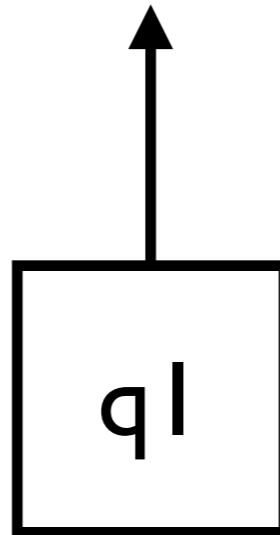
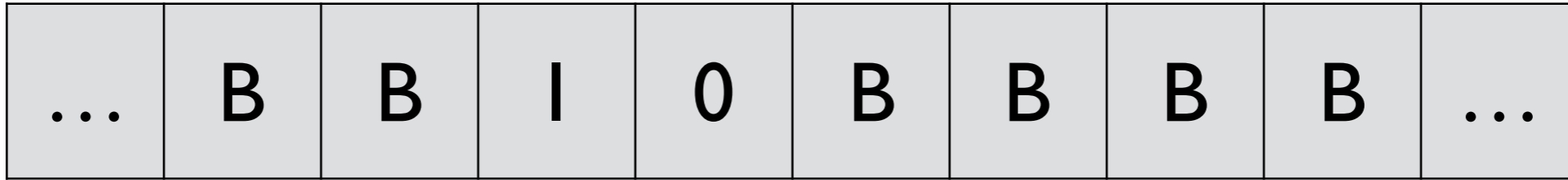
	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			

e.g.,  $f(1, 1)$



- change 0 by B
- enter q1

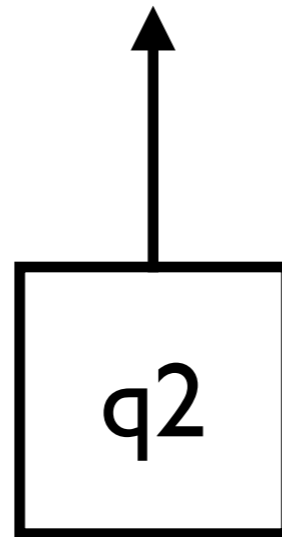
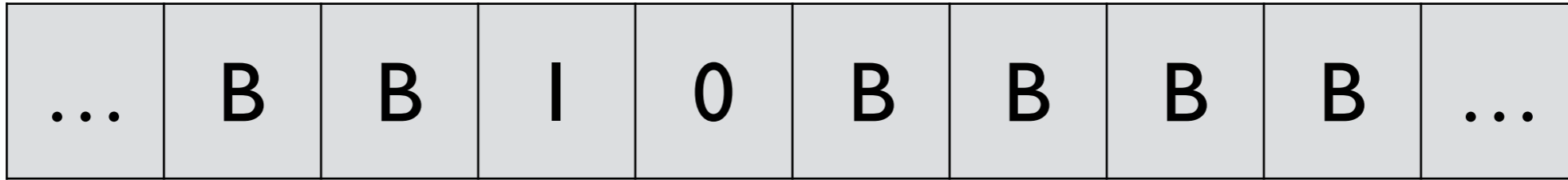
	0	1	<i>B</i>
<i>q</i> <sub>0</sub>	( <i>q</i> <sub>1</sub> , <i>B</i> , <i>R</i> )	( <i>q</i> <sub>5</sub> , <i>B</i> , <i>R</i> )	
<i>q</i> <sub>1</sub>	( <i>q</i> <sub>1</sub> , 0, <i>R</i> )	( <i>q</i> <sub>2</sub> , 1, <i>R</i> )	
<i>q</i> <sub>2</sub>	( <i>q</i> <sub>3</sub> , 1, <i>L</i> )	( <i>q</i> <sub>2</sub> , 1, <i>R</i> )	( <i>q</i> <sub>4</sub> , <i>B</i> , <i>L</i> )
<i>q</i> <sub>3</sub>	( <i>q</i> <sub>3</sub> , 0, <i>L</i> )	( <i>q</i> <sub>3</sub> , 1, <i>L</i> )	( <i>q</i> <sub>0</sub> , <i>B</i> , <i>R</i> )
<i>q</i> <sub>4</sub>	( <i>q</i> <sub>4</sub> , 0, <i>L</i> )	( <i>q</i> <sub>4</sub> , <i>B</i> , <i>L</i> )	( <i>q</i> <sub>6</sub> , 0, <i>R</i> )
<i>q</i> <sub>5</sub>	( <i>q</i> <sub>5</sub> , <i>B</i> , <i>R</i> )	( <i>q</i> <sub>5</sub> , <i>B</i> , <i>R</i> )	( <i>q</i> <sub>6</sub> , <i>B</i> , <i>R</i> )
<i>q</i> <sub>6</sub>			



