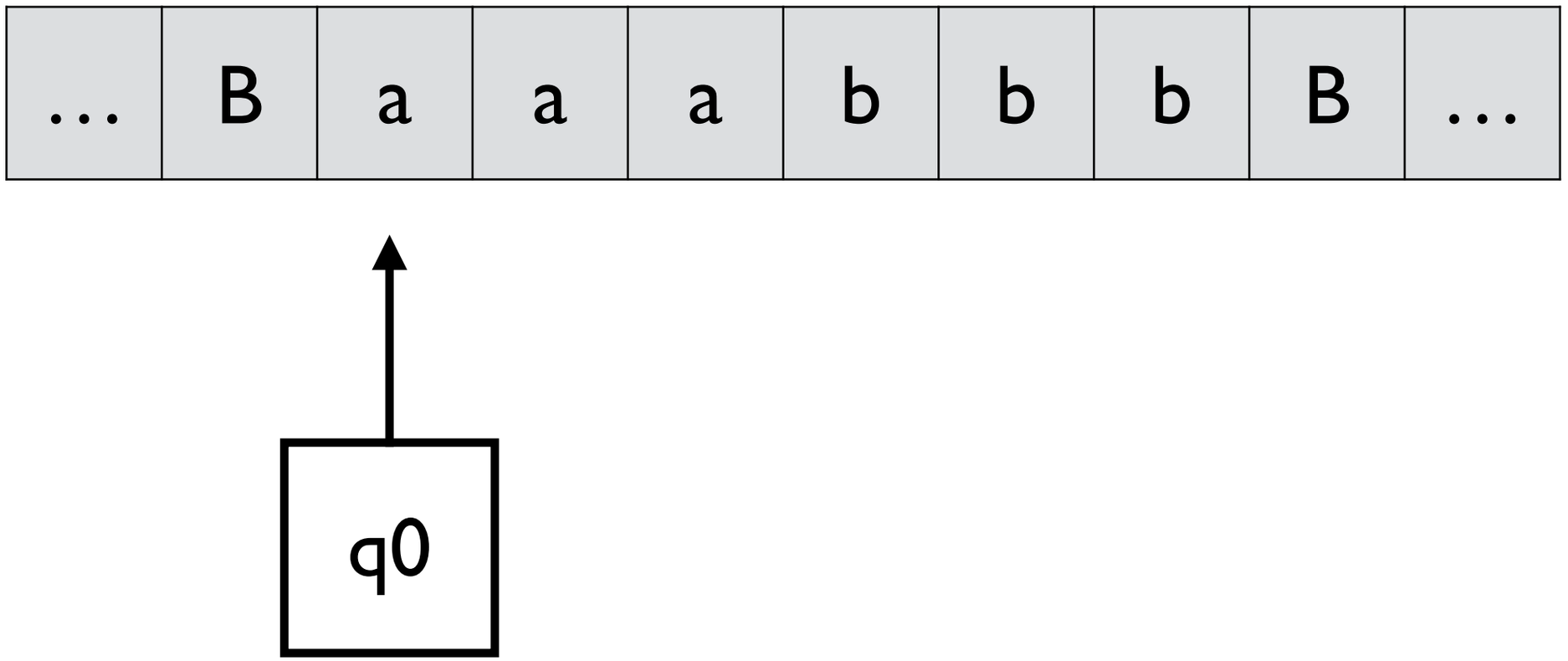
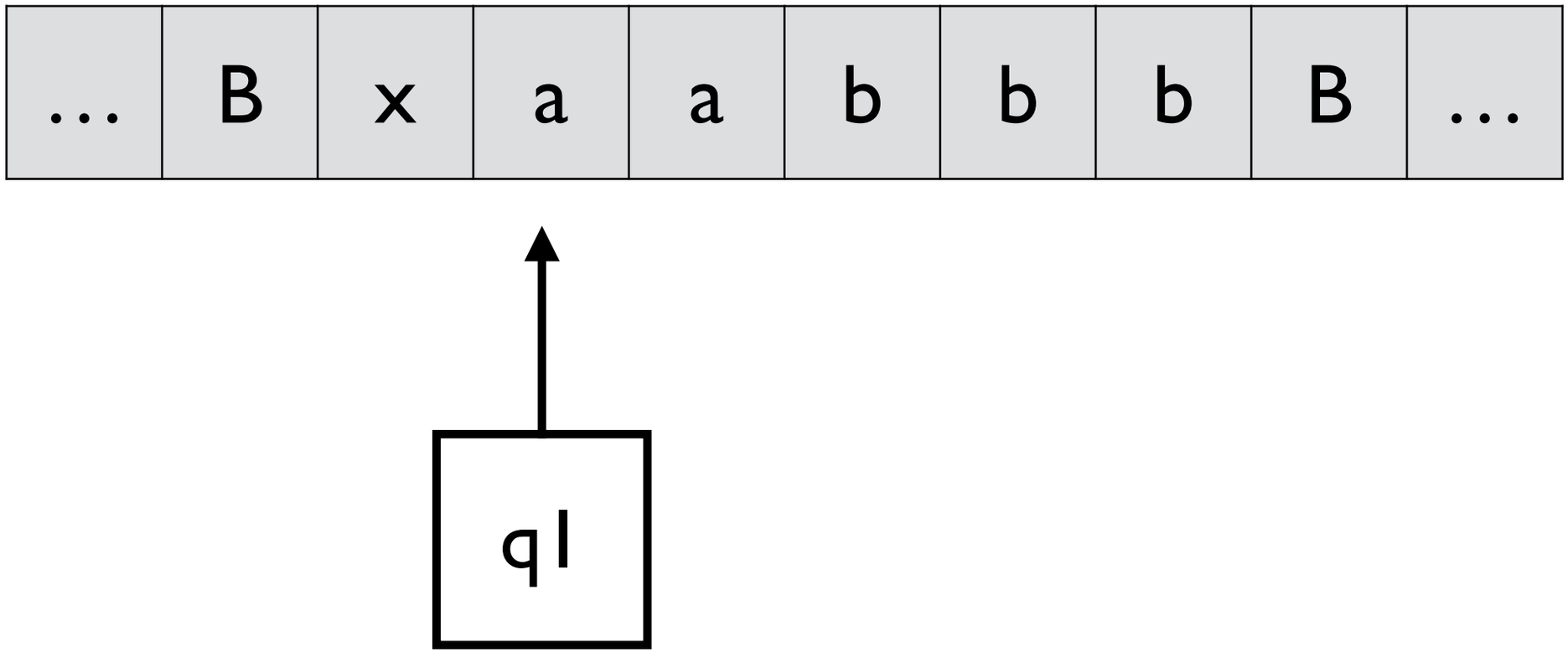
Example 1. Design a Turing machine that accepts $L = \{a^n b^n \mid n \geq 1\}$.

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$$

$$\begin{array}{lll}
\delta(q_0, a) = (q_1, x, R) & \delta(q_2, y) = (q_2, y, L) & \delta(q_0, y) = (q_3, y, R) \\
\delta(q_1, a) = (q_1, a, R) & \delta(q_2, a) = (q_2, a, L) & \delta(q_3, y) = (q_3, y, R) \\
\delta(q_1, y) = (q_1, y, R) & \delta(q_2, x) = (q_0, x, R) & \delta(q_3, B) = (q_4, B, R) \\
\delta(q_1, b) = (q_2, y, L) & &
\end{array}$$

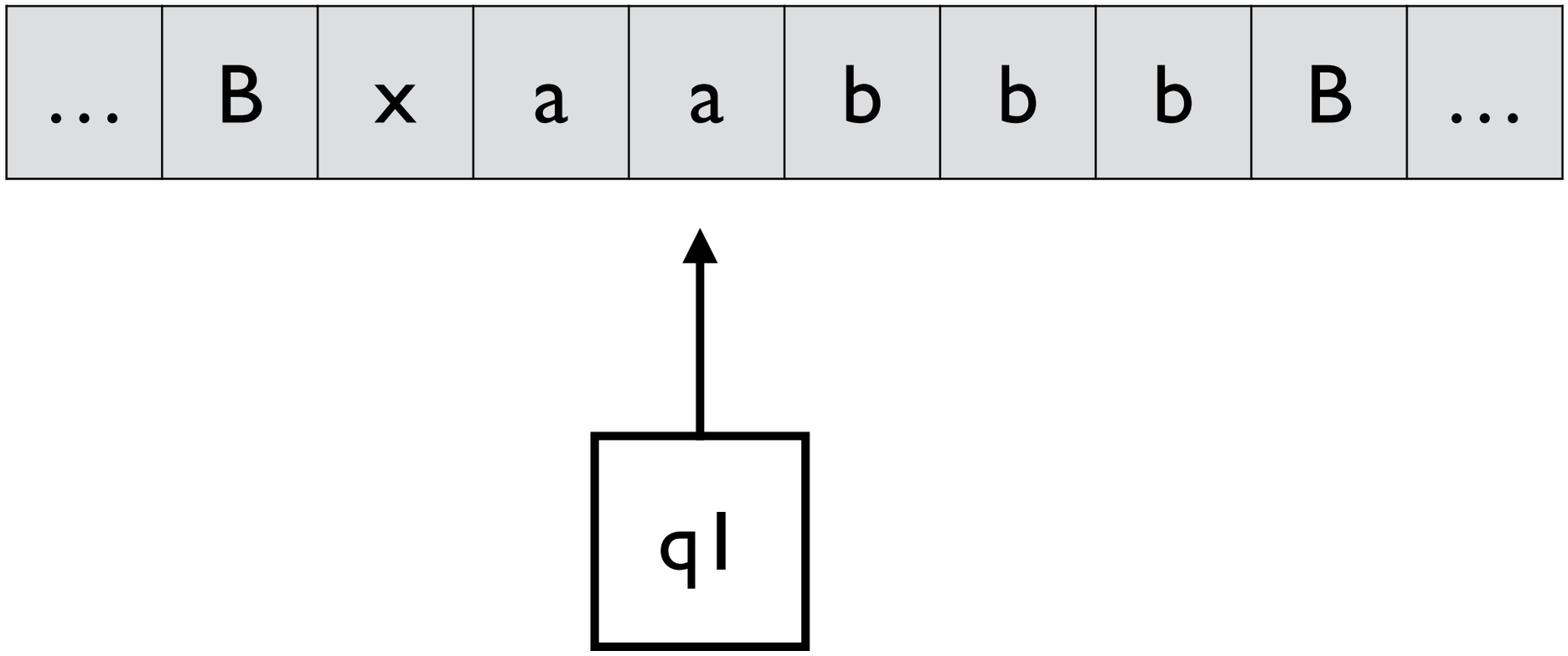


$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

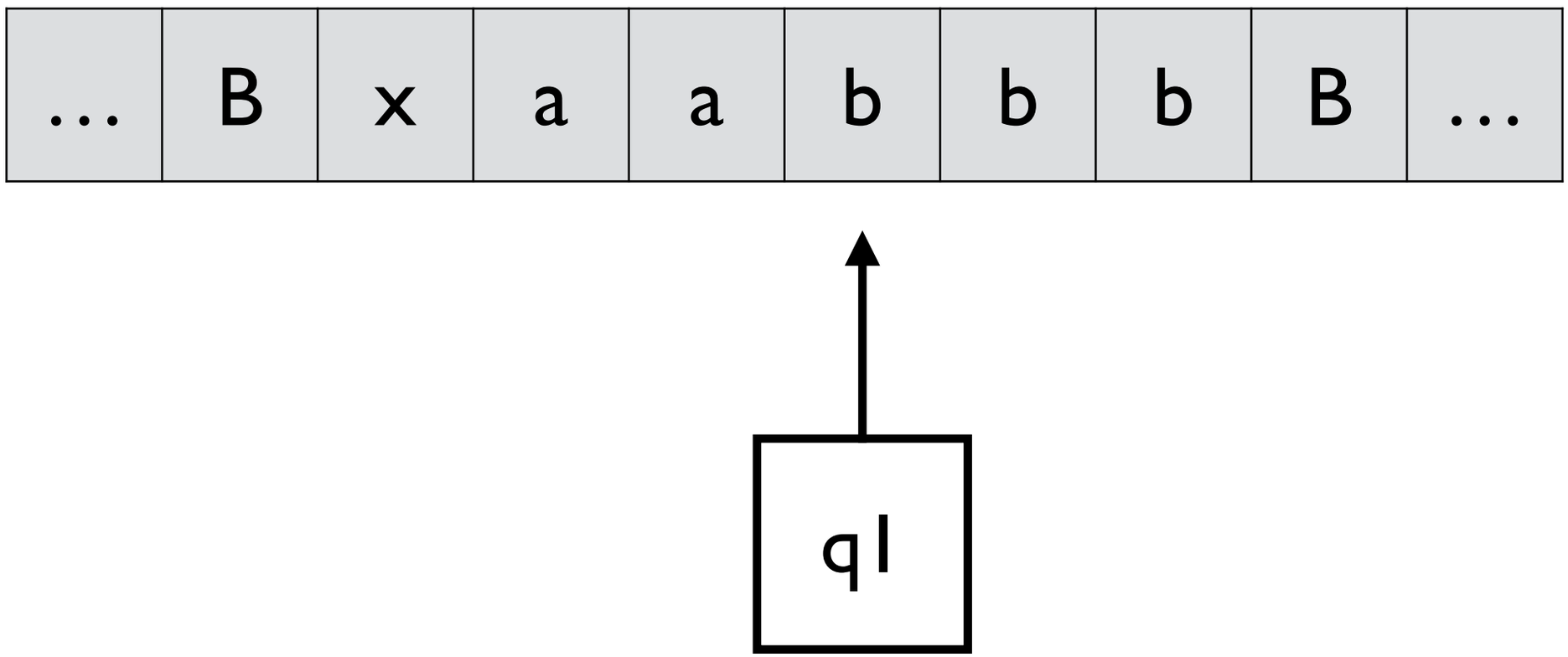


(In q1, move right to search for the leftmost 'b')

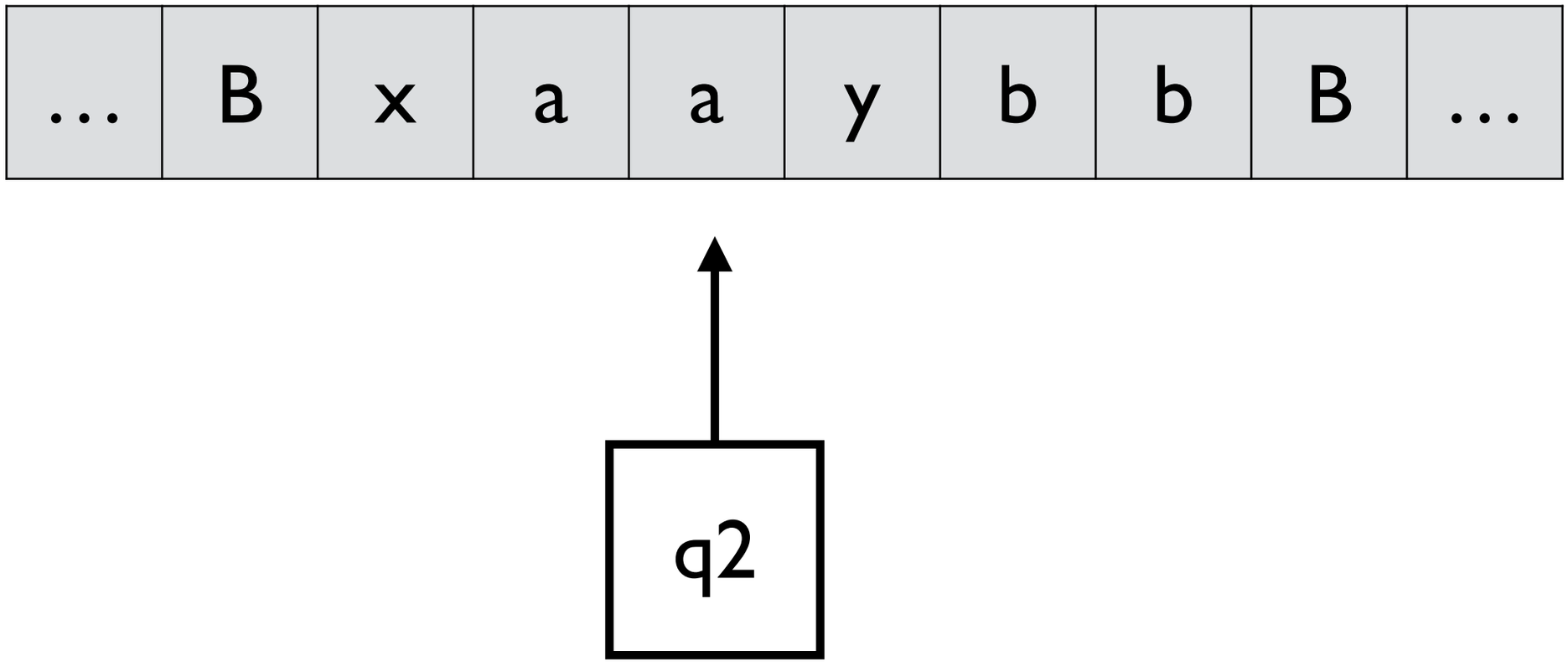
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$

