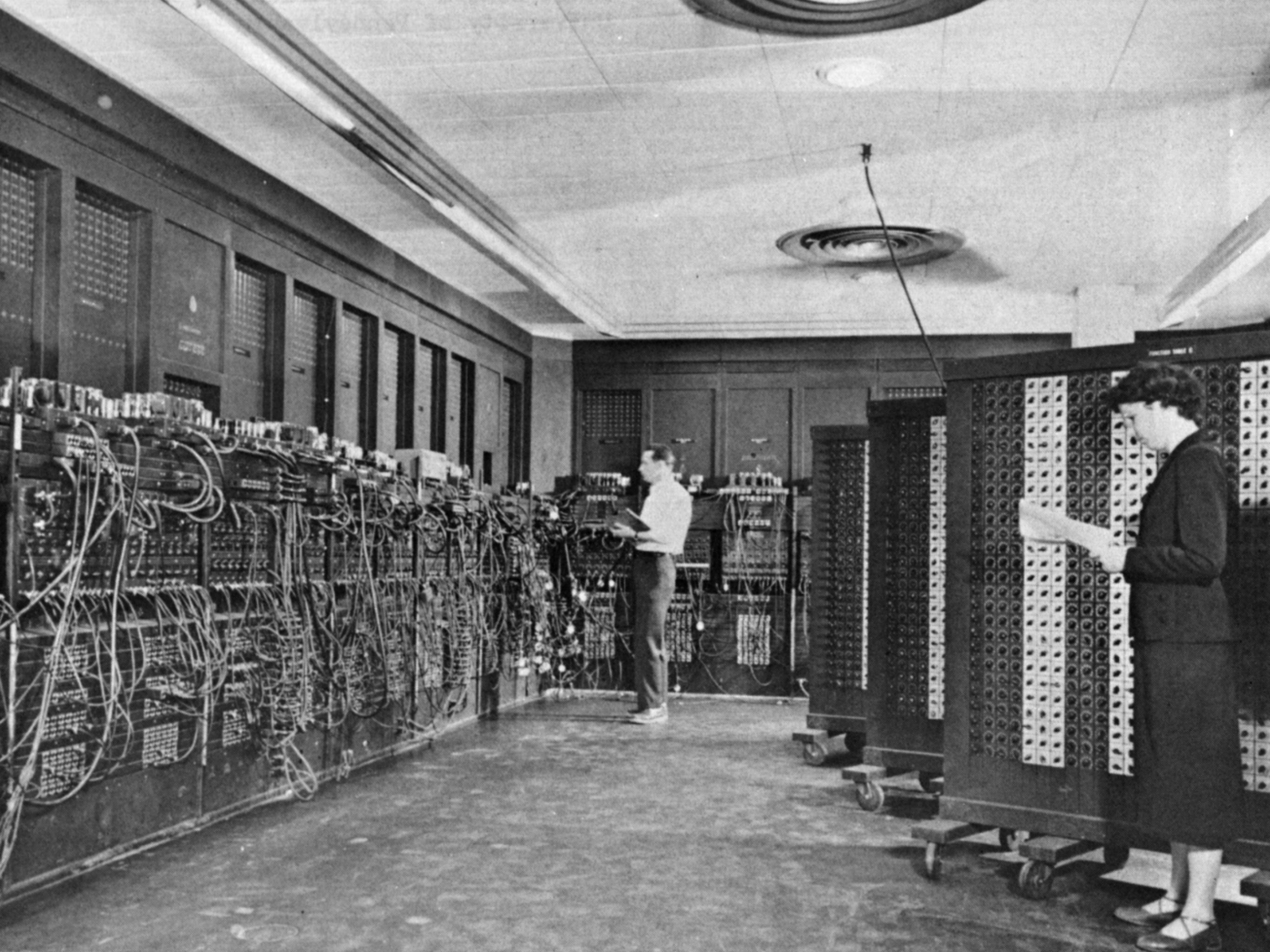
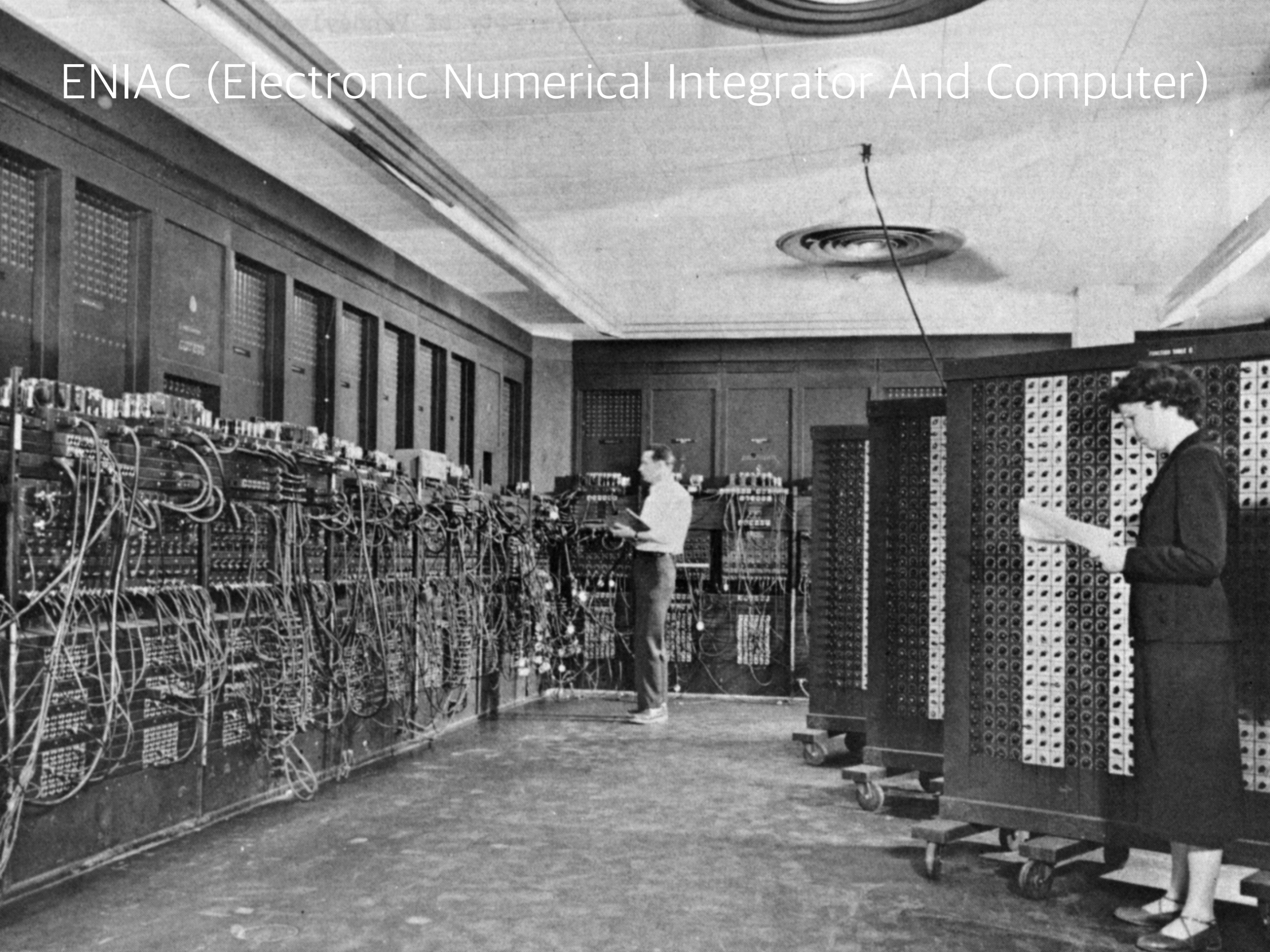