- look for the leftmost I
- when found, enter q2

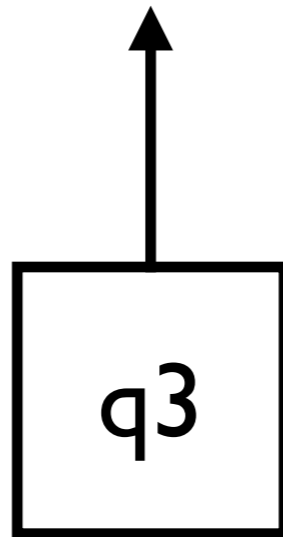
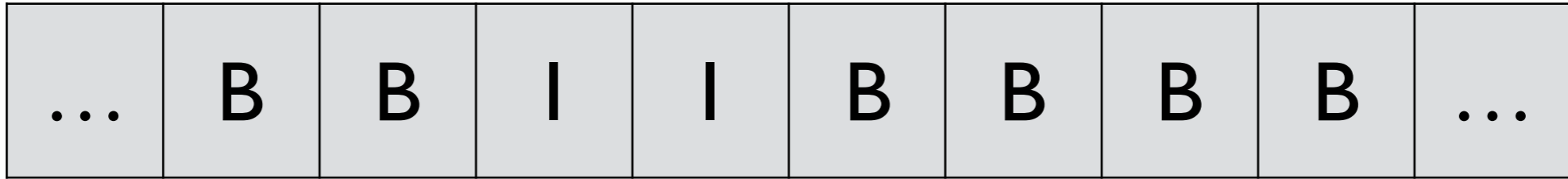
	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			





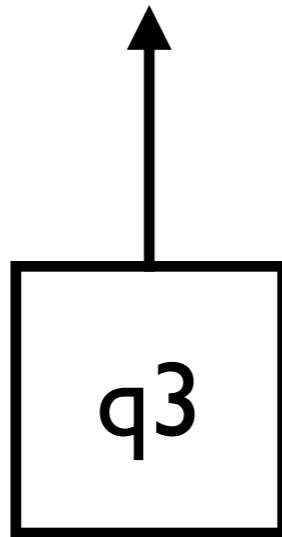
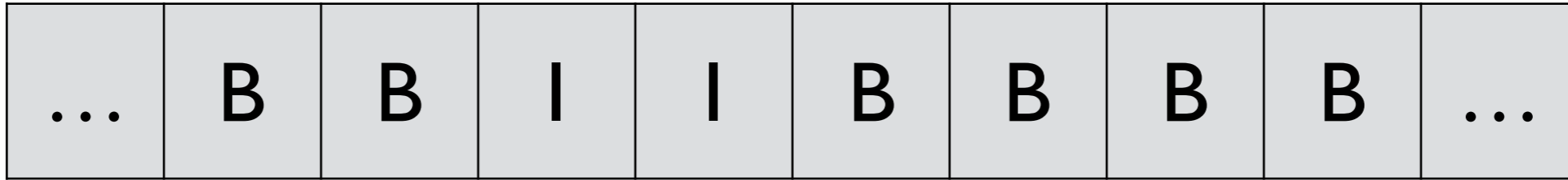
- move right until it finds 0
- when found, that 0 by 1
- enter q3

	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



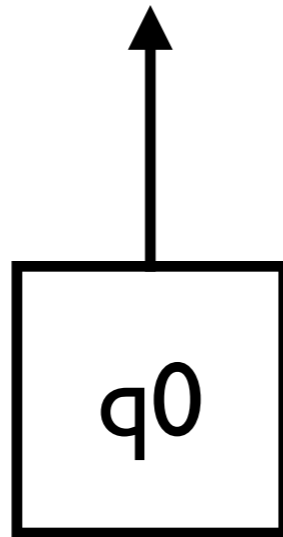
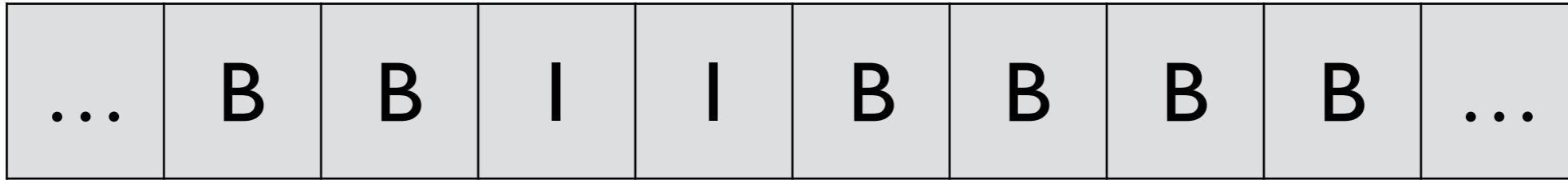
move left until it finds B