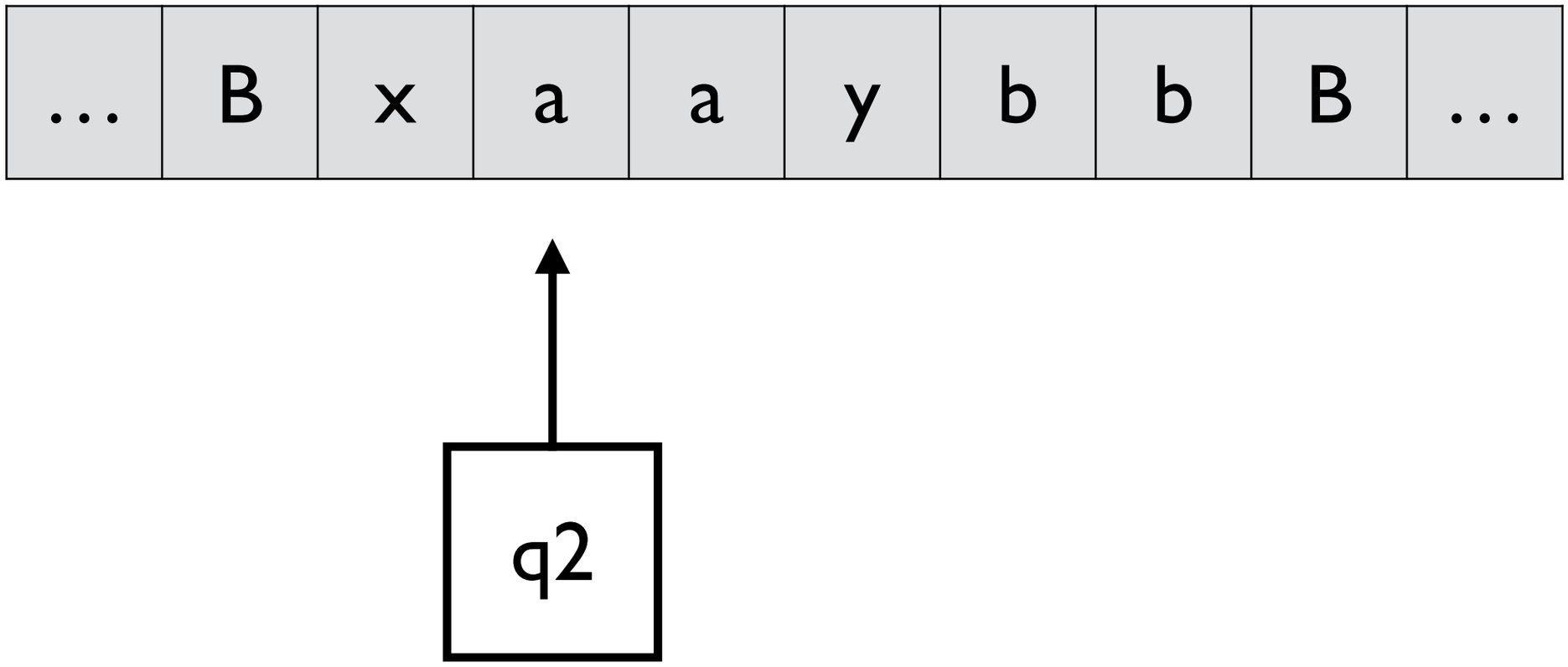


$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$

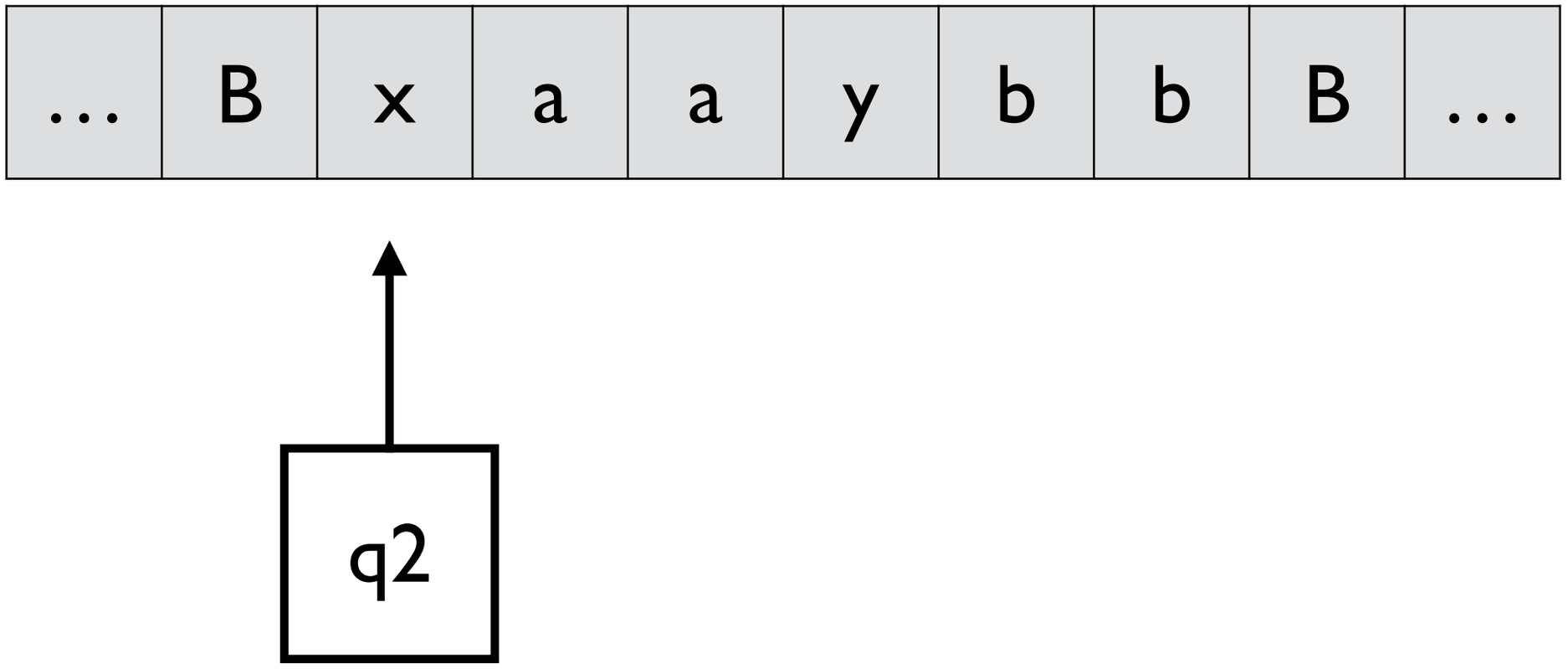


(In q2, move left to search for the leftmost ‘a’)

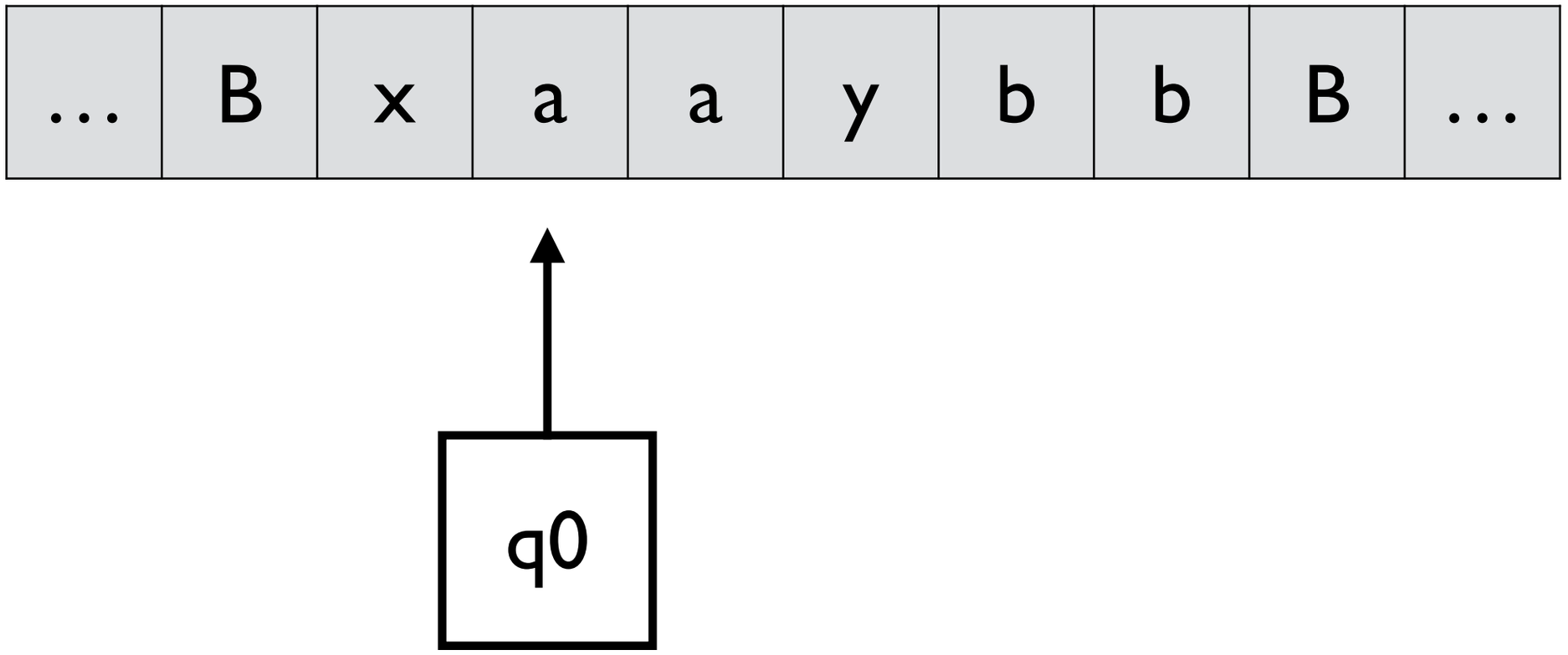
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



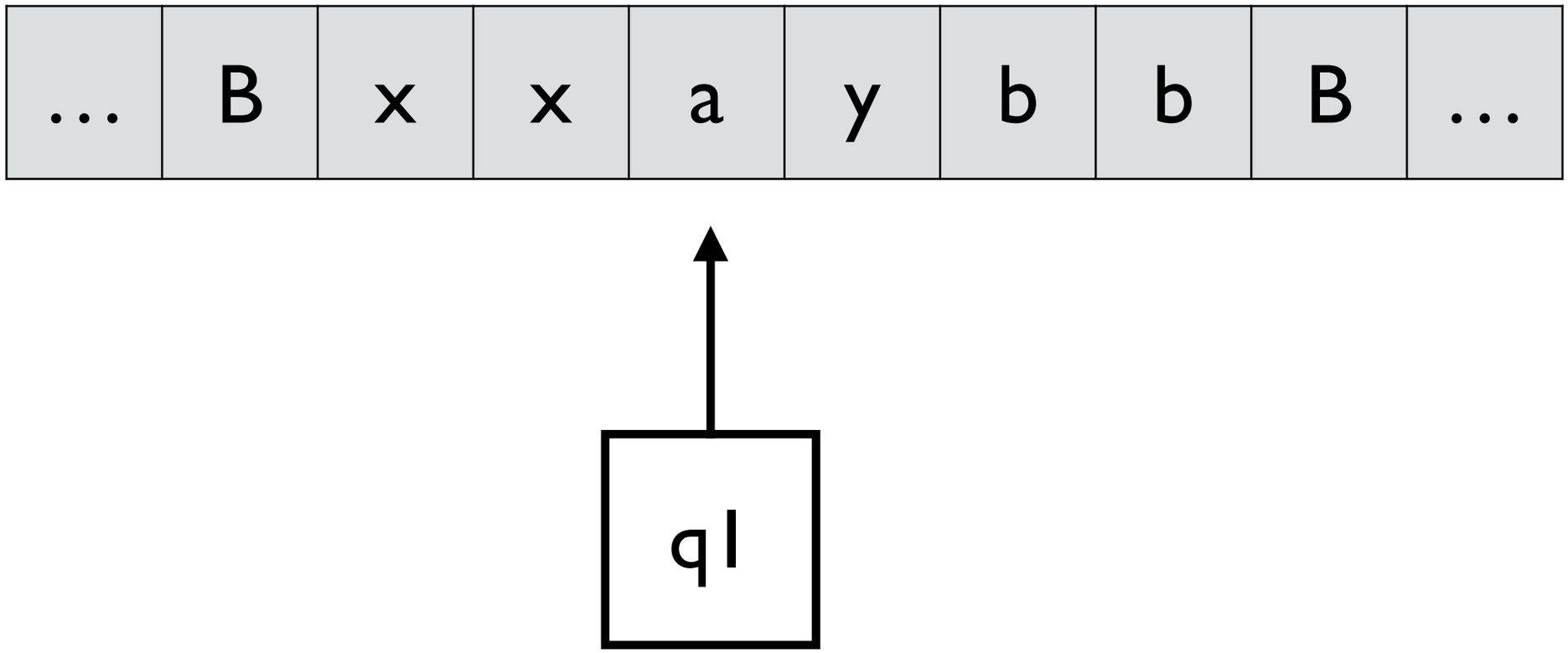
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



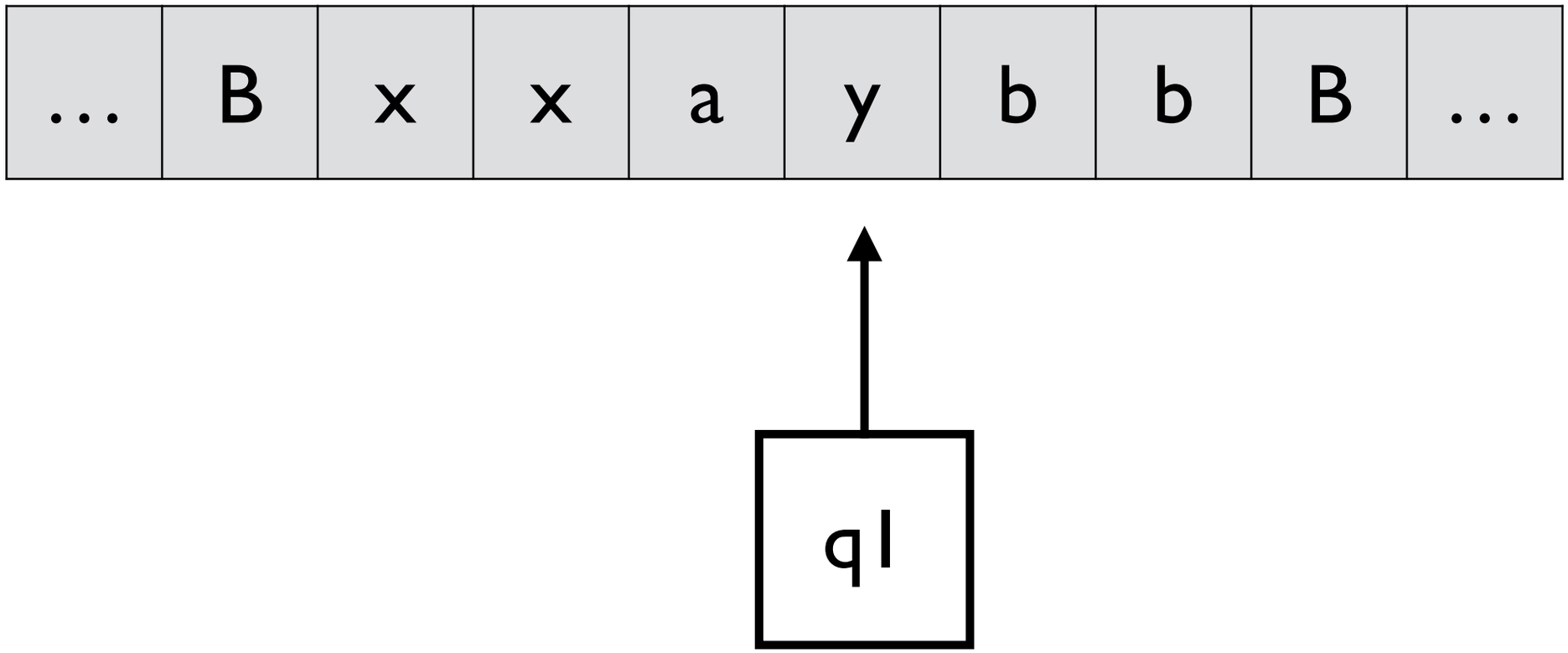
$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



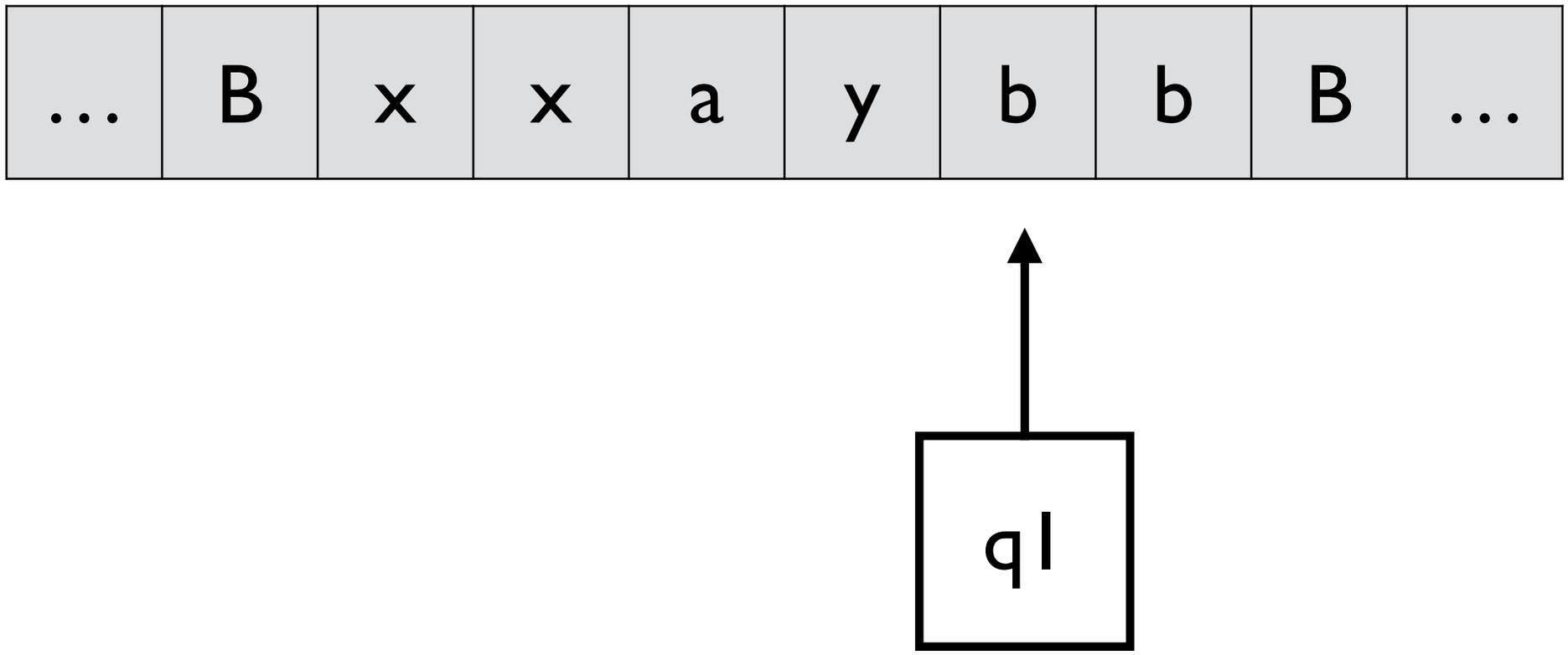
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