# Review

- Automata theory
  - Finite automata
  - Pushdown automata
  - Turing machines
- Computability
  - Computable
  - Incomputable problems



# ENIAC (Electronic Numerical Integrator And Computer)



# ENIAC (Electronic Numerical Integrator And Computer)

1 x 2.5m x 25m

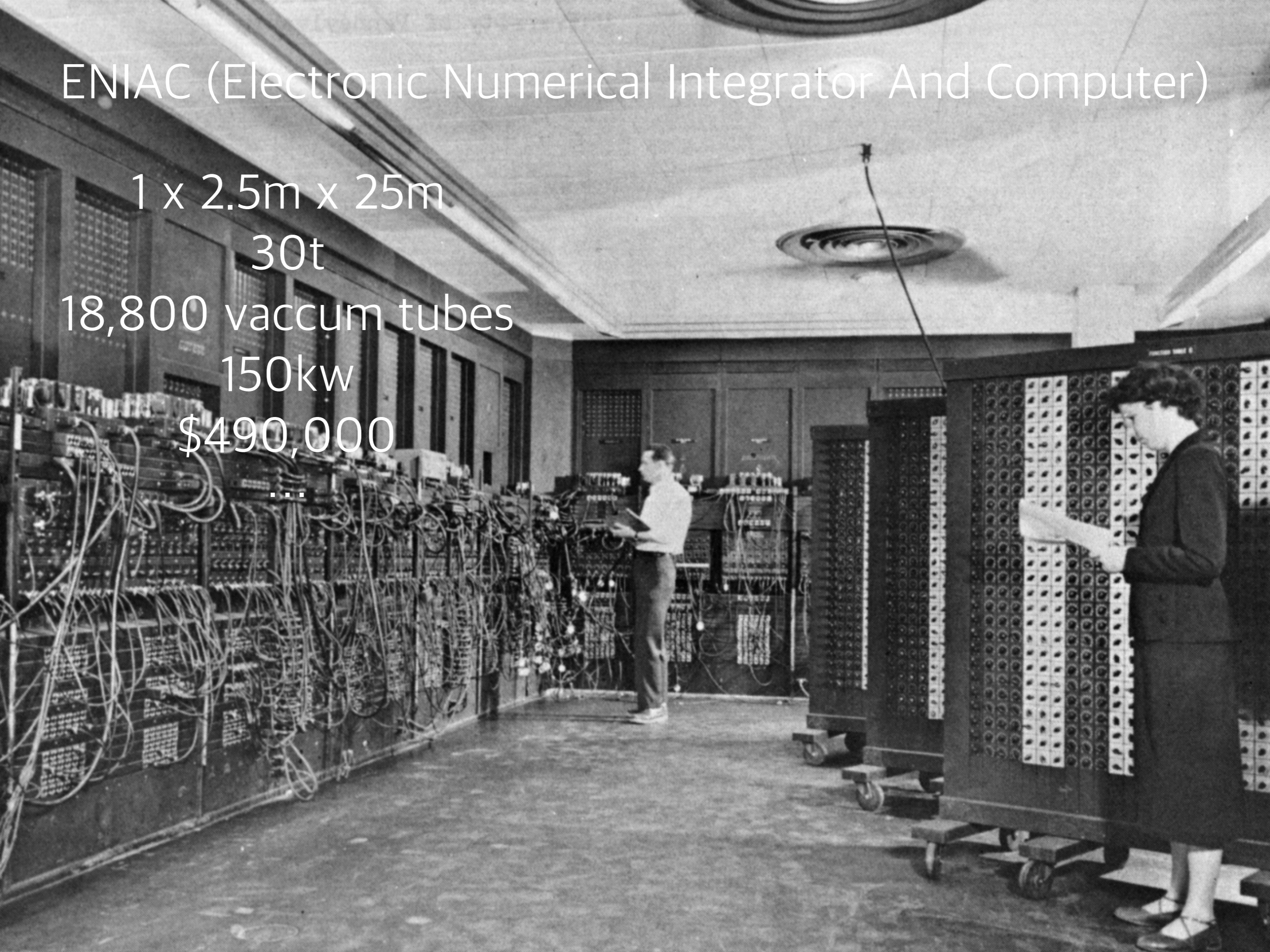
30t

18,800 vacuum tubes

150kw

\$490,000

...



# ENIAC (Electronic Numerical Integrator And Computer)

1 x 2.5m x 25m

30t

18,800 vacuum tubes

150kw

\$490,000

...

Lots of engineering



ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO  
THE ENTSCHIEDUNGSPROBLEM

By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

The “computable” numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth. The fundamental problems involved are, however, the same in each case, and I have chosen the computable numbers for explicit treatment as involving the least cumbersome technique. I hope shortly to give an account of the relations of the computable numbers, functions, and so forth to one another. This will include a development of the theory of functions of a real variable expressed in terms of computable numbers. According to my definition, a number is computable if its decimal can be written down by a machine.

In §§ 9, 10 I give some arguments with the intention of showing that the computable numbers include all numbers which could naturally be regarded as computable. In particular, I show that certain large classes of numbers are computable. They include, for instance, the real parts of all algebraic numbers, the real parts of the zeros of the Bessel functions, the numbers  $\pi$ ,  $e$ , etc. The computable numbers do not, however, include all definable numbers, and an example is given of a definable number which is not computable.

Although the class of computable numbers is so great, and in many ways similar to the class of real numbers, it is nevertheless enumerable. In § 8 I examine certain arguments which would seem to prove the contrary. By the correct application of one of these arguments, conclusions are reached which are superficially similar to those of Gödel†. These results

† Gödel, “Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I”, *Monatshefte Math. Phys.*, 38 (1931), 173–198.

