COSE215: Theory of Computation

Lecture 10 — Pushdown Automata (1)

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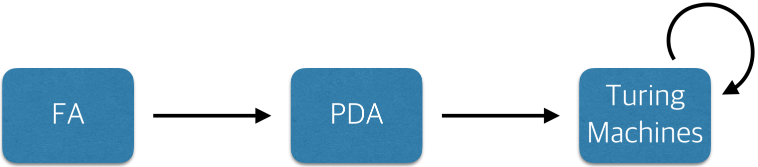
Roadmap of This Course

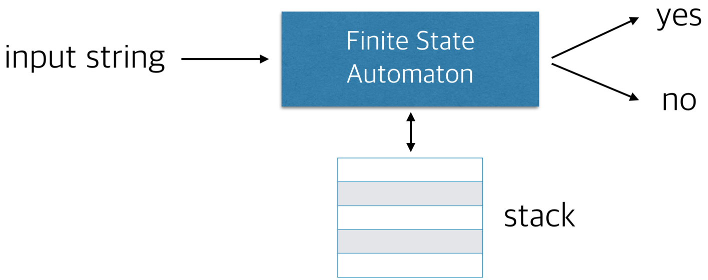


- Finite automata (FA): the basic model of computation
- Pushdown automata (PDA): an extension of FA
- Turing machines: an extension of PDA

Pushdown Automata

Essentially, an ϵ-NFA with a stack:

- In FA, the next state is determined by the current state and the input symbol.
- In PDA, the next state is determined by the current state, the input symbol, and the stack contents.



Formal Definition of Pushdown Automata

Definition (Pushdown Automata)

A pushdown automaton (PDA) is defined as

$$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

- Q: A finite set of *states*
- Σ: A finite set of *input symbols*
- Γ: A finite set of *stack alphabets*
- $\delta \in Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \to 2^{Q \times \Gamma^*}$: the *transition function*
- $q_0 \in Q$: the initial state (the state the PDA is in before making any transitions)
- $Z_0 \in \Gamma$: the start stack symbol. Initially, the PDA's stack consists of only this symbol.
- $F \subseteq Q$: the set of final states

The Transition Function

$$\delta \in Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \rightarrow 2^{Q \times \Gamma^*}$$

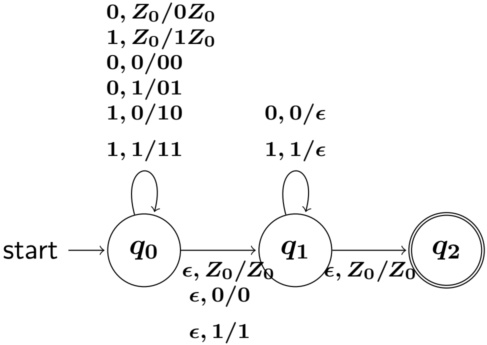
- δ takes a triple (q, a, X):
  - q: the current state
  - a: the current input symbol
  - X: the current symbol on top of the stack
- The output of δ is a finite set of paris (p, γ):
  - p: the next state
  - γ: the string of stack symbols that replaces the top of the stack

Example

$$P = (\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1, Z_0\}, \delta, q_0, Z_0, \{q_2\})$$

$$\begin{aligned}
\delta(q_0, 0, Z_0) &= \{(q_0, 0Z_0)\} \\
\delta(q_0, 1, Z_0) &= \{(q_0, 1Z_0)\} \\
\delta(q_0, 0, 0) &= \{(q_0, 00)\} \\
\delta(q_0, 0, 1) &= \{(q_0, 01)\} \\
\delta(q_0, 1, 0) &= \{(q_0, 10)\} \\
\delta(q_0, 1, 1) &= \{(q_0, 11)\} \\
\delta(q_0, \epsilon, Z_0) &= \{(q_1, Z_0)\} \\
\delta(q_0, \epsilon, 0) &= \{(q_1, 0)\} \\
\delta(q_0, \epsilon, 1) &= \{(q_1, 1)\} \\
\delta(q_1, 0, 0) &= \{(q_1, \epsilon)\} \\
\delta(q_1, 1, 1) &= \{(q_1, \epsilon)\} \\
\delta(q_1, \epsilon, Z_0) &= \{(q_2, Z_0)\}
\end{aligned}$$

Transition Graph



Exercises

1. $L = \{a^n b^n \mid n \geq 0\}$

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1. $L = \{a^n b^n \mid n \geq 0\}$
2. $L = \{w \in \{a, b\}^* \mid n_a(w) = n_b(w)\}$
3. $L = \{a^i b^j c^k \mid i, j, k \geq 0 \wedge (i = j \vee i = k)\}$