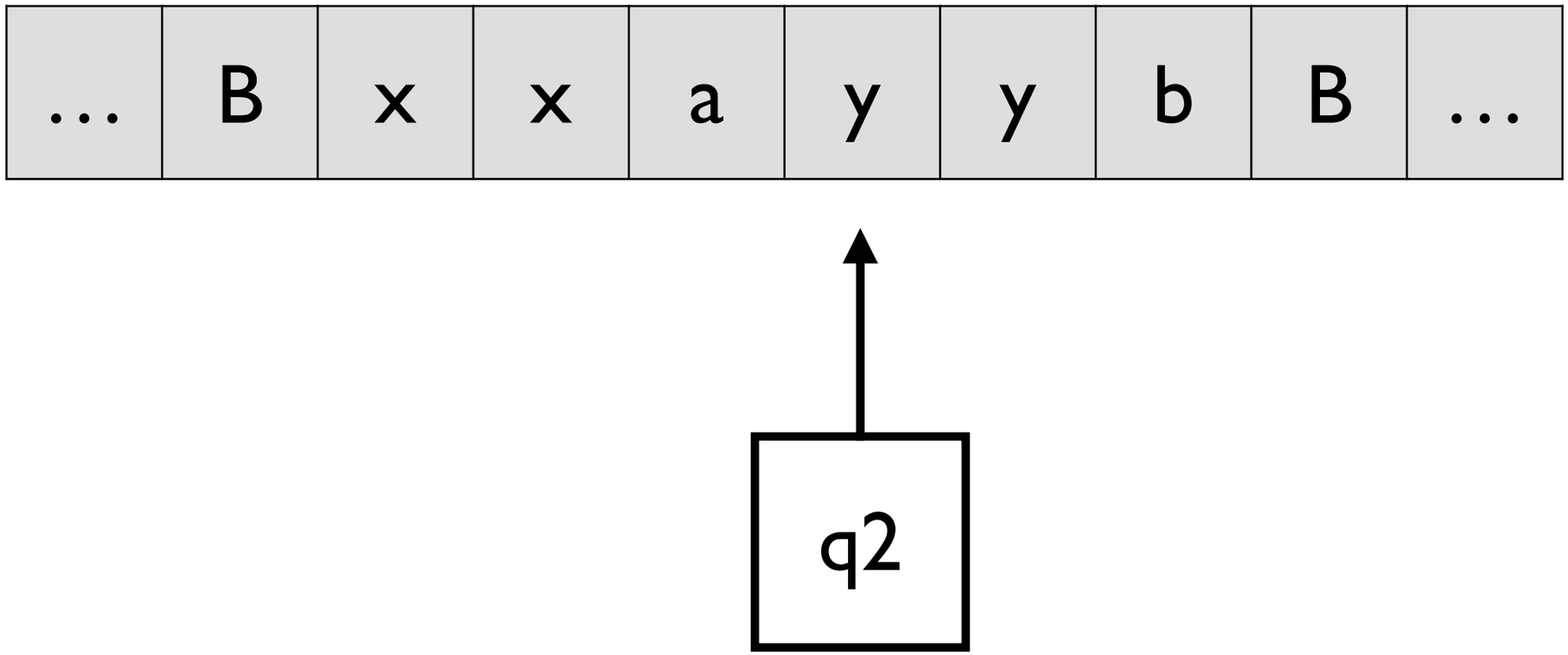
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
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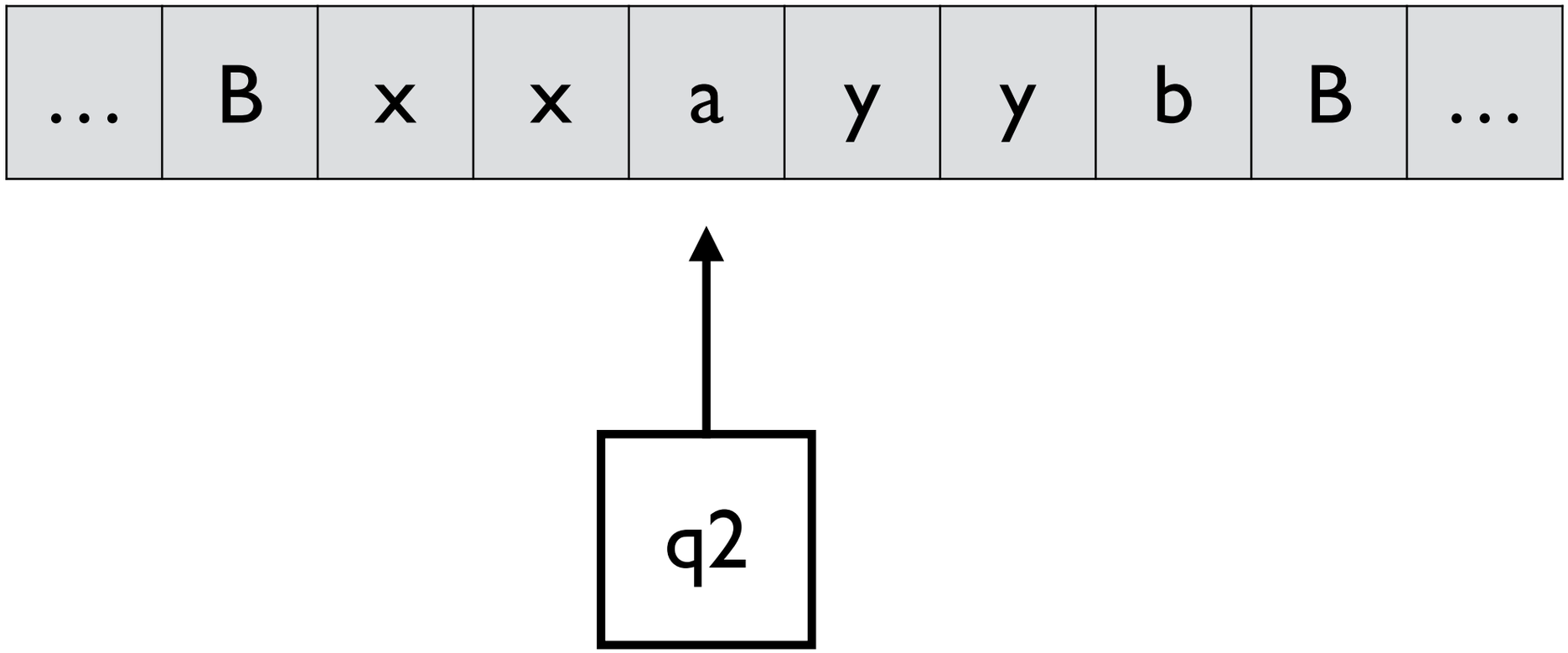
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
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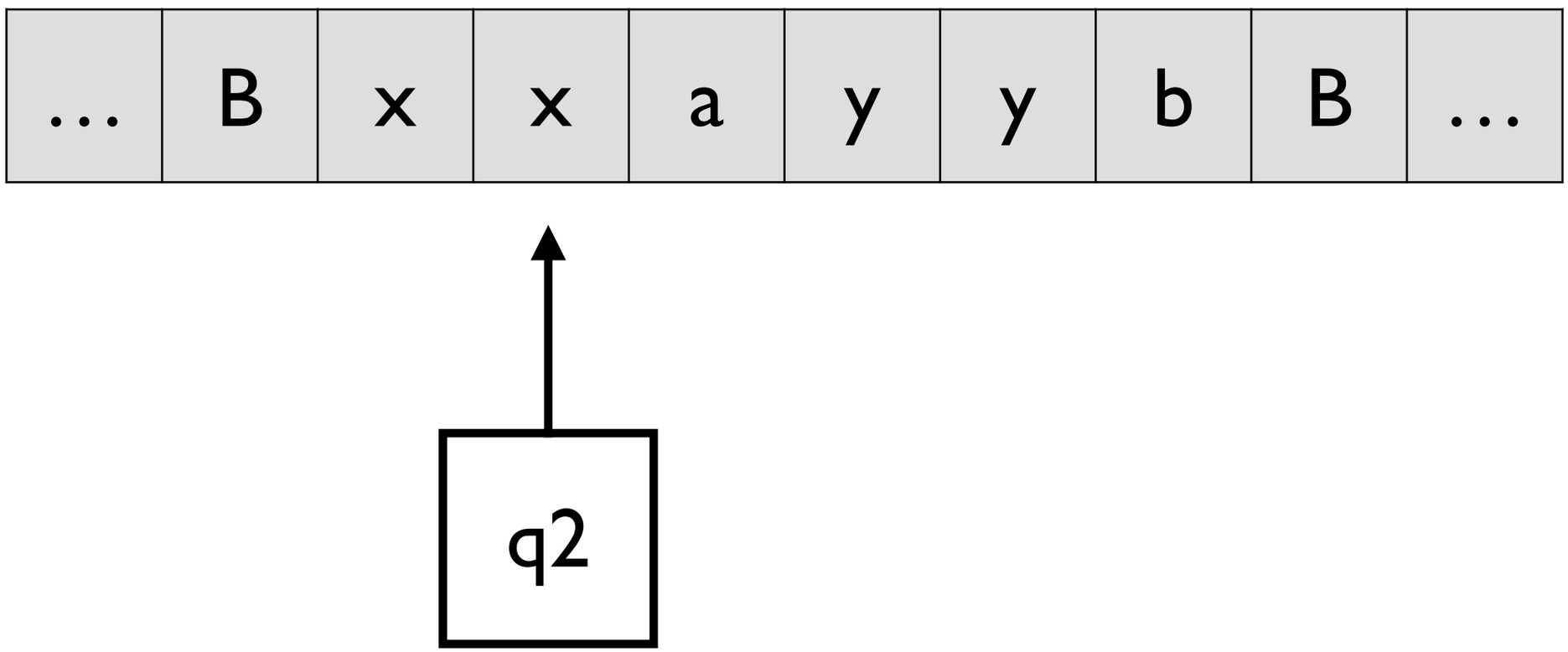
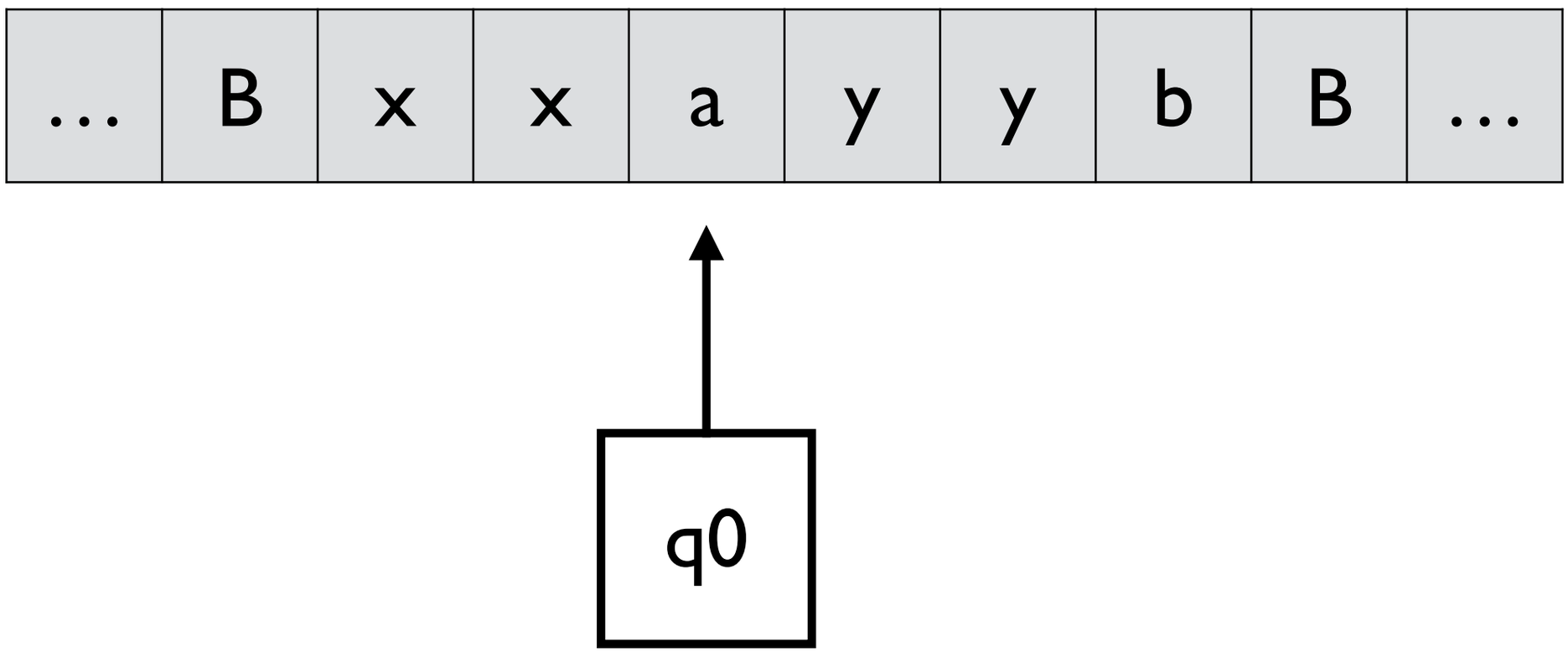
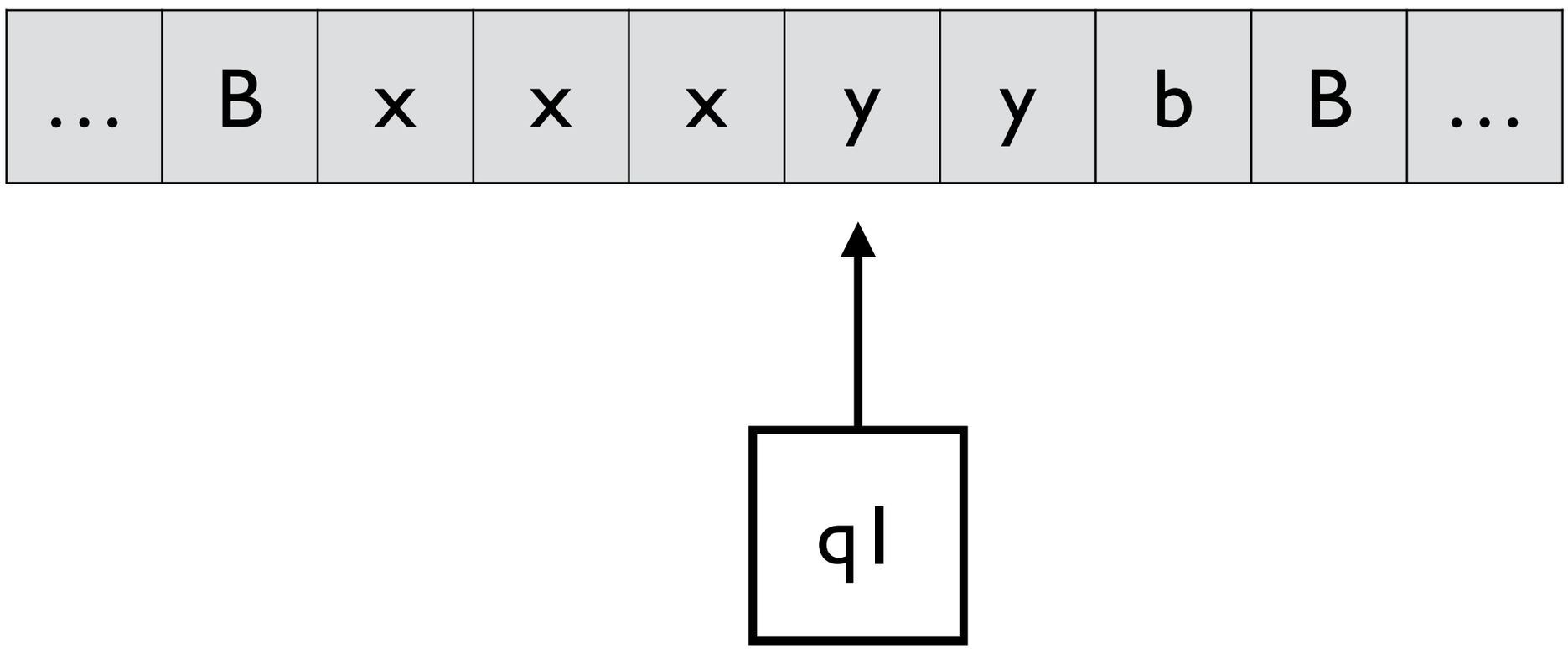
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