	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



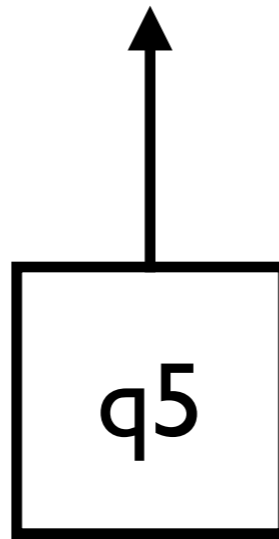
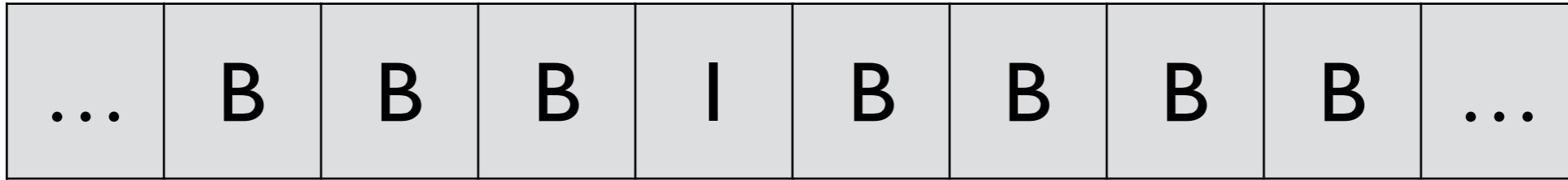
when B found, begin the cycle

	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



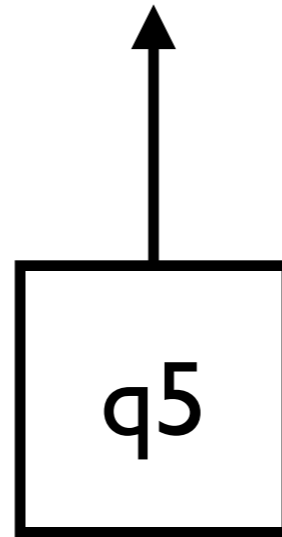
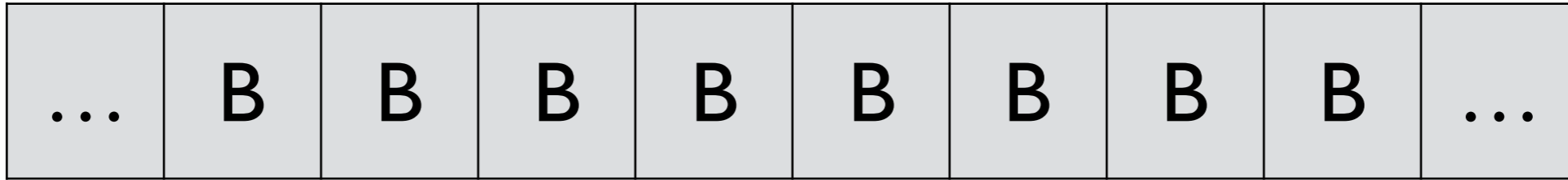
- cannot find 0 in  $q_0$
- $n \geq m$
- replace I by B
- enter  $q_5$

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



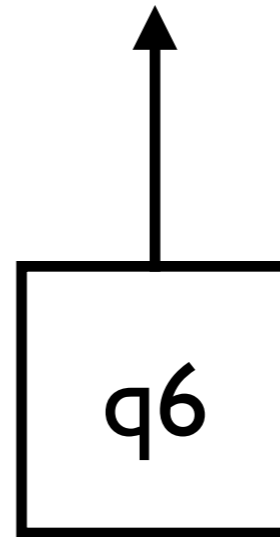
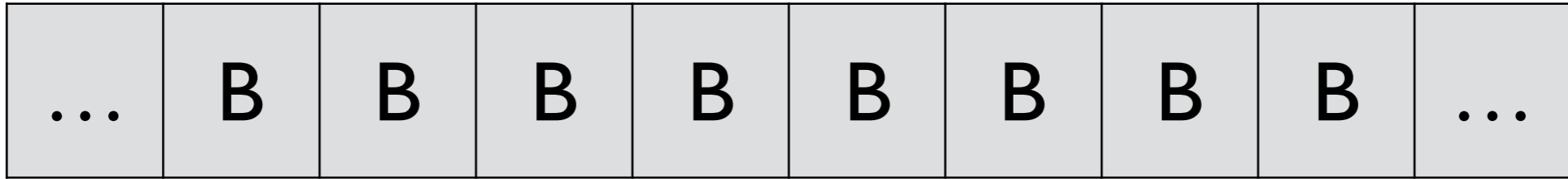
In  $q_5$ , change all 0's and 1's to B

	0	1	$B$
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



enter q6

	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			



	0	1	<i>B</i>
$q_0$	$(q_1, B, R)$	$(q_5, B, R)$	
$q_1$	$(q_1, 0, R)$	$(q_2, 1, R)$	
$q_2$	$(q_3, 1, L)$	$(q_2, 1, R)$	$(q_4, B, L)$
$q_3$	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_0, B, R)$
$q_4$	$(q_4, 0, L)$	$(q_4, B, L)$	$(q_6, 0, R)$
$q_5$	$(q_5, B, R)$	$(q_5, B, R)$	$(q_6, B, R)$
$q_6$			