## foundations of computing

- what computing is
- what computers can do and cannot do





EDVAC

45.5m<sup>2</sup>

6,000 vaccum tubes

150kw

\$100,000

5.5KB memory

1~3sec / +x



EDVAC

45.5m<sup>2</sup>

6,000 vacuum tubes

150kw

\$100,000

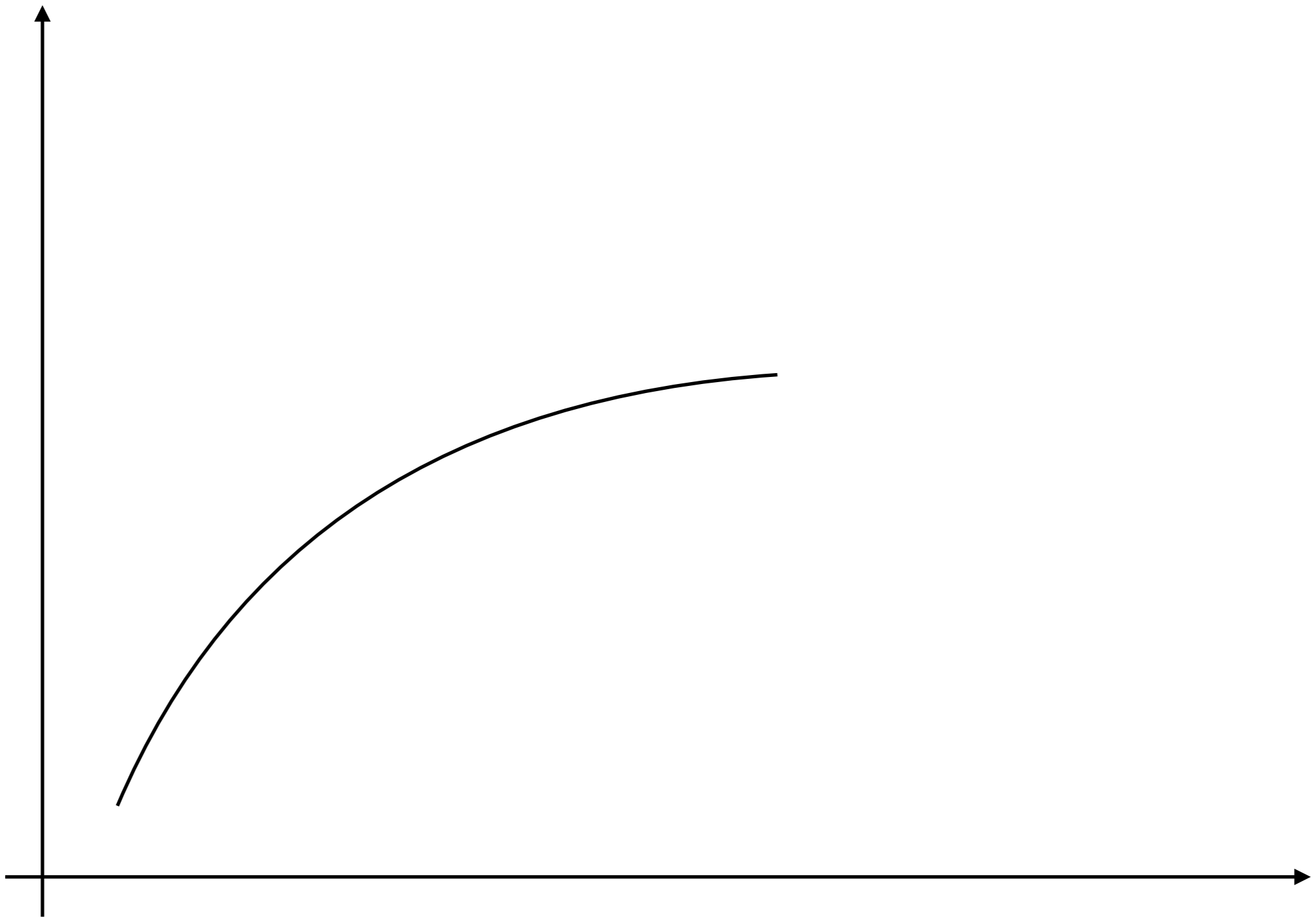
5.5KB memory

1~3sec / +x

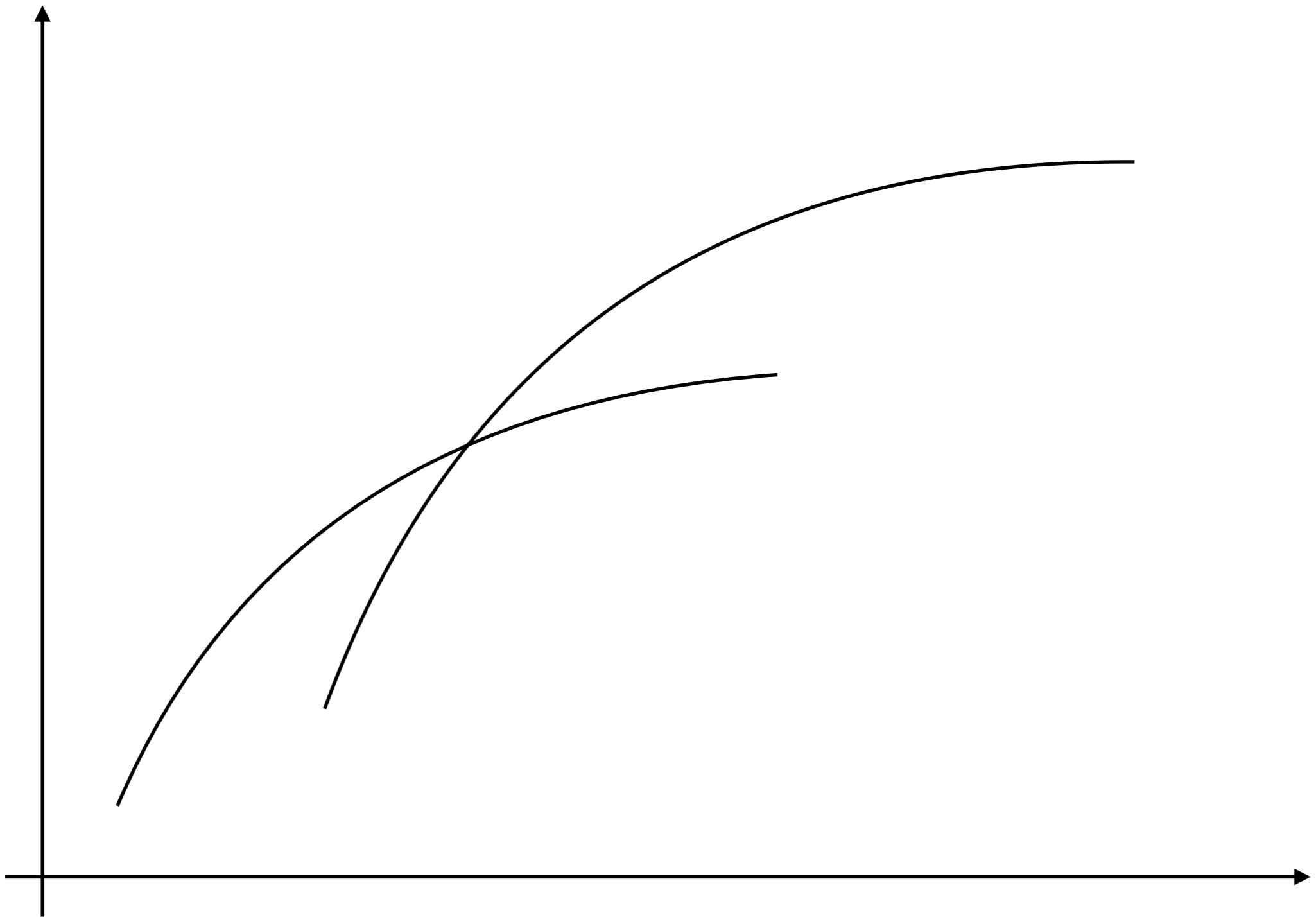
**first stored program  
(universal) computer**