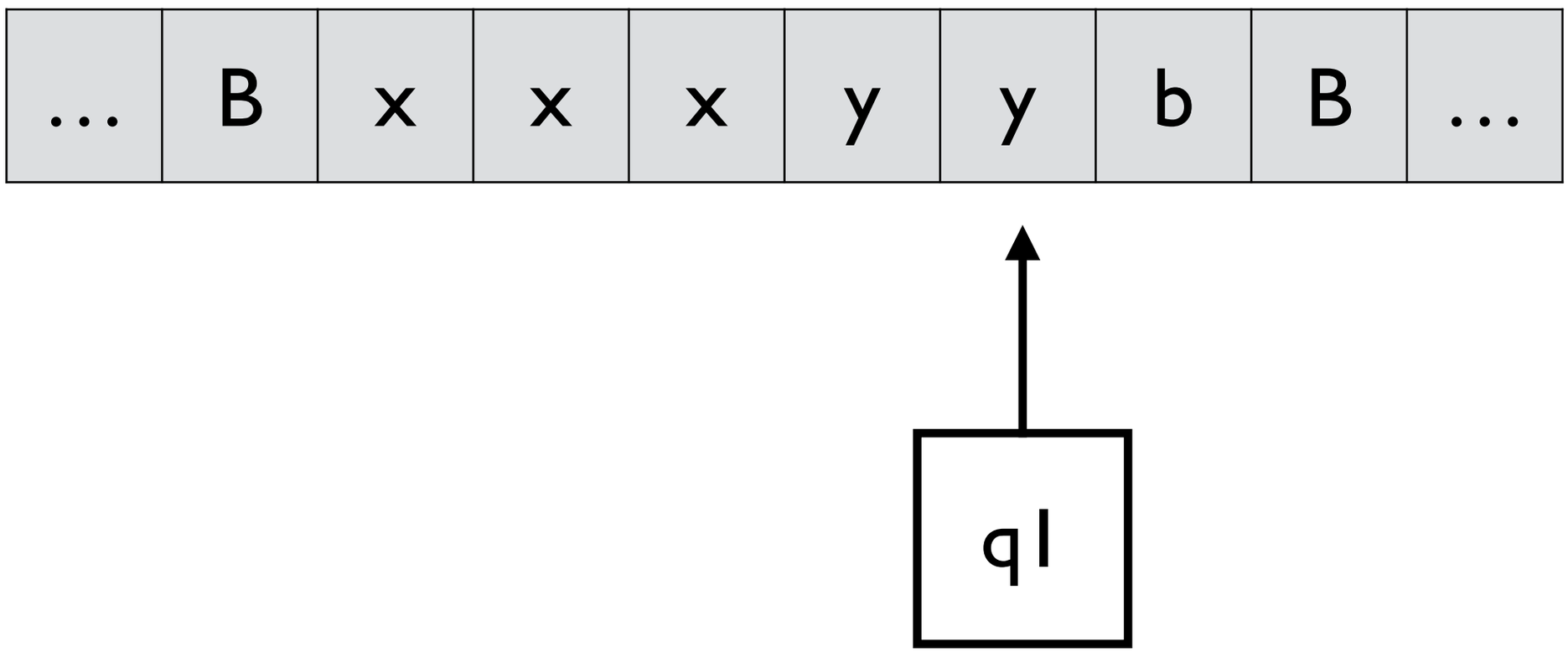
$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



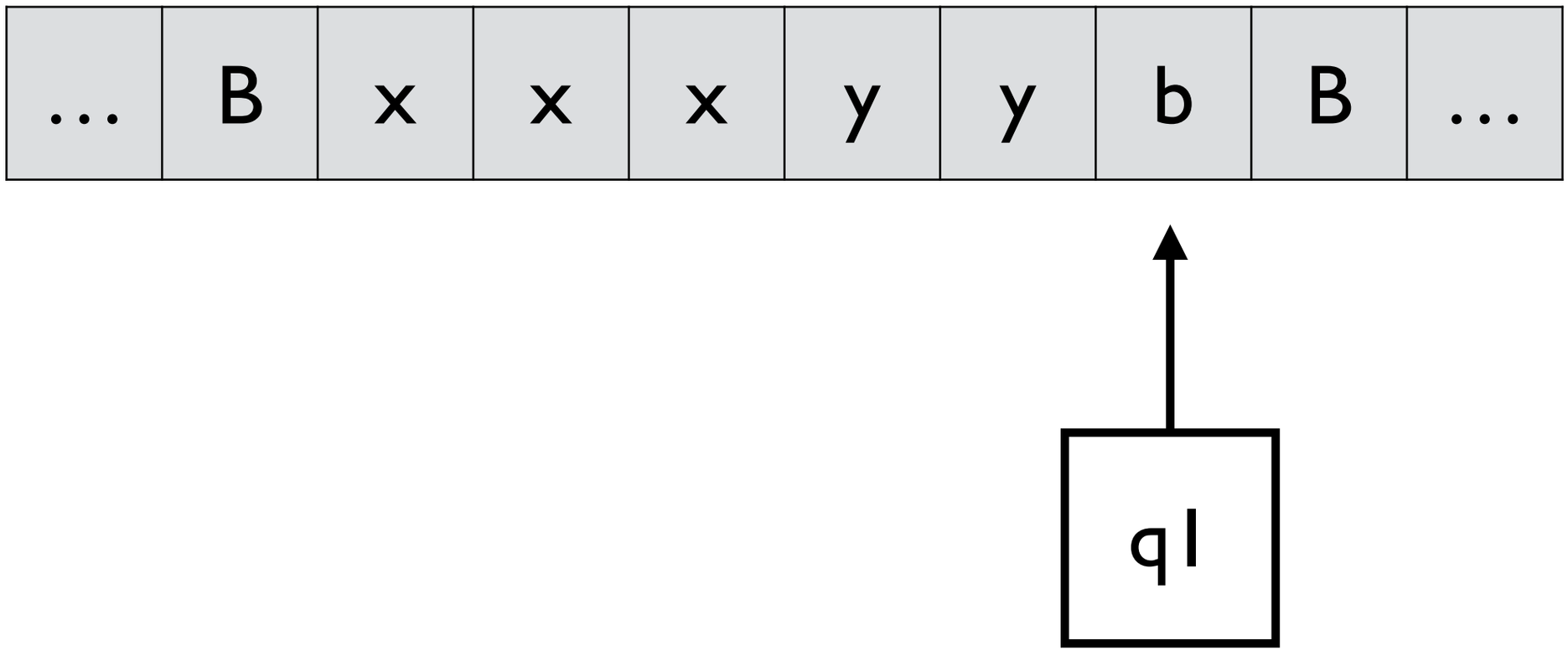
$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



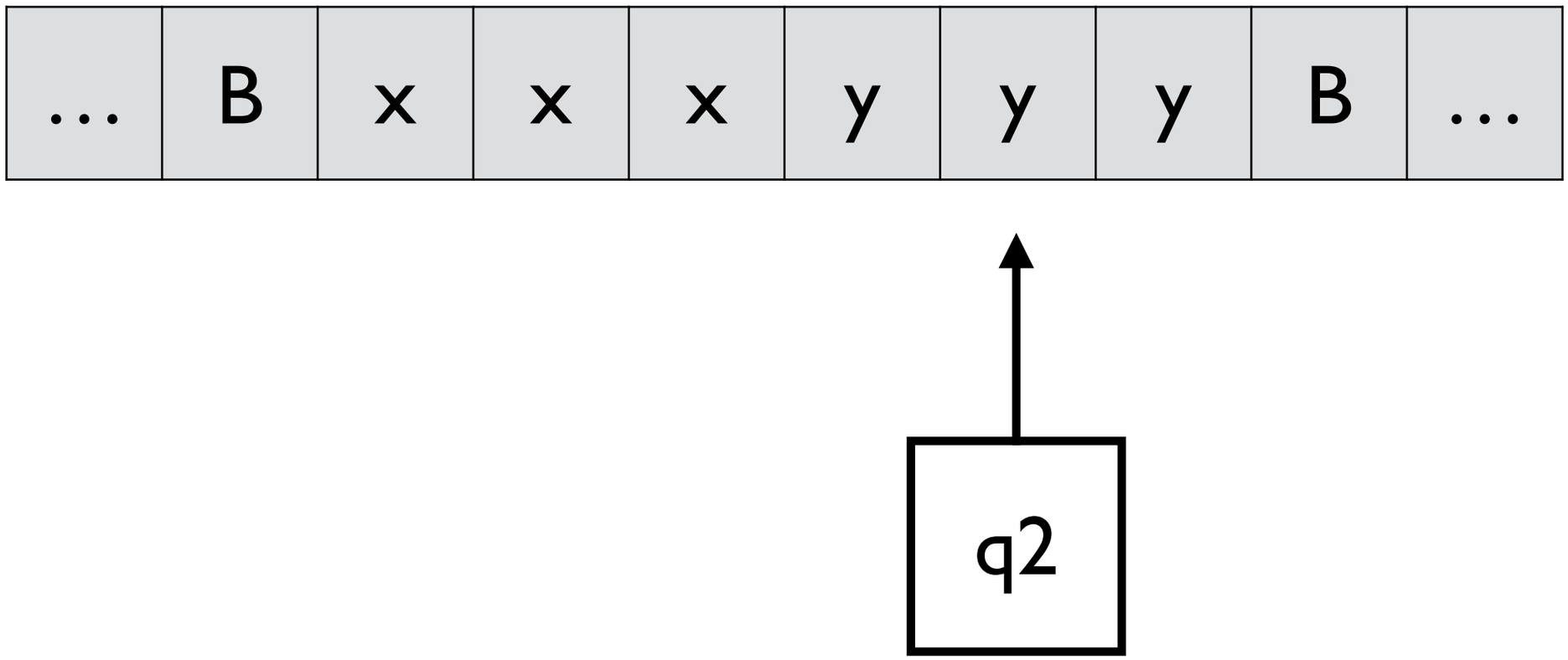
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



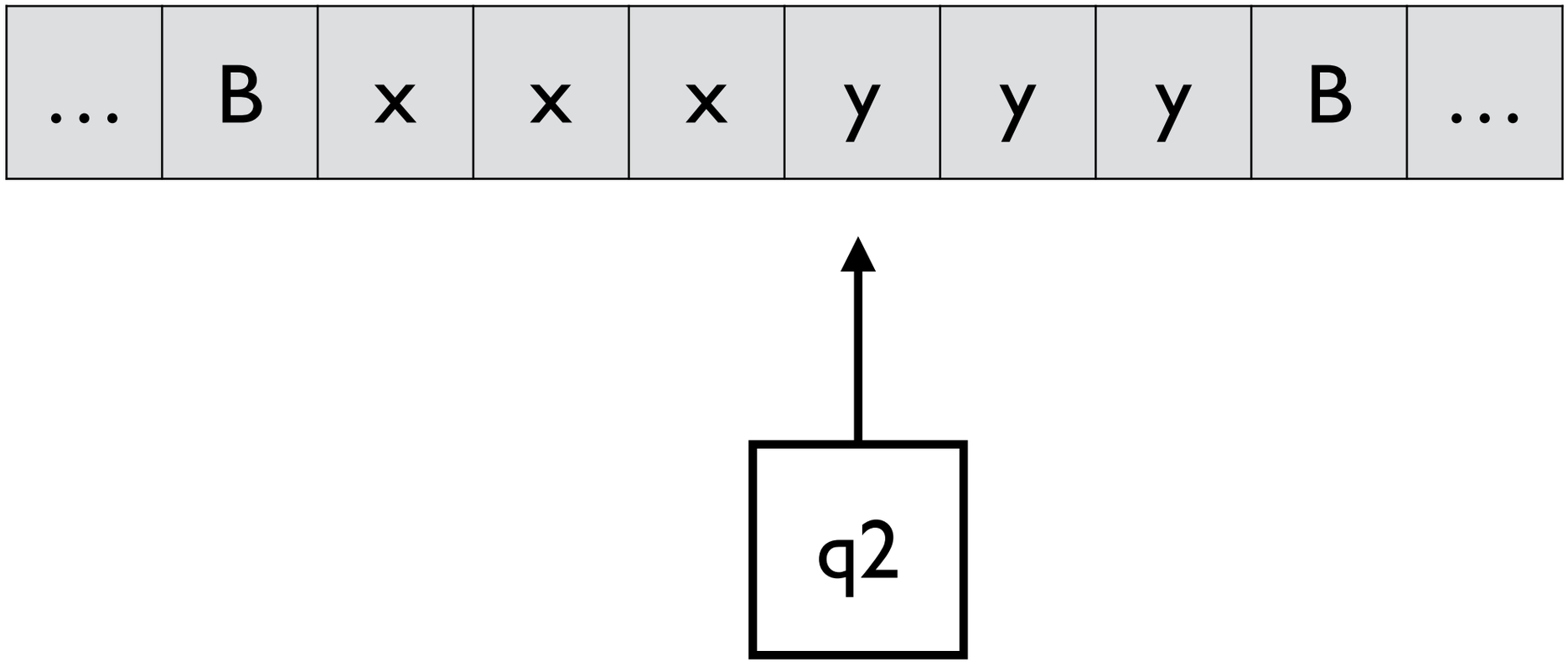
$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



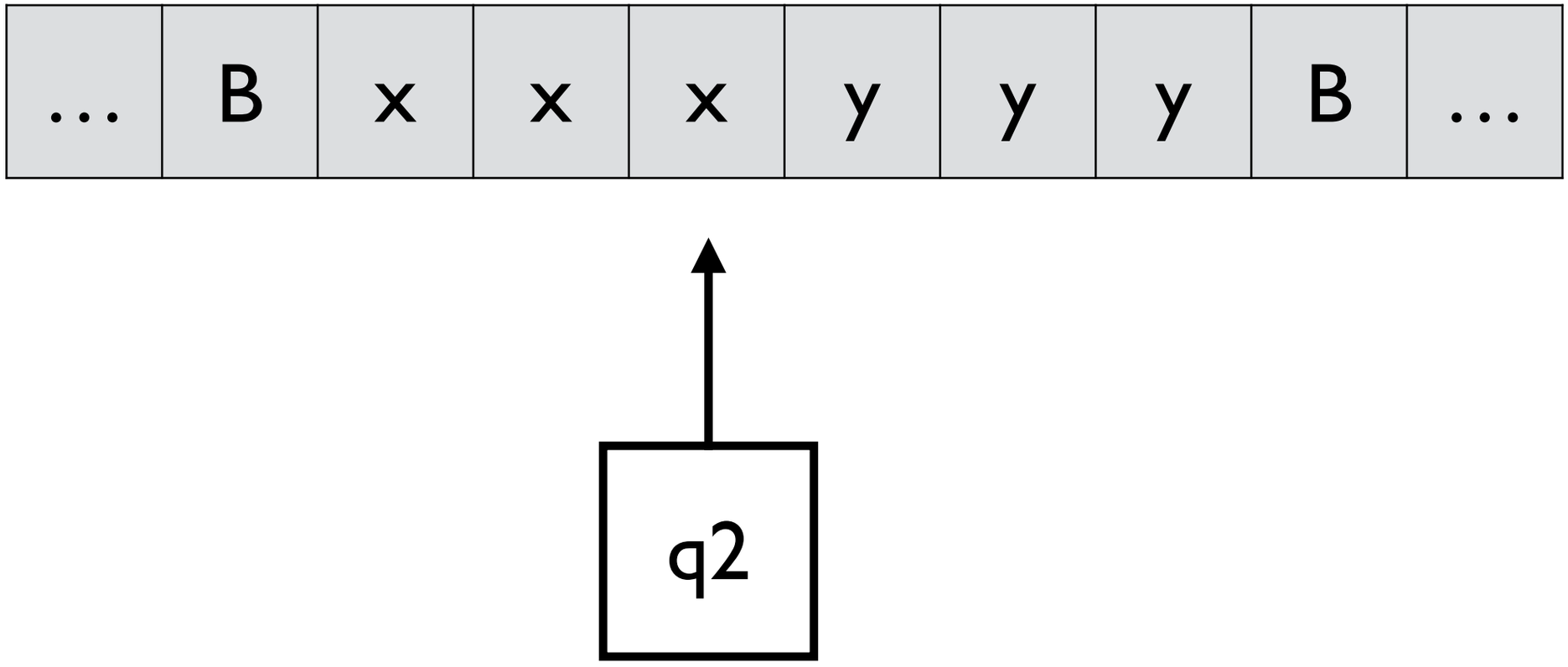
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



