Homework 2
COSE215, Spring 2017

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Due: 4/17 (Mon), 09:00 (in class)

Problem 1 (10pts) Consider the following *extended* regular expression:

$$\begin{array}{rcl} R & \rightarrow & \emptyset \\ & | & \epsilon \\ & | & a \in \Sigma \\ & | & R_1 + R_2 \\ & | & R_1 R_2 \\ & | & R^* \\ & | & R^+ \\ & | & R? \\ & | & (R) \end{array}$$

The semantics of the regular expression is defined as follows:

$$\begin{array}{rcl} L(\emptyset) & = & \emptyset \\ L(\epsilon) & = & \{\epsilon\} \\ L(a) & = & \{a\} \\ L(R_1 + R_2) & = & L(R_1) \cup L(R_2) \\ L(R_1 R_2) & = & L(R_1)L(R_2) \\ L(R^*) & = & (L(R))^* \\ L(R^+) & = & L(R)L(R^*) \\ L(R?) & = & \{\epsilon\} \cup L(R) \\ L((R)) & = & L(R) \end{array}$$

Evaluate the following regular expressions according to the semantics. Show the full evaluation sequences. In all cases, assume $\Sigma = \{a, b\}$.

1. $L((a + b)^+ a)$
2. $L(((a + b)?)^*)$

Problem 2 (25pts, 5pts each) Find regular expressions for the following languages and explain why.

1. $L = \{w \in \{a, b, c\}^* \mid w$ contains at least one a and at least one $b\}$
2. $L = \{w \in \{0, 1\}^* \mid$ The fifth symbol of w from the right end is 1 $\}$
3. $L = \{w \in \{0, 1\}^* \mid$ the number of 0's in w is divisible by three$\}$
4. $L = \{a^n b^m \mid n \geq 1, m \geq 1, nm \geq 3\}$
5. $L = \{w \in \{0, 1\}^* \mid w$ has exactly one pair of consecutive zeros$\}$

Problem 3 (10pts) Convert the following regular expressions to finite automata (ϵ-NFA):

1. $ab^*aa + bba^*ab$
2. $(a + b)^*b(a + bb)^*$

Problem 4 (10pts) Find an ϵ-NFA that accepts language $L(ab^*a^*) \cap L(a^*b^*a)$.

Problem 5 (15pts) Suppose h is a homomorphism. Are the following statements true? If so, prove it, otherwise give a counter-example.

1. $h(L_1 \cup L_2) = h(L_1) \cup h(L_2)$
2. $h(L_1 \cap L_2) = h(L_1) \cap h(L_2)$
3. $h(L_1 L_2) = h(L_1)h(L_2)$

Problem 6 (30pts) Use the pumping lemma and prove that the following languages are not regular.

1. $L = \{0^i \mid i$ is a prime$\}$:
2. $L = \{ww \mid w \in \{0, 1\}^*\}$
3. $L = \{w \in \{a, b\}^* \mid n_a(w) = n_b(w)\}$ ($n_a(w)$ and $n_b(w)$ denote the number of a's and b's in w, respectively)