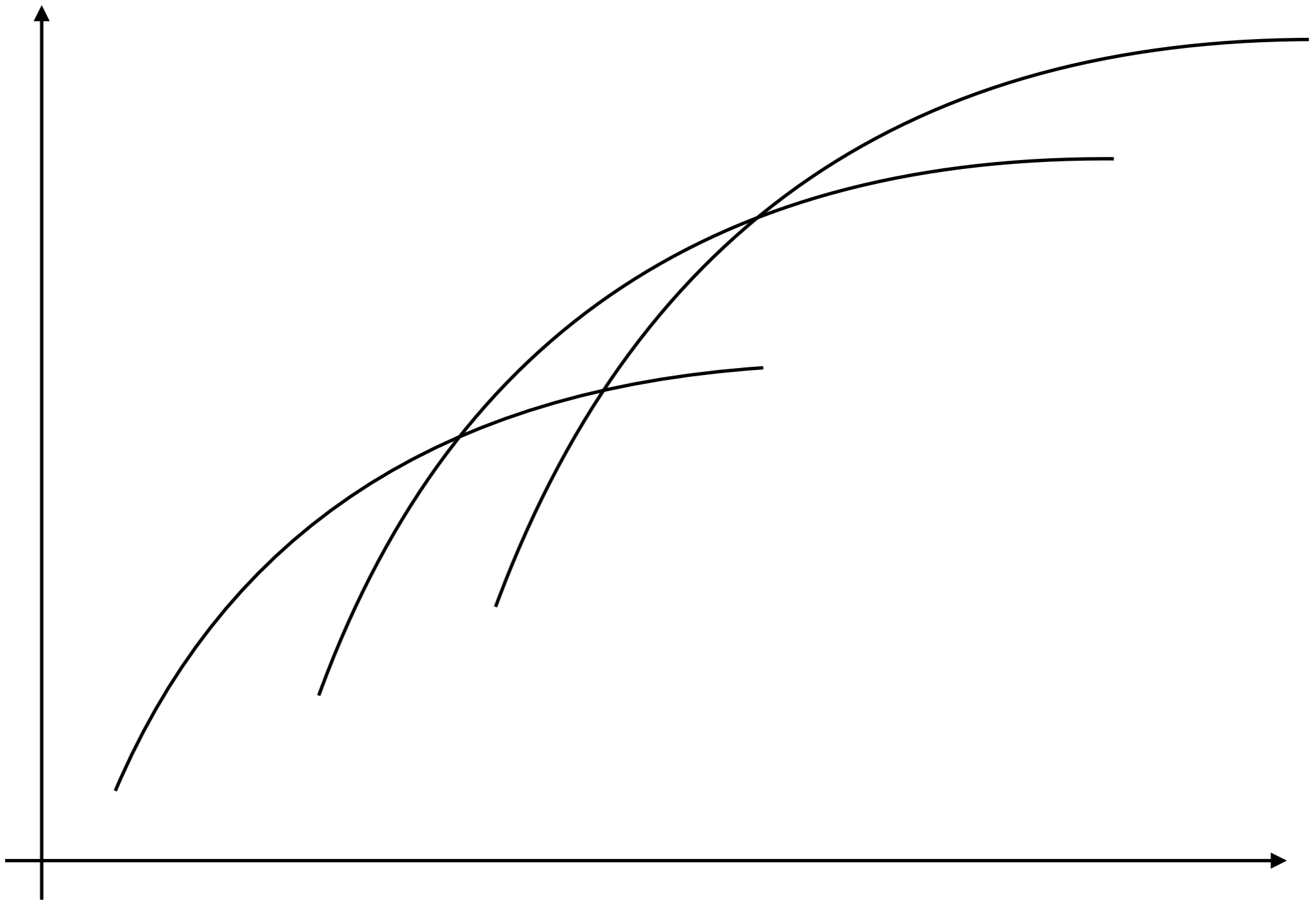
# Advances of technologies



# Advances of technologies



# Advances of technologies



# Mathematics & Logics

Algorithm

The diagram consists of two vertical rectangular blocks, one green on the left and one red on the right. The green block contains the word 'Algorithm' and the red block contains the word 'Language'. Below these two blocks is a blue curved shape that contains the text 'Mathematics & Logics'. The entire diagram is set against a white background.

Language

Mathematics & Logics

The diagram is a stylized house. At the base is a blue semi-circle labeled 'Mathematics & Logics'. Two vertical pillars support a yellow triangular roof. The left pillar is green and labeled 'Algorithm'. The right pillar is red and labeled 'Language'. The roof is labeled 'SW techs'.