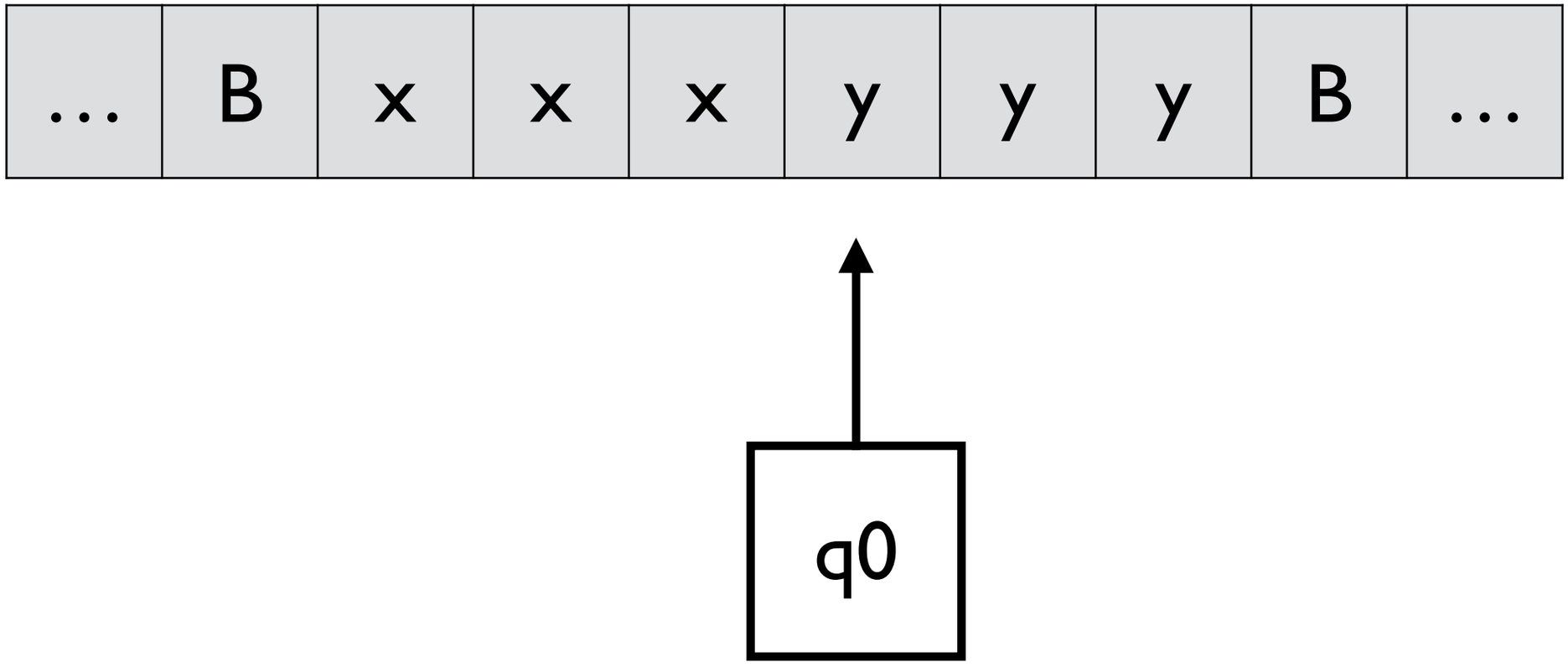
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



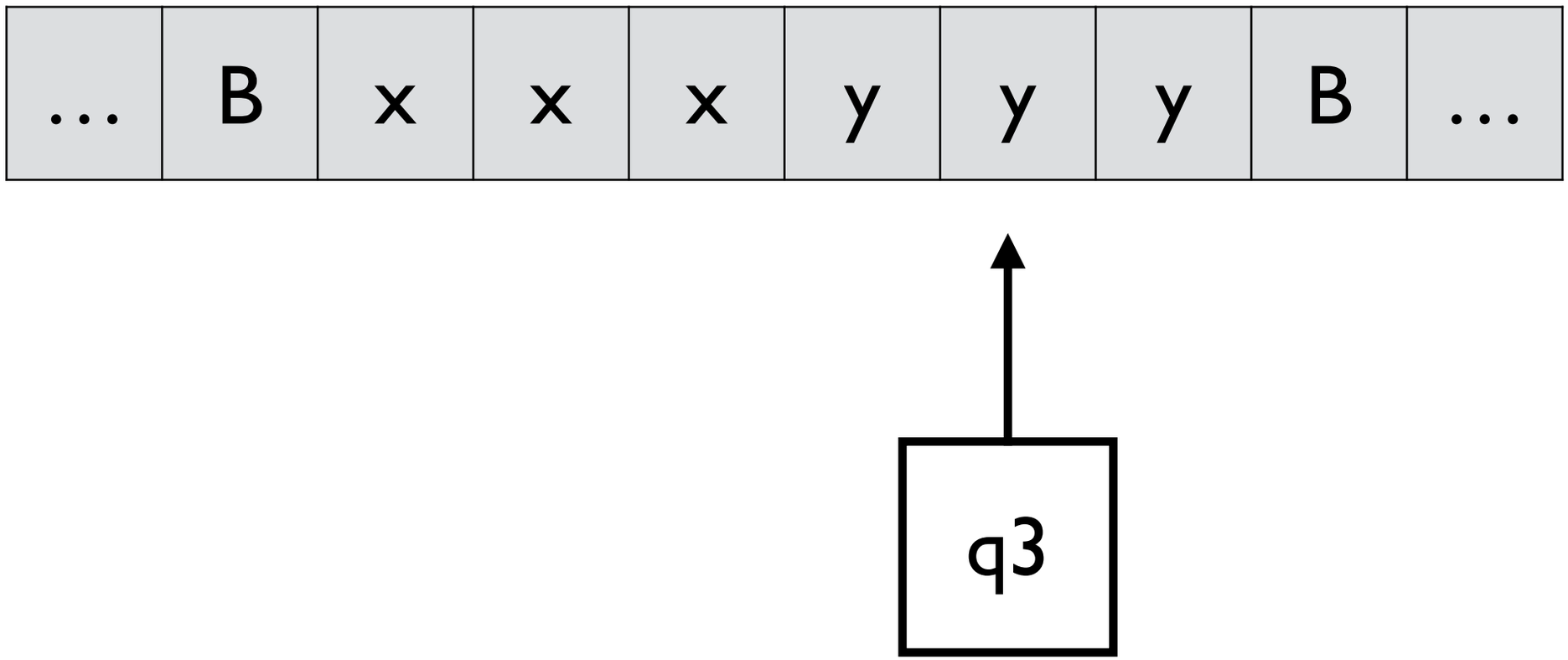
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$

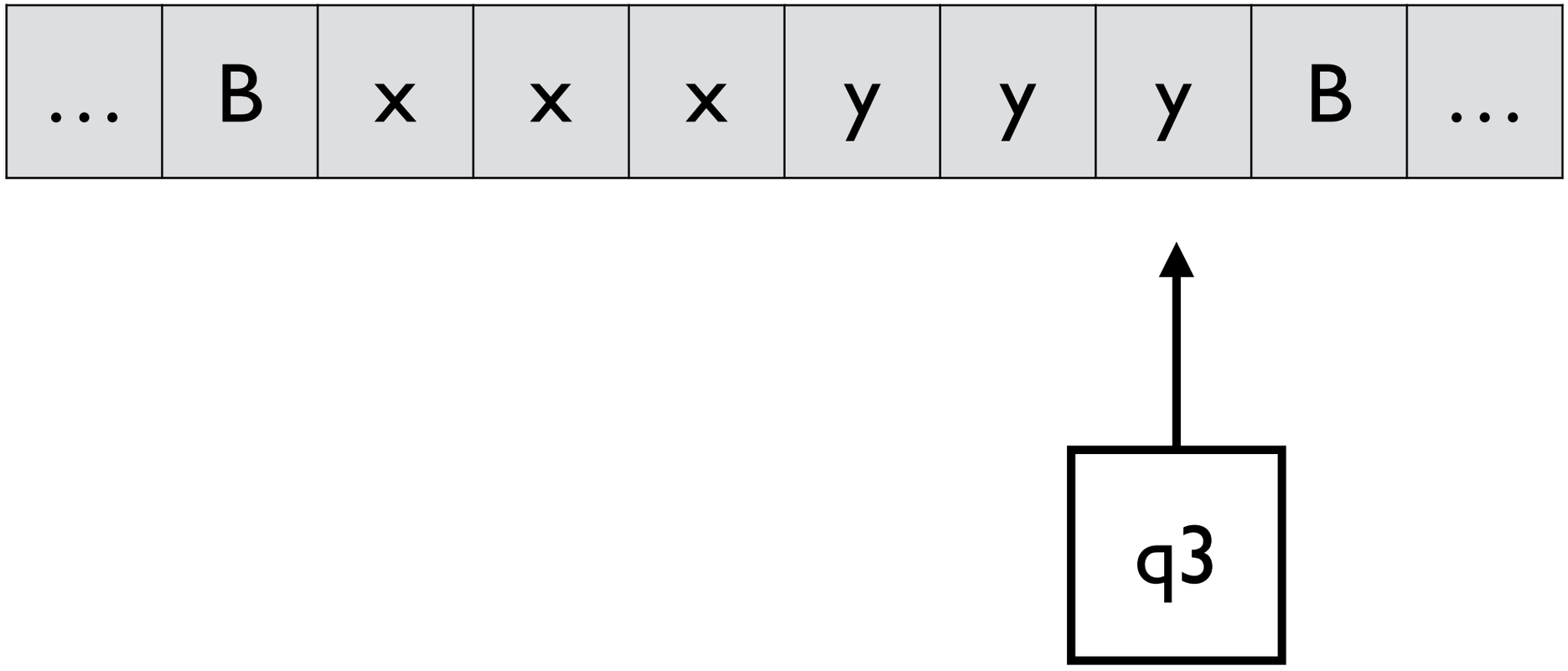


$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

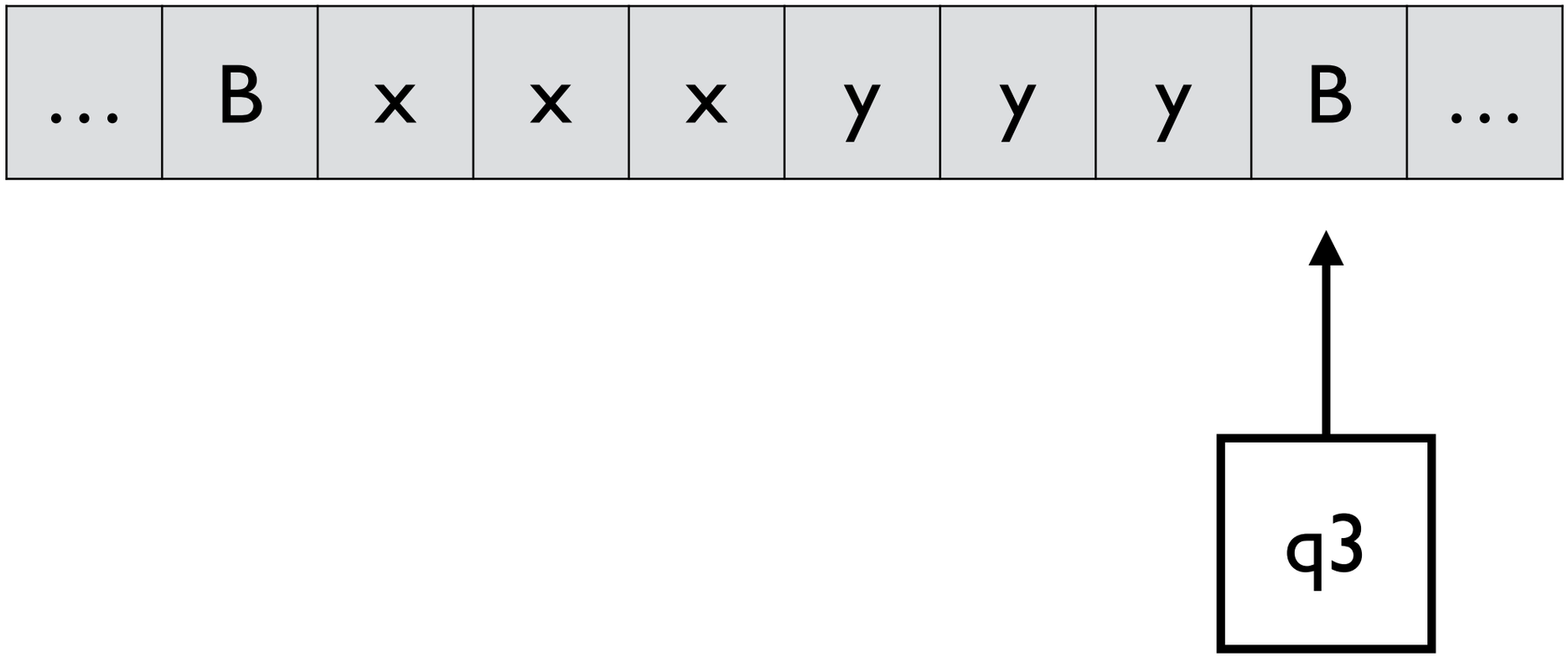


(In q3, move right to check that there are no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

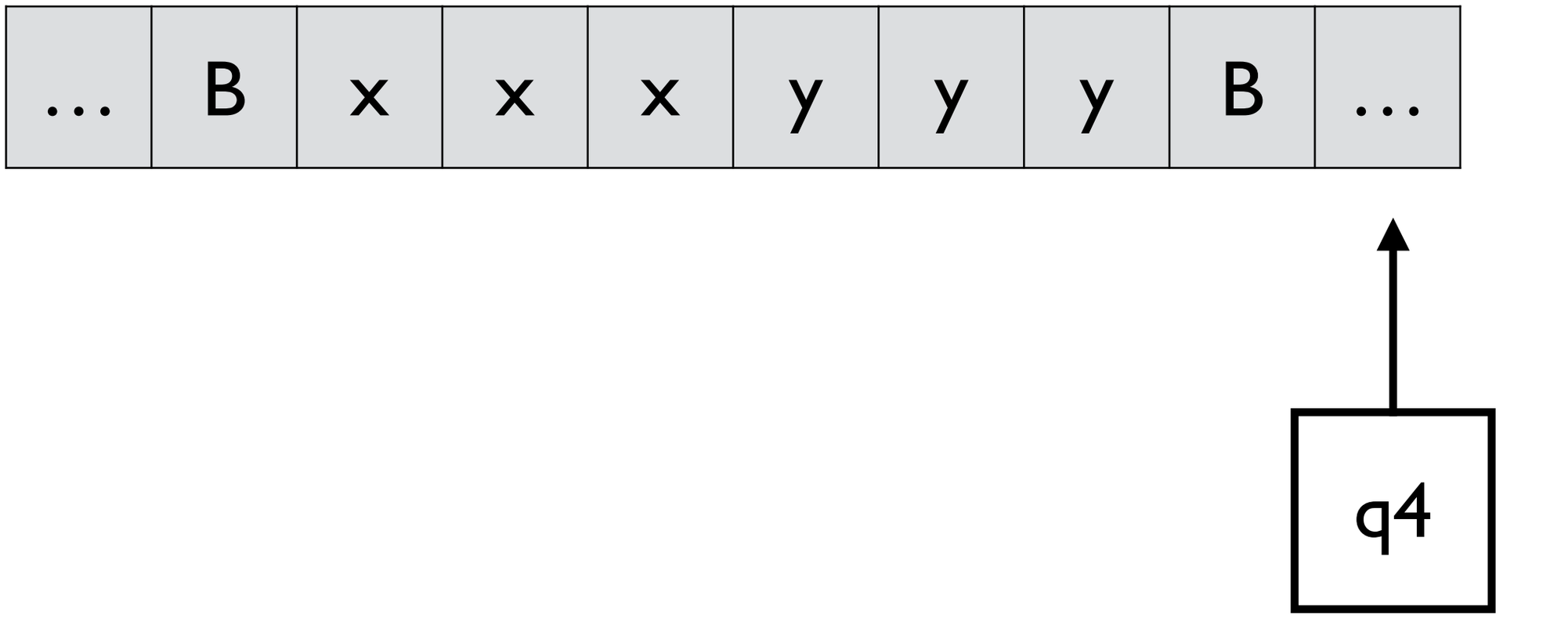


$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



(no more b's)

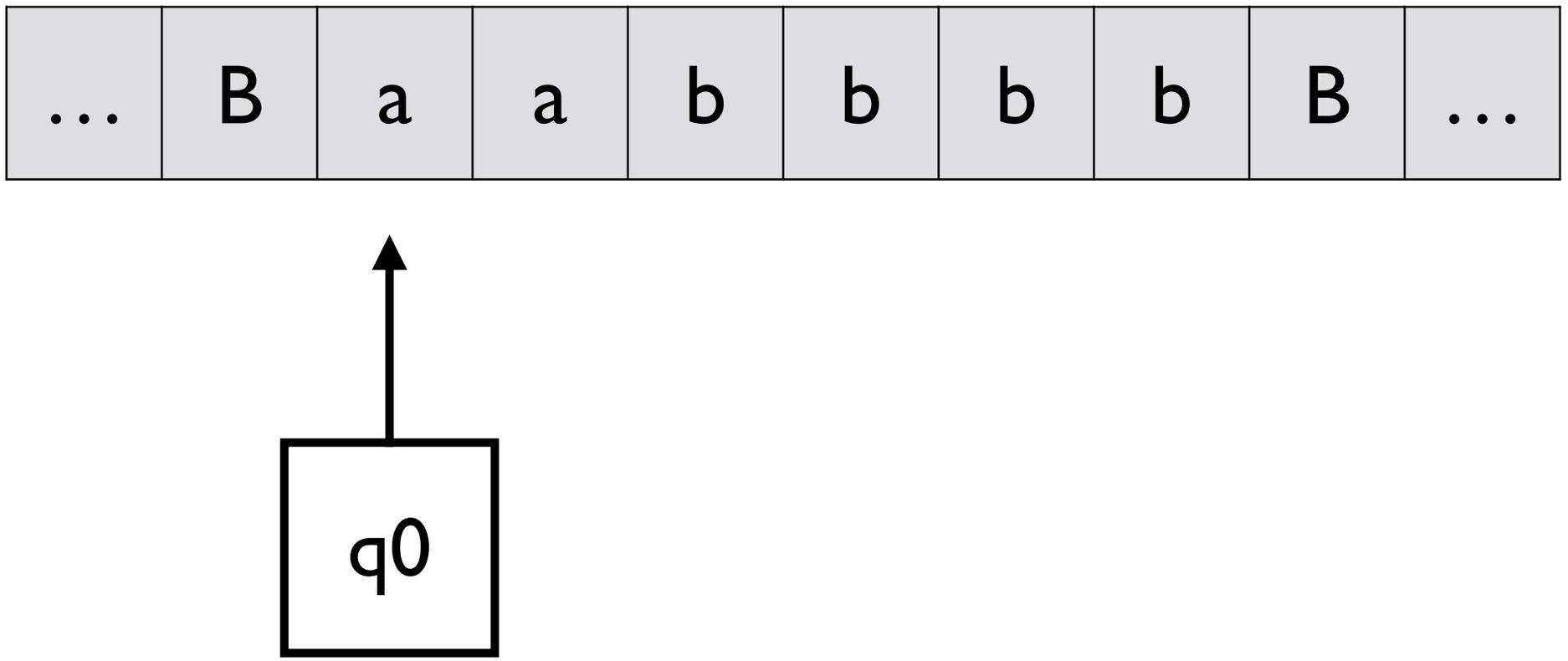
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



