COSE215: Theory of Computation

Lecture 15 — Extensions of Turing Machines

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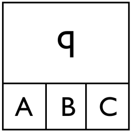
Extensions

Extend the standard Turing machine with

1. storage in the state
2. multiple tracks
3. a stay-option
4. multiple tapes
5. non-determinism

Storage in the state

The finite control stores a finite amount of data:



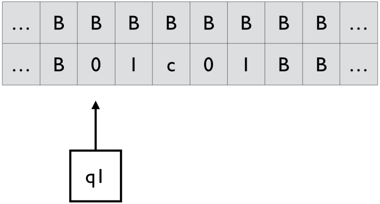
Example

A Turing machine that accepts $\mathbf{01^* + 10^*}$:

$(\{q_0, q_1\} \times \{0, 1, B\}, \{0, 1\}, \{0, 1, B\}, \delta, (q_0, B), B, \{(q_1, B)\})$

1. $\delta((q_0, B), a) = ((q_1, a), a, R)$ for $a = 0$ or $a = 1$
2. $\delta((q_1, a), \bar{a}) = ((q_1, a), \bar{a}, R)$
3. $\delta((q_1, a), B) = ((q_1, B), B, R)$

Multiple Tracks



Example

A Turing machine that accepts $L = \{wcw \mid w \in \{0,1\}^+\}$.
$(Q, \Sigma, \Gamma, \delta, (q_1, B), (B, B), \{q_9, B\})$

- $\{q_1, q_2, \ldots, q_9\} \times \{0, 1, B\}$
- $\Gamma = \{B, *\} \times \{0, 1, c, B\}$
- $\Sigma = \{(B, 0), (B, 1), (B, c)\}$

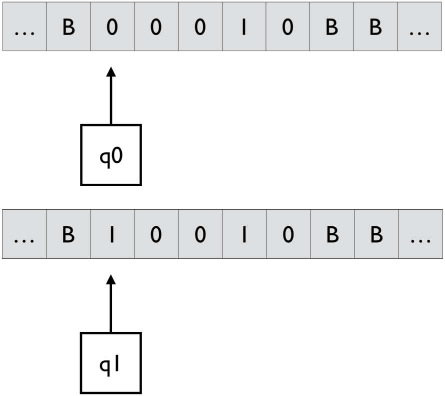
Turing Machines with a Stay-Option

The tape head can be stationary:

$$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

ex) $\delta(q_0, 0) = (q_1, 1, S)$



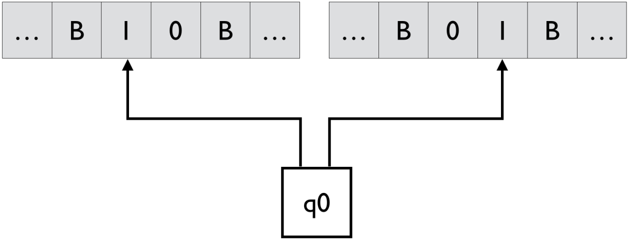
Equivalence

1. Is the TM with a stay-option is as powerful as the standard TM?
2. Is the standard TM is as powerful as the TM with a stay-option?

Multitape Turing Machines

Turing machine with

- multiple tapes
- each tape has its own tape head



Multitape Turing Machines

$$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$\delta : Q \times \Gamma^n \to Q \times \Gamma^n \times \{L, R\}^n$$

Initially,

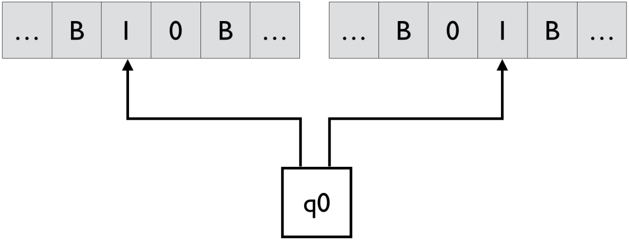
1. The input is placed on the first tape.
2. All other cells of all the tapes hold the blank.
3. The finite control is in the initial state.
4. The head of the first tape is at the left end of the input.
5. All other tape heads are at some arbitrary cell.

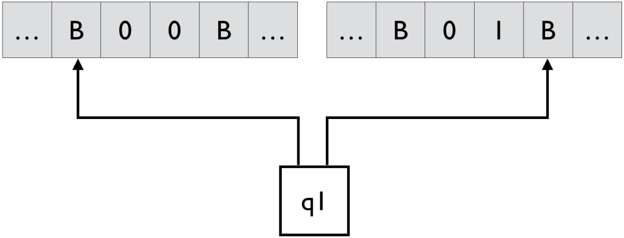
In one move, the multitape TM does the following:

1. The control enters a new state.
2. On each tape, a new tape symbol is written on the cell scanned.
3. Each of the tape heads makes a move independently of each other.

Multitape Turing Machines

ex) $\delta(q_0, 1, 1) = (q_1, 0, 1, L, R)$



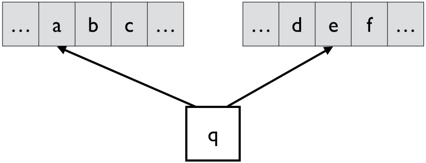


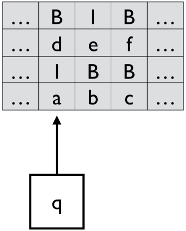
Equivalence

To represent a MTM by a standard TM, we need to represent

- the contents of multiple tapes, and
- the positions of multiple tape heads.

Represent them by a tape with multiple tracks: e.g.,





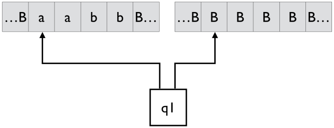
cf) Efficiency

Although the expressiveness is the same, MTM can be more efficient than the standard TM.

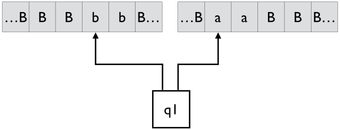
Example

Design a multitape Turing machine that accepts $L = \{a^n b^n \mid n \geq 1\}$.

- In standard TM, repeated back-and-forth movements are required.
- In MTM, copy all a's to tape 2



and then match b's on tape 1 against a's on tape 2



Non-deterministic Turing Machines

$$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$\delta : Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L,R\}}$$

- E.g., $\delta(q_0, a) = \{(q_1, b, R), (q_2, c, L)\}$
- A NTM accepts w if there is a sequence s.t.

$$q_0 w \vdash^* x_1 q_f x_2$$

  with $q_f \in F$.
- Still, equivalent.

Summary

No matter how we extend the standard Turing machines, the expressiveness remains the same.