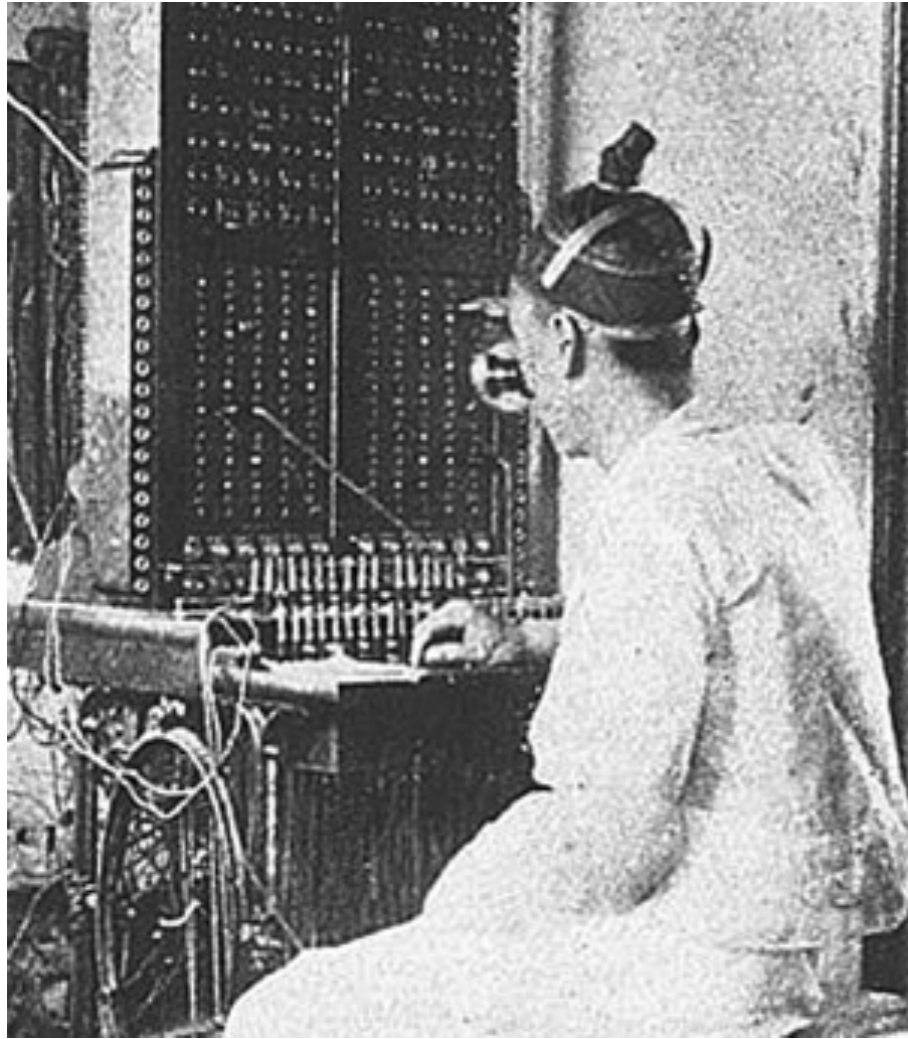
SW techs

Algorithm

Language