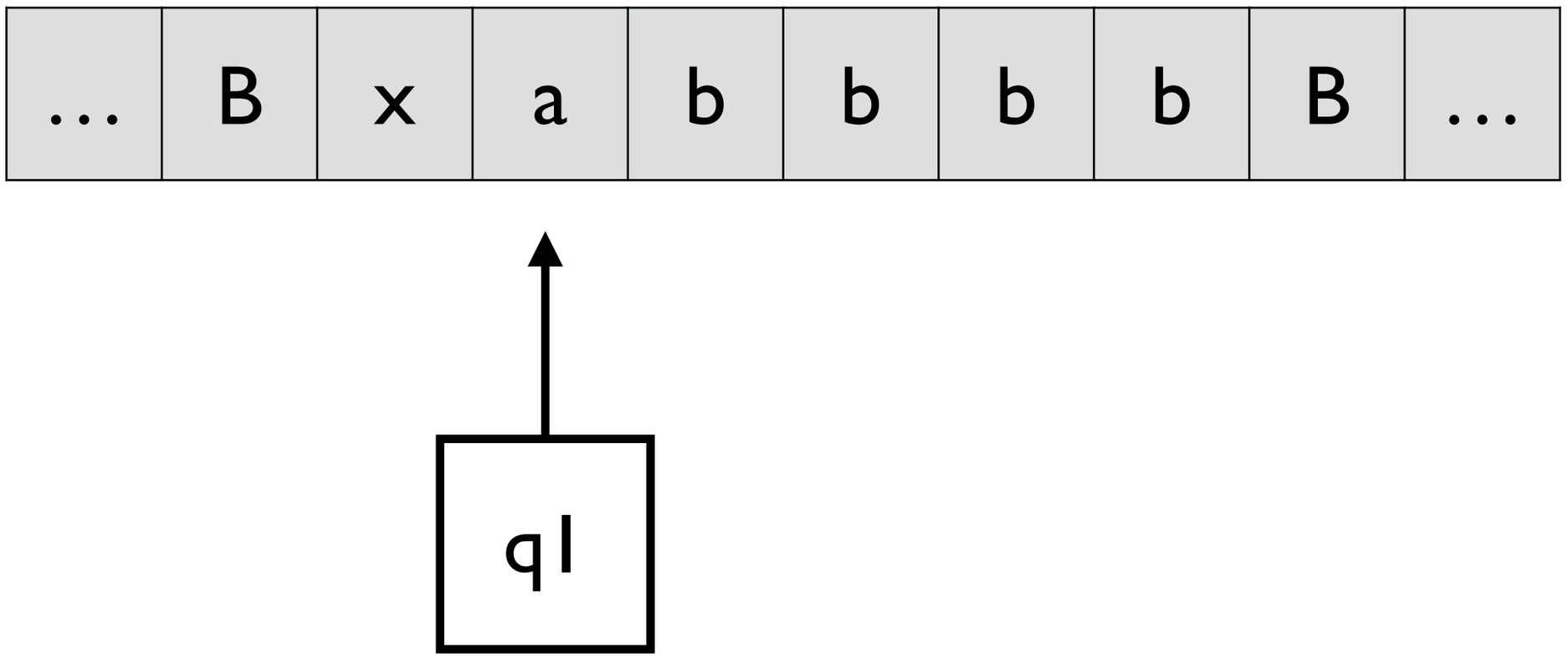
"final state"

$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

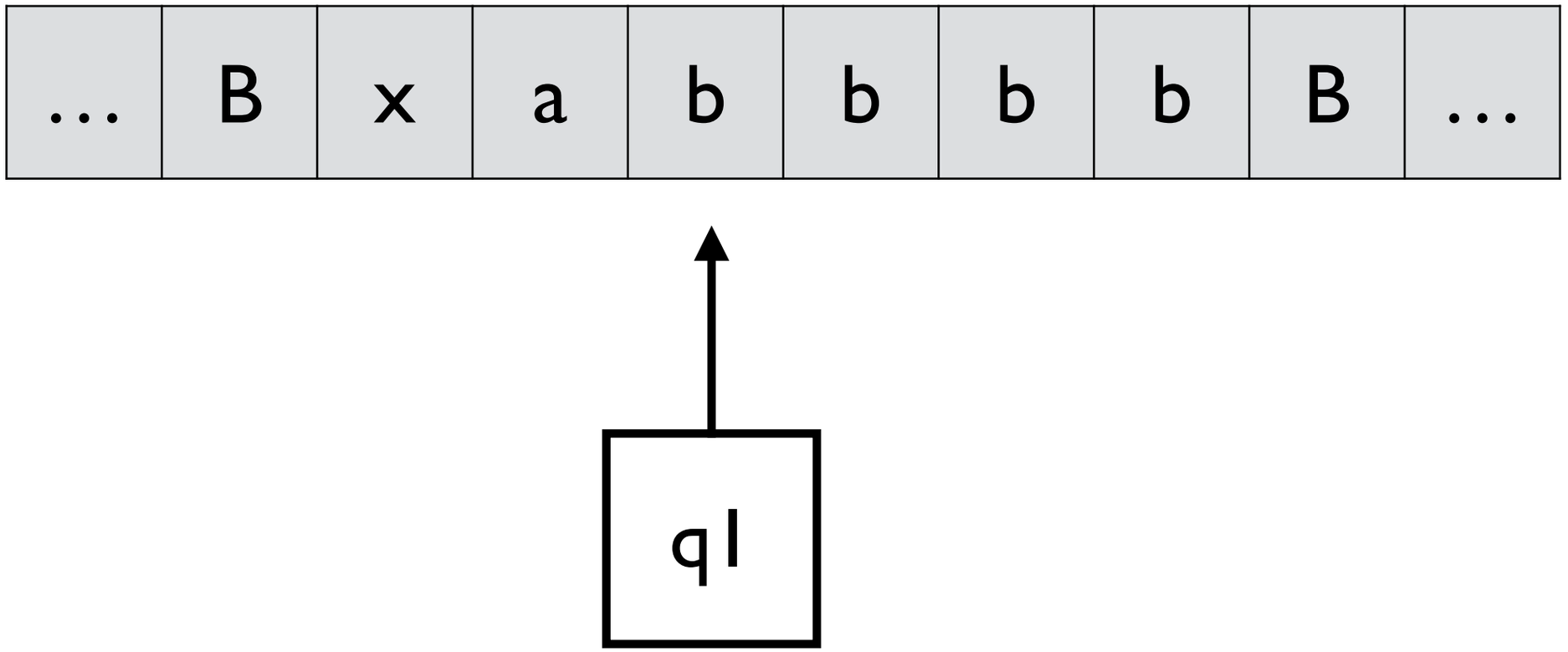
When the input string is not in the language:



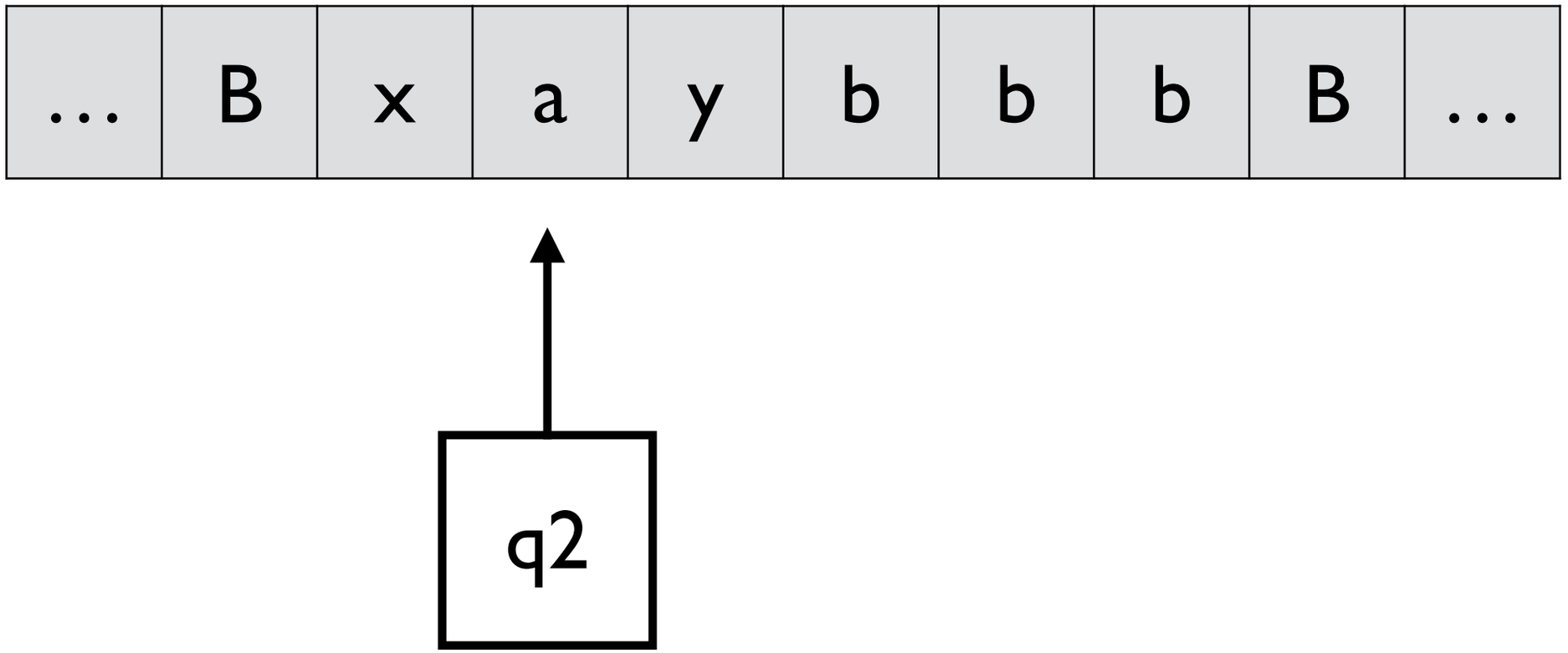
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



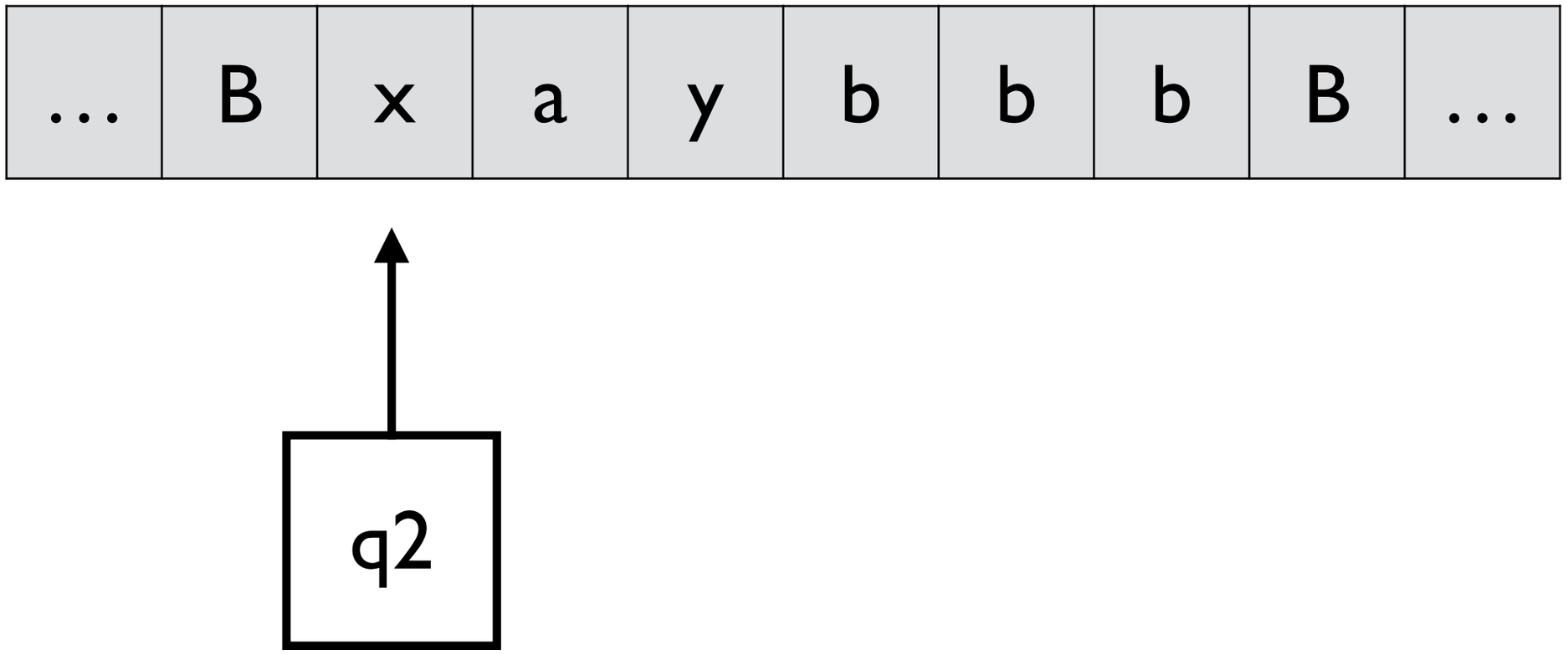
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



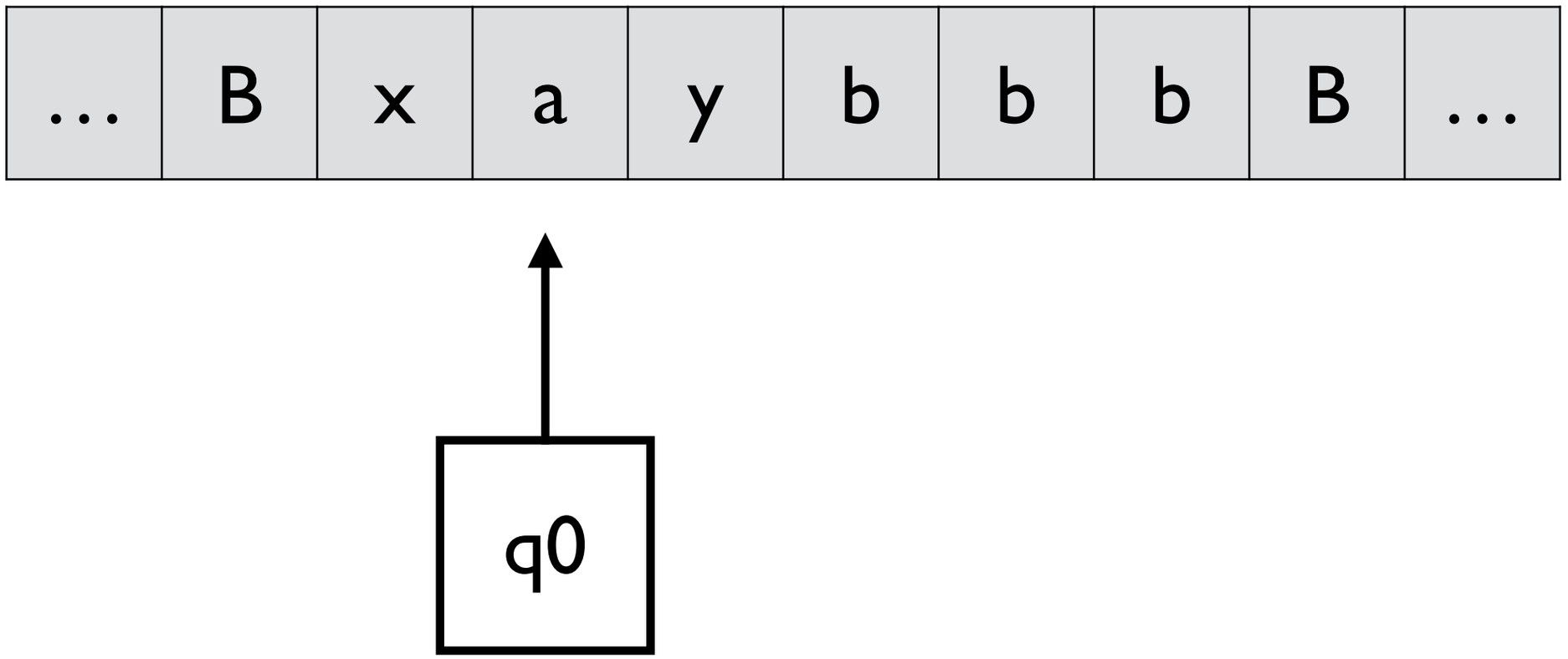
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