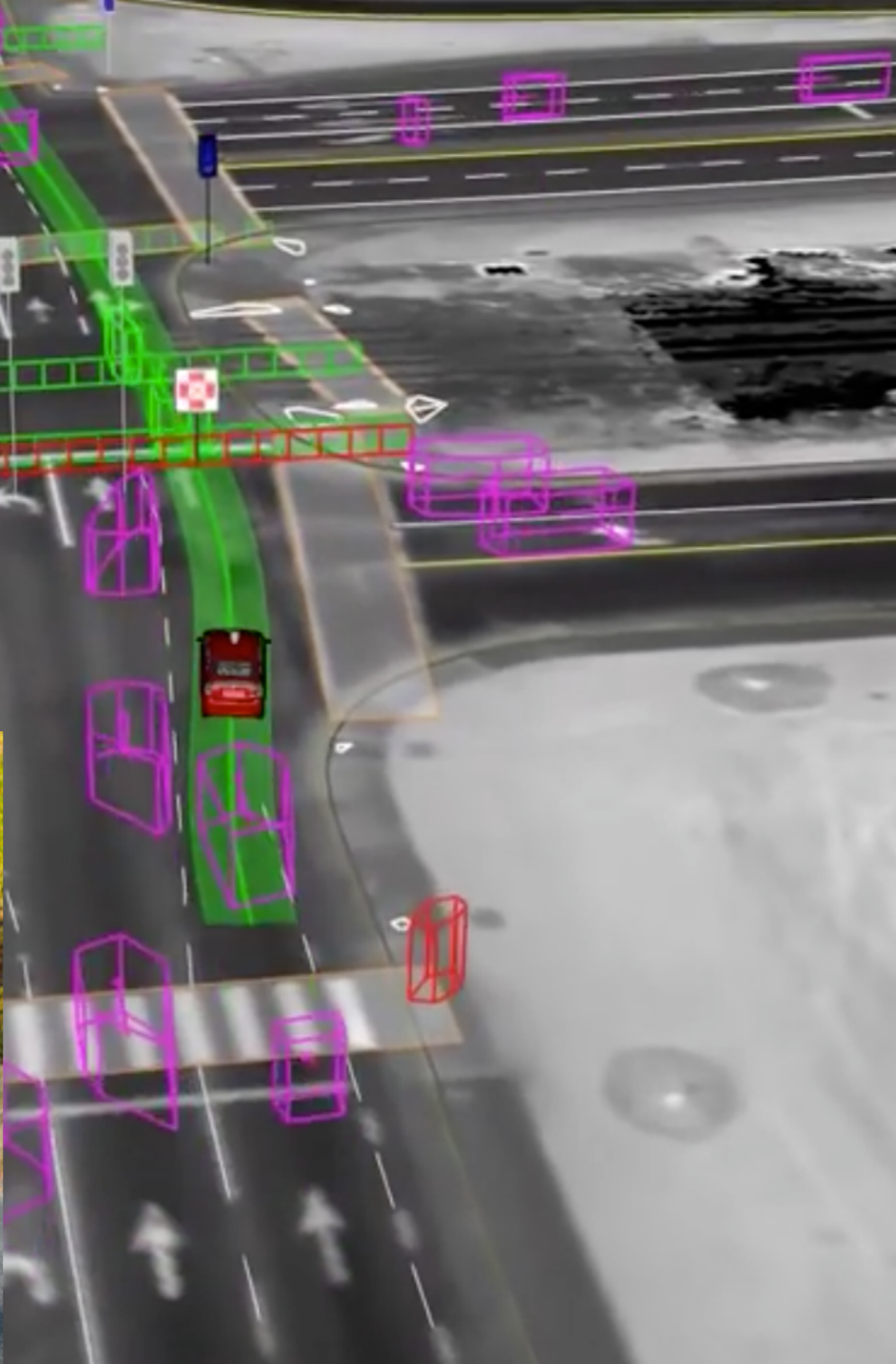
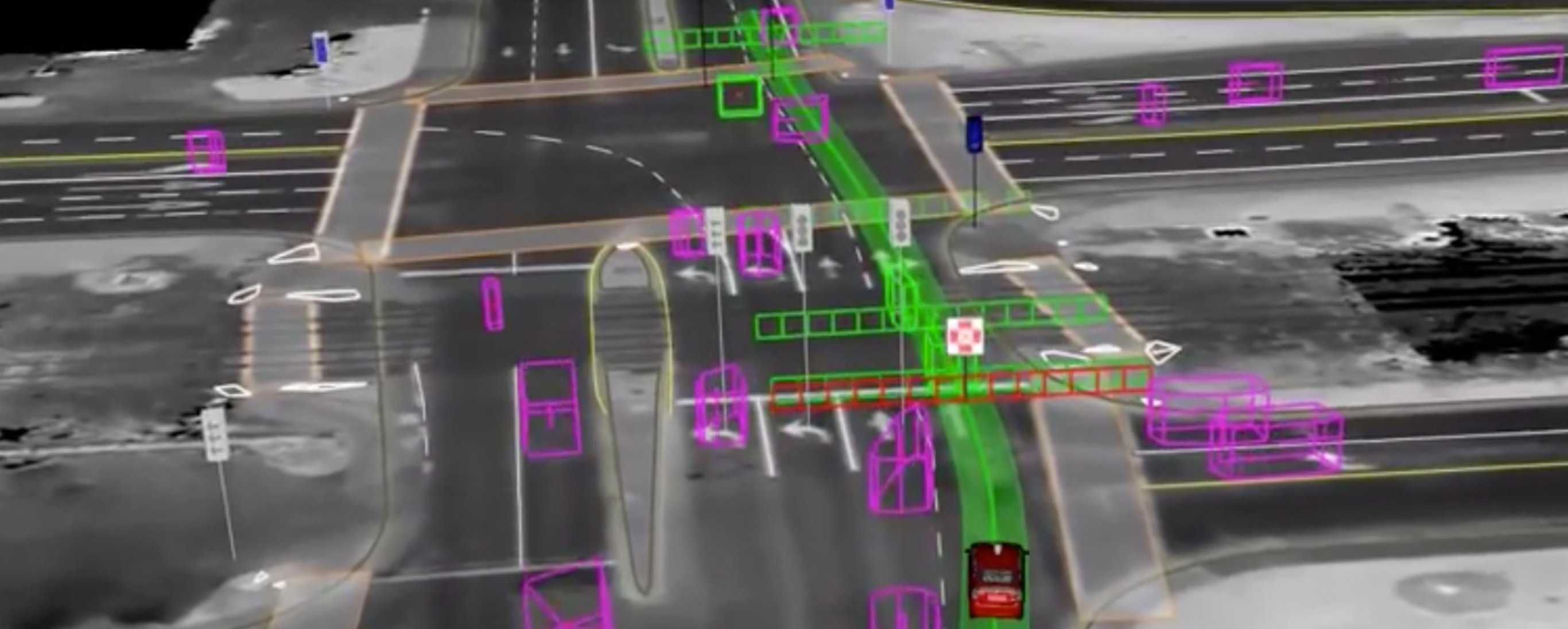
Mathematics & Logics