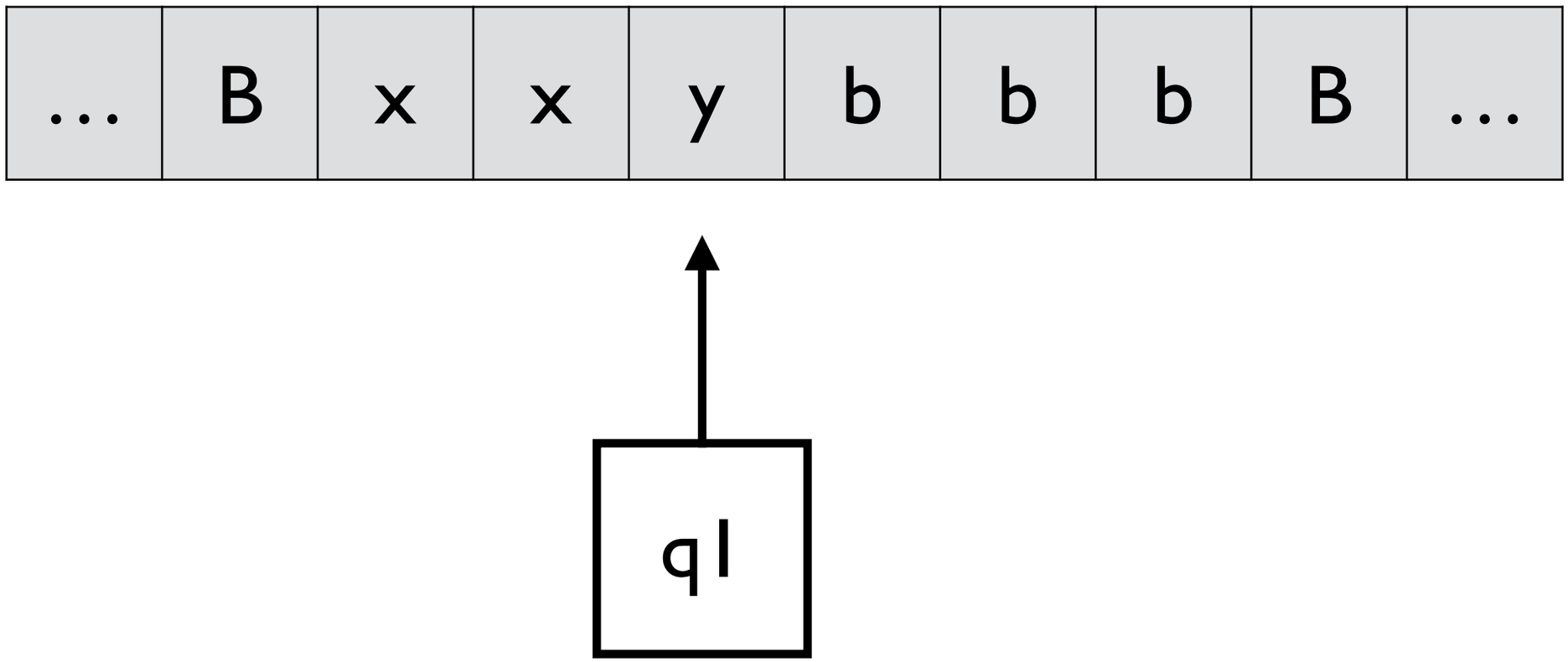
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



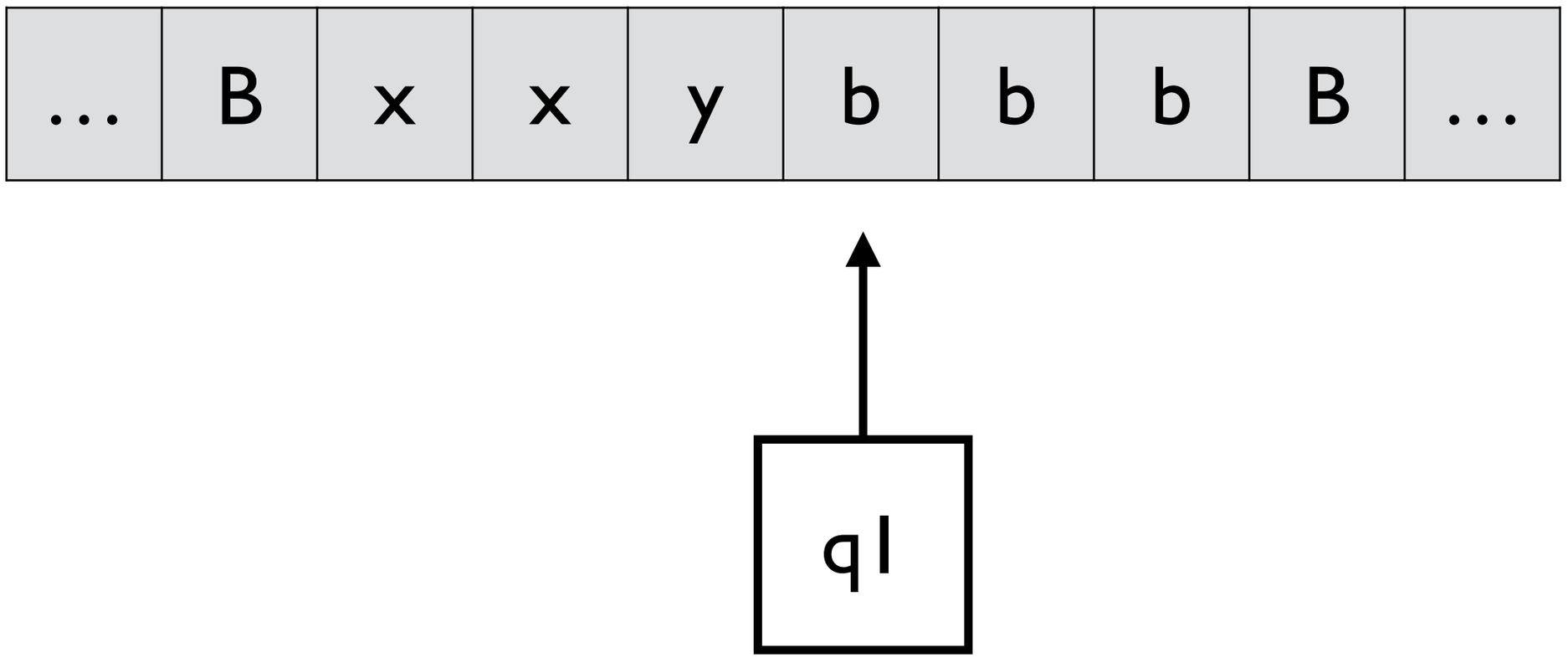
$$\delta(q_0, a) = (q_1, x, R) \qquad \delta(q_2, y) = (q_2, y, L) \qquad \delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, a) = (q_2, a, L) \qquad \delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_1, y) = (q_1, y, R) \qquad \delta(q_2, x) = (q_0, x, R) \qquad \delta(q_3, B) = (q_4, B, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$



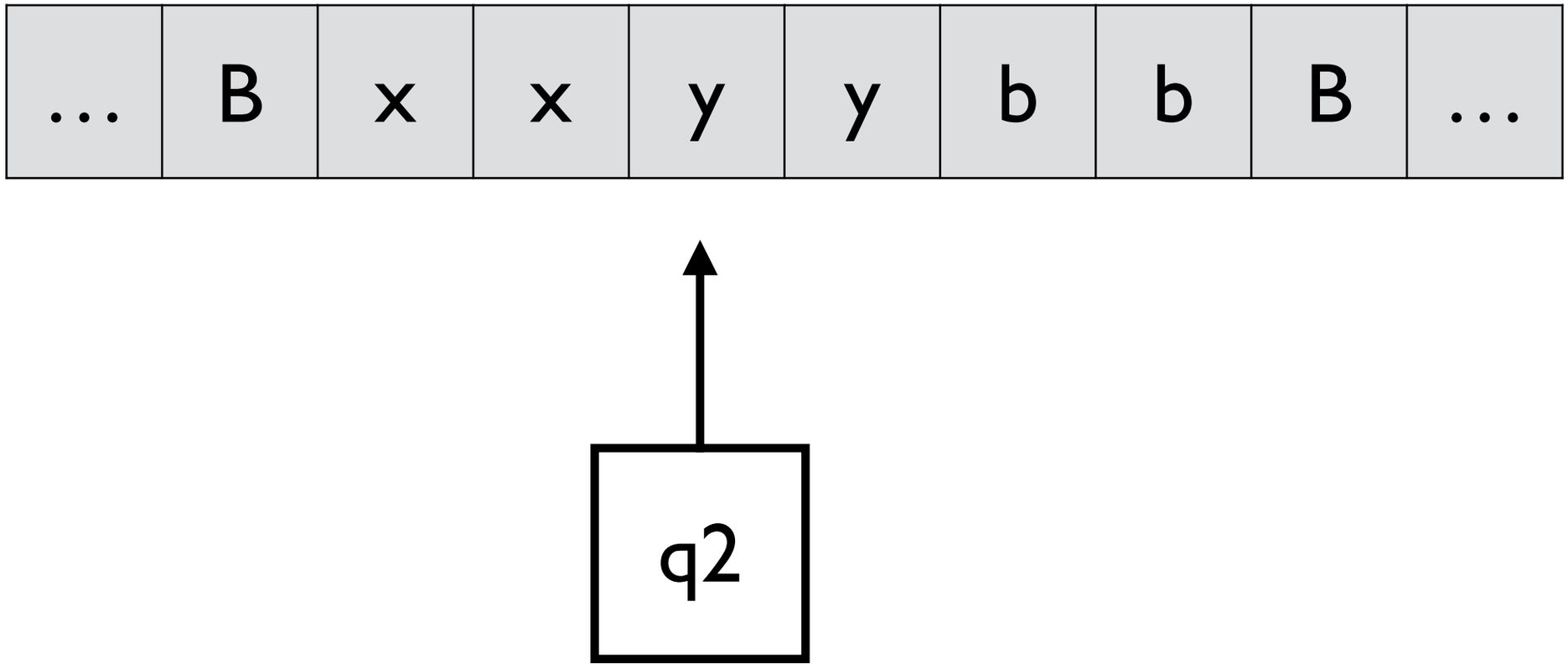
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



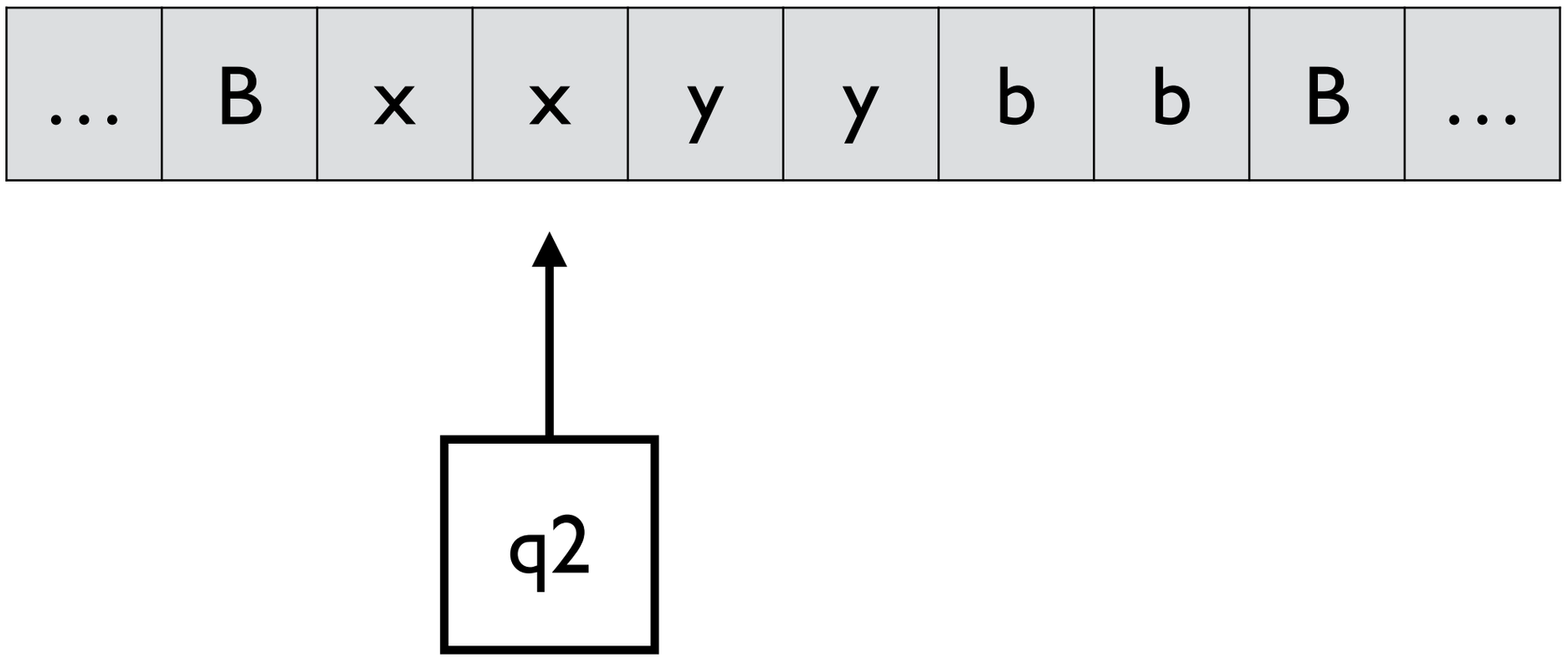
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



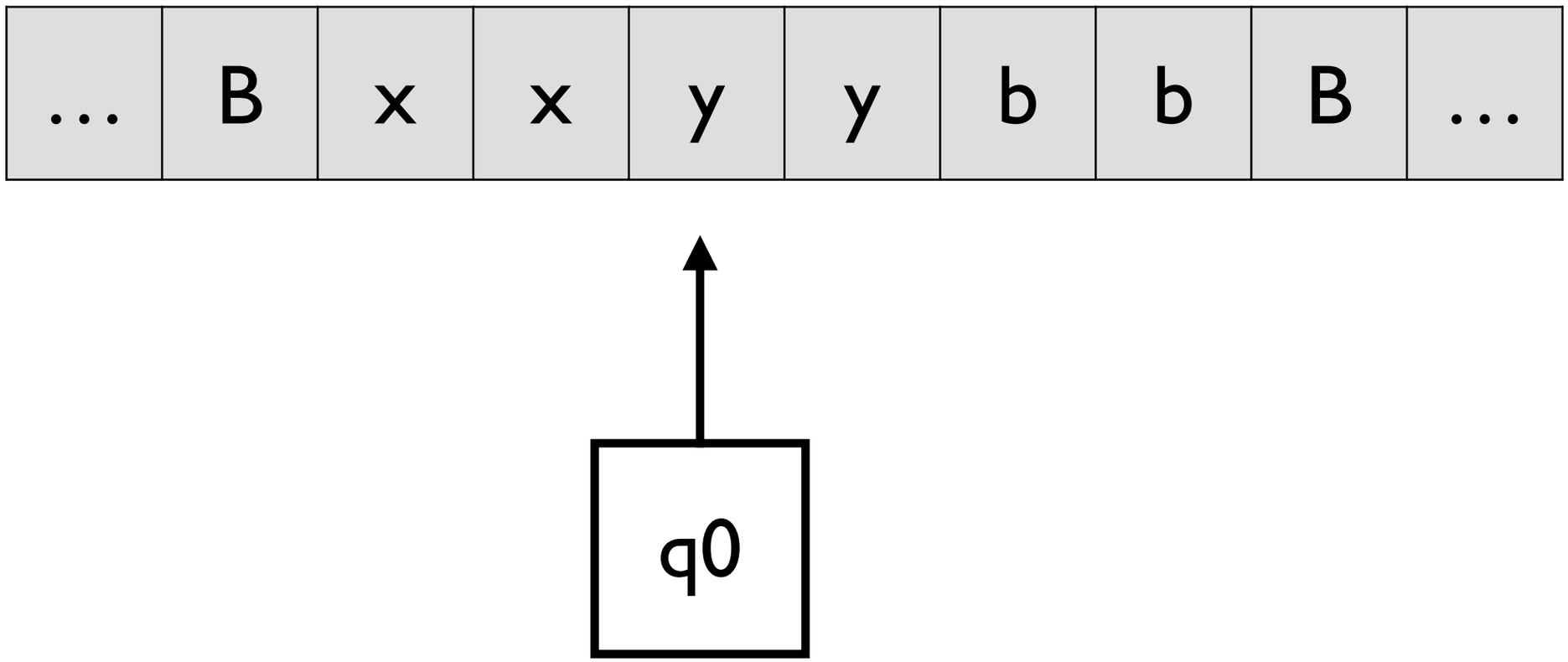
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



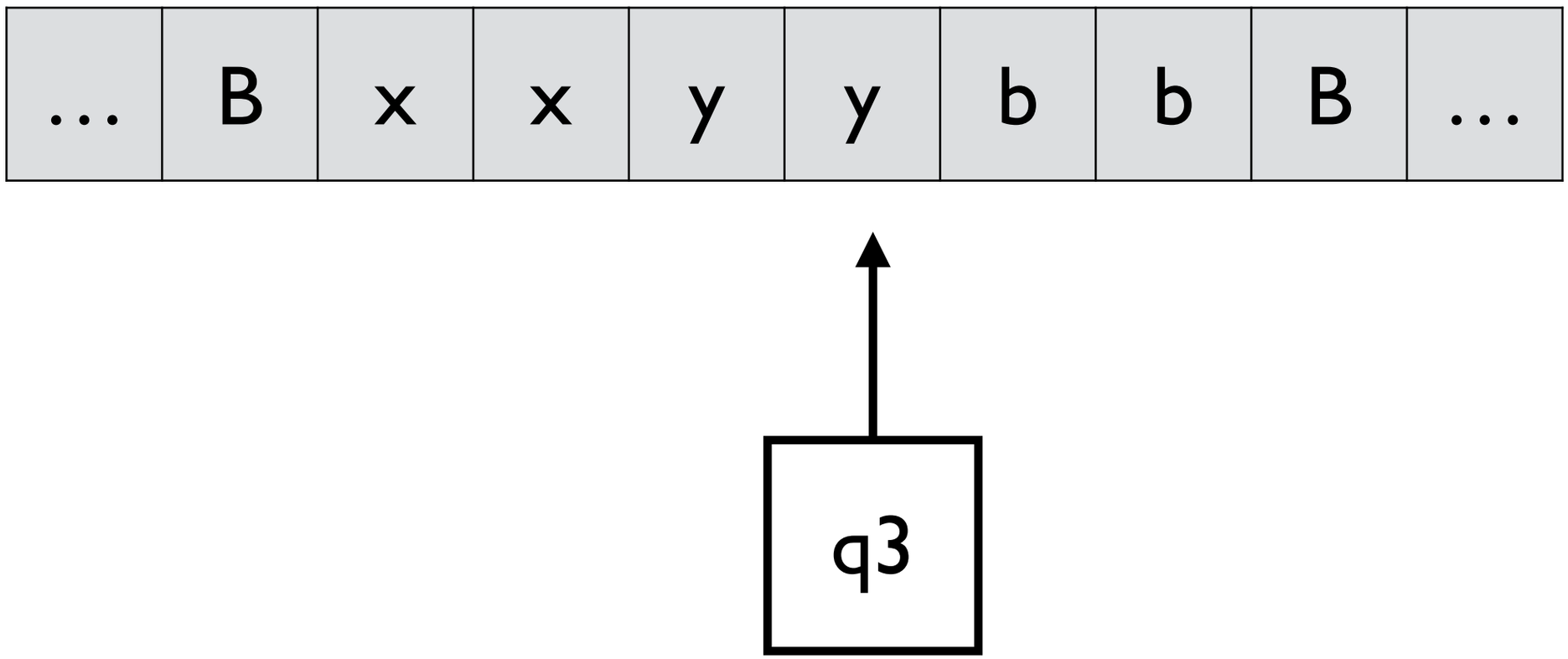
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