# Human vs. Computer











AlphaGo

Lee Sedol

AlphaGo

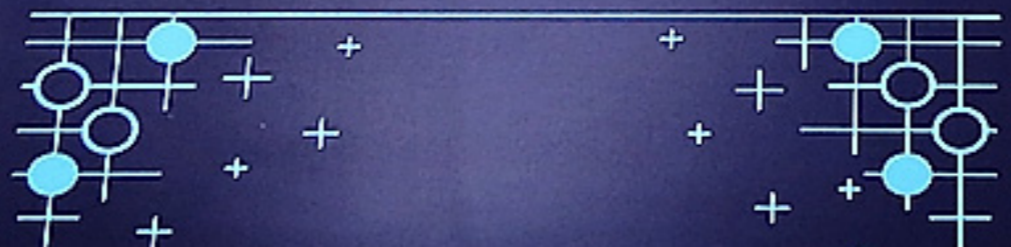


ALPHAGO  
01:59:50

LEE SEDOL  
01:59:44

AlphaGo

Lee Sedol



한국어



영어

말하기

한국어



취소



support convenience technology innovation  
learning new  
online enrolling  
everyone project  
teaching  
thought  
possibility  
change open people reach special courses participate accessible everyone  
goals simple think online hope social discussion digital questions  
experience education complete attention challenge

**What's unique in  
Human Intelligence?**

# Final Exam

- 6/9 (Thr), Rooms 611, 615 in Science Library
- Check your room on blackboard
- Coverage: pushdown automata ~ undecidability  
(lec10.pdf ~ lec17.pdf)

한학기 수고 많았습니다!