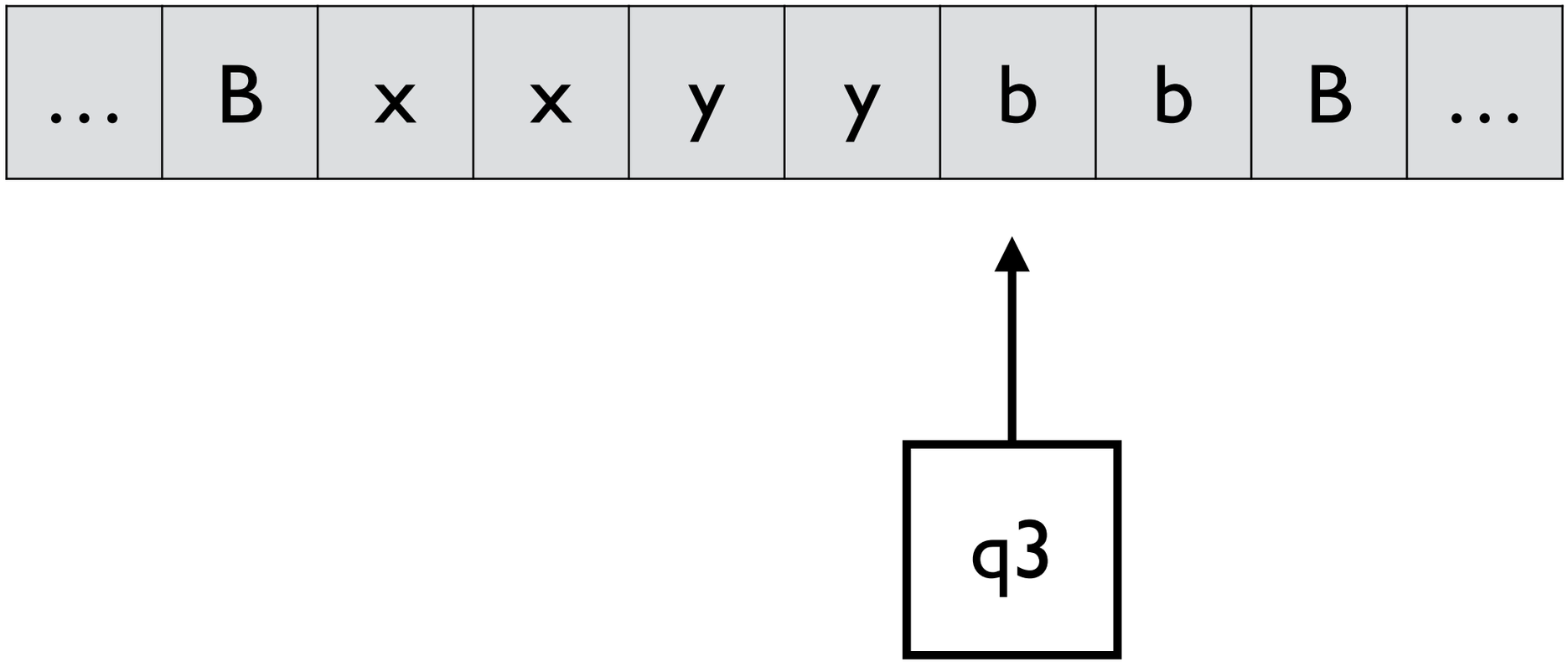
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$



(undefined, halt)

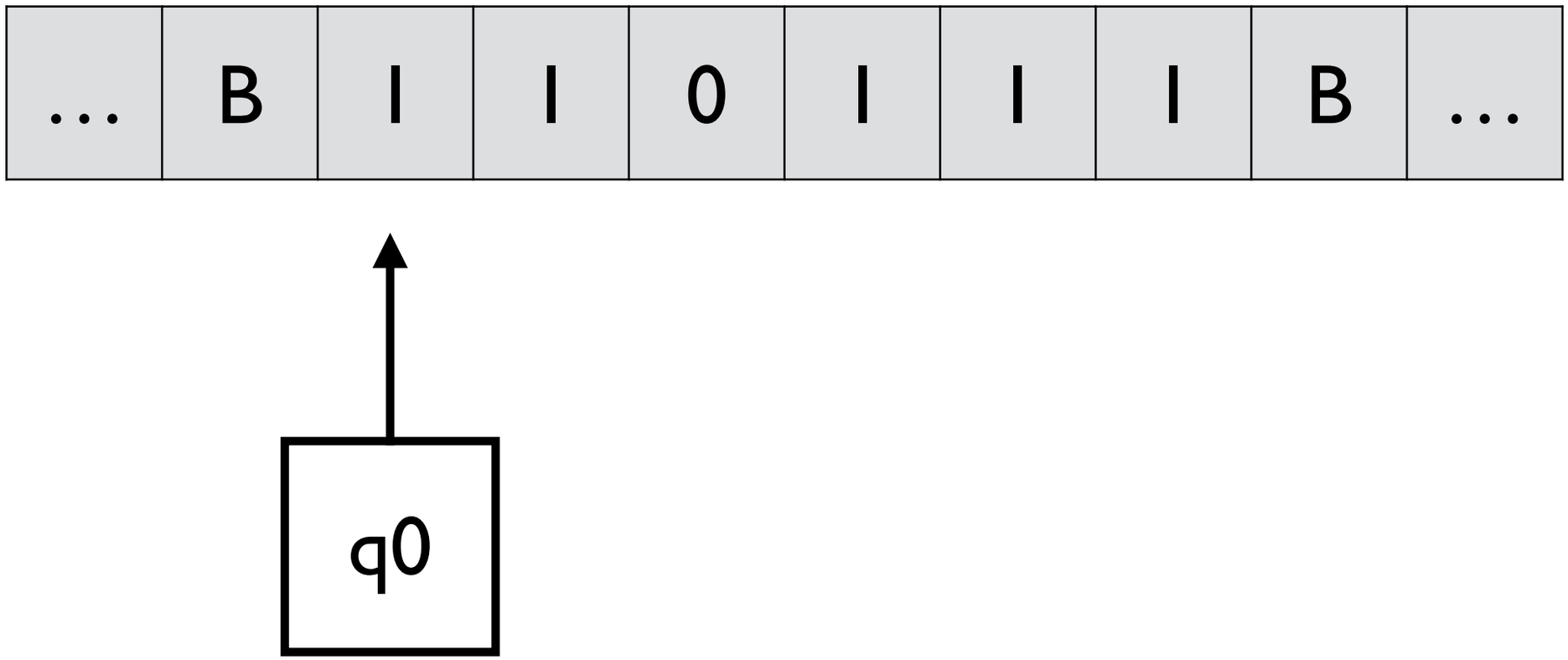
$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$
$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$
$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

Example 2. Given x and y, design a Turing machine that computes $x + y$.

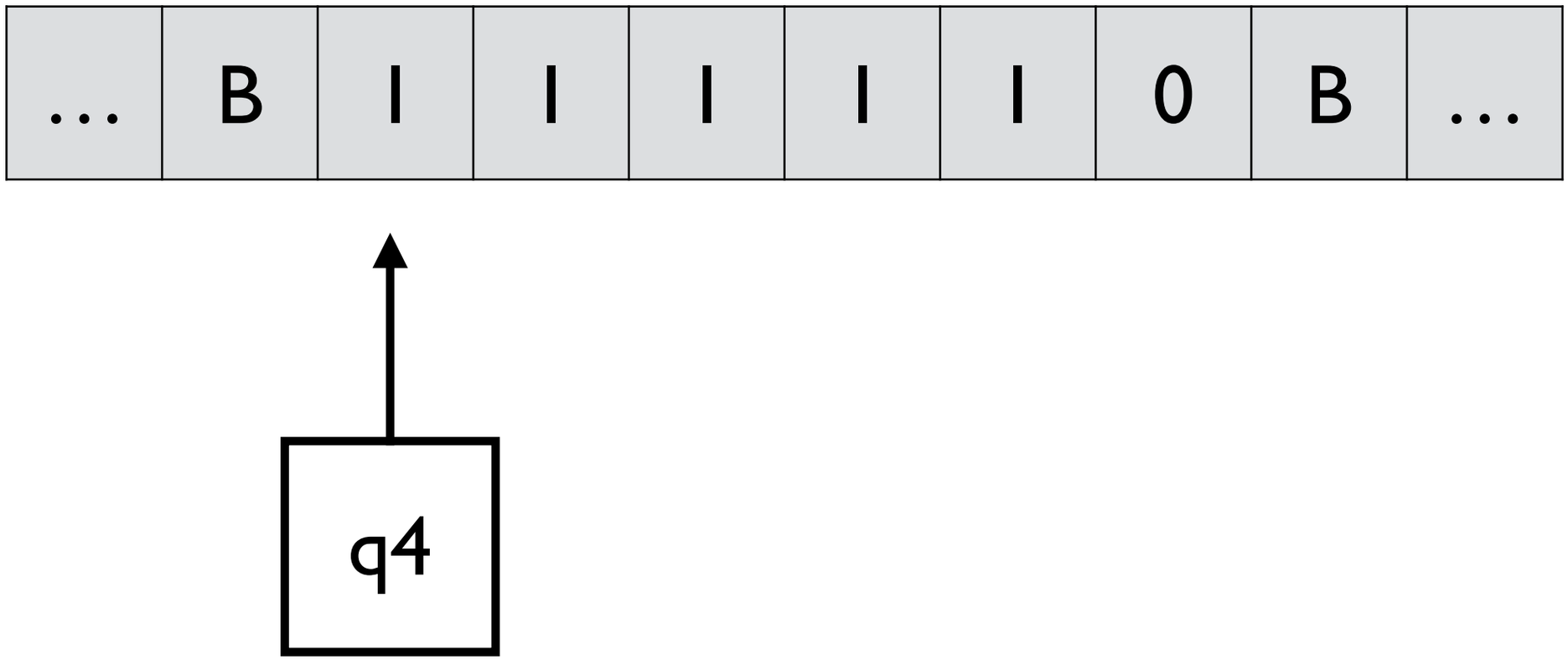
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_4\})$$

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}$$

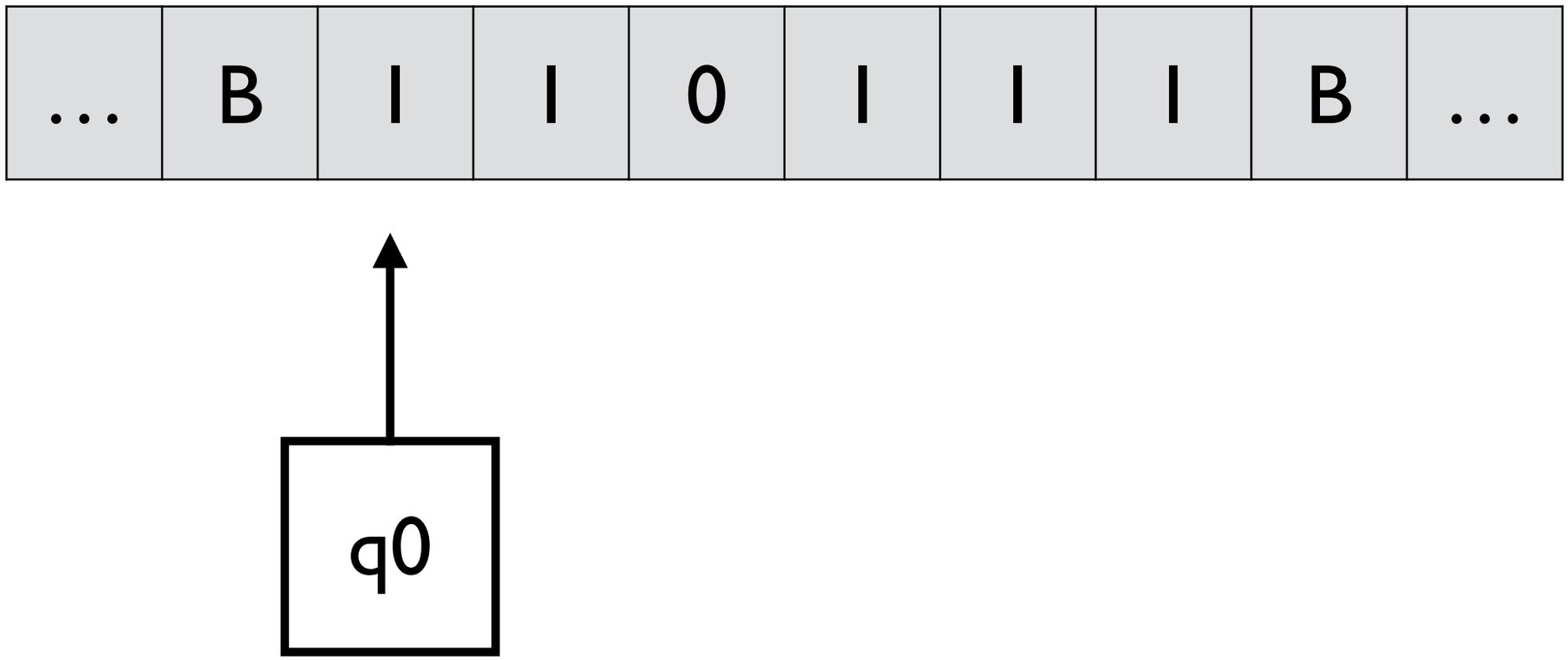
Initial machine configuration:



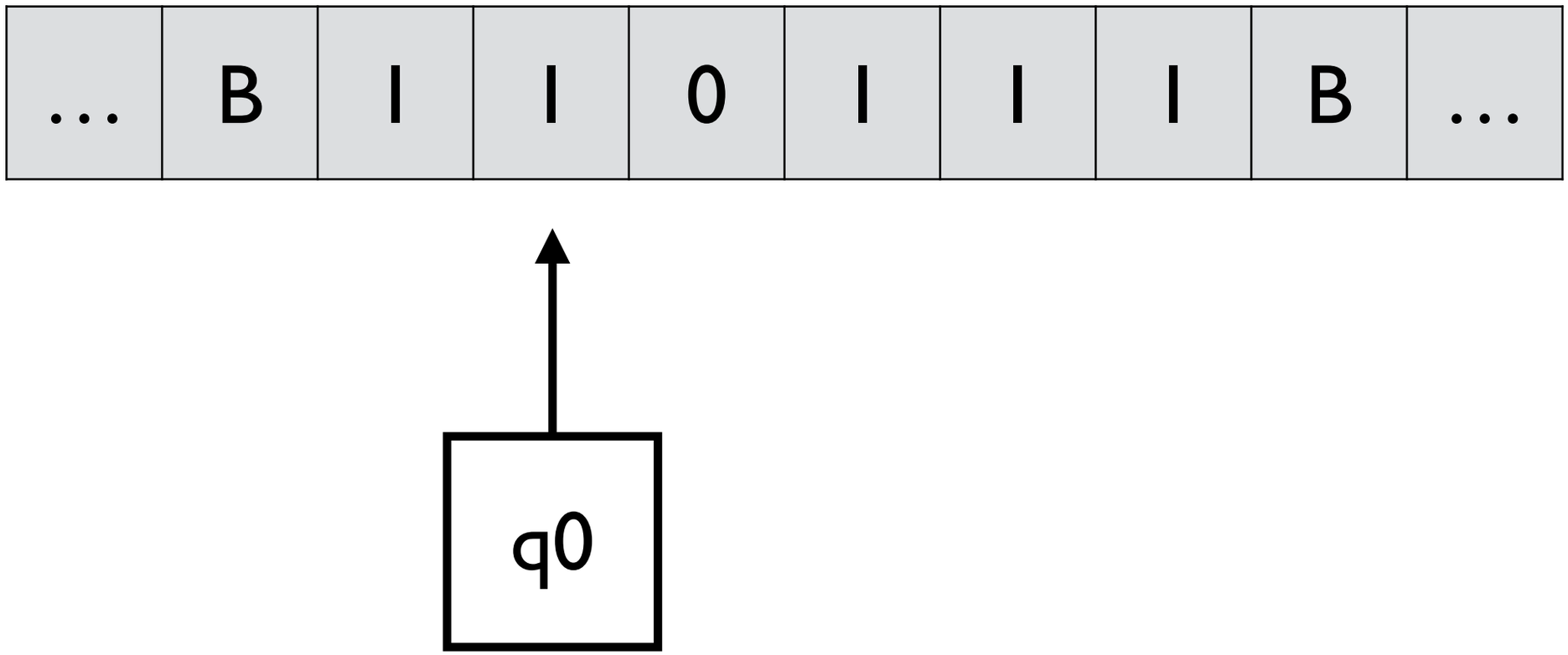
Final machine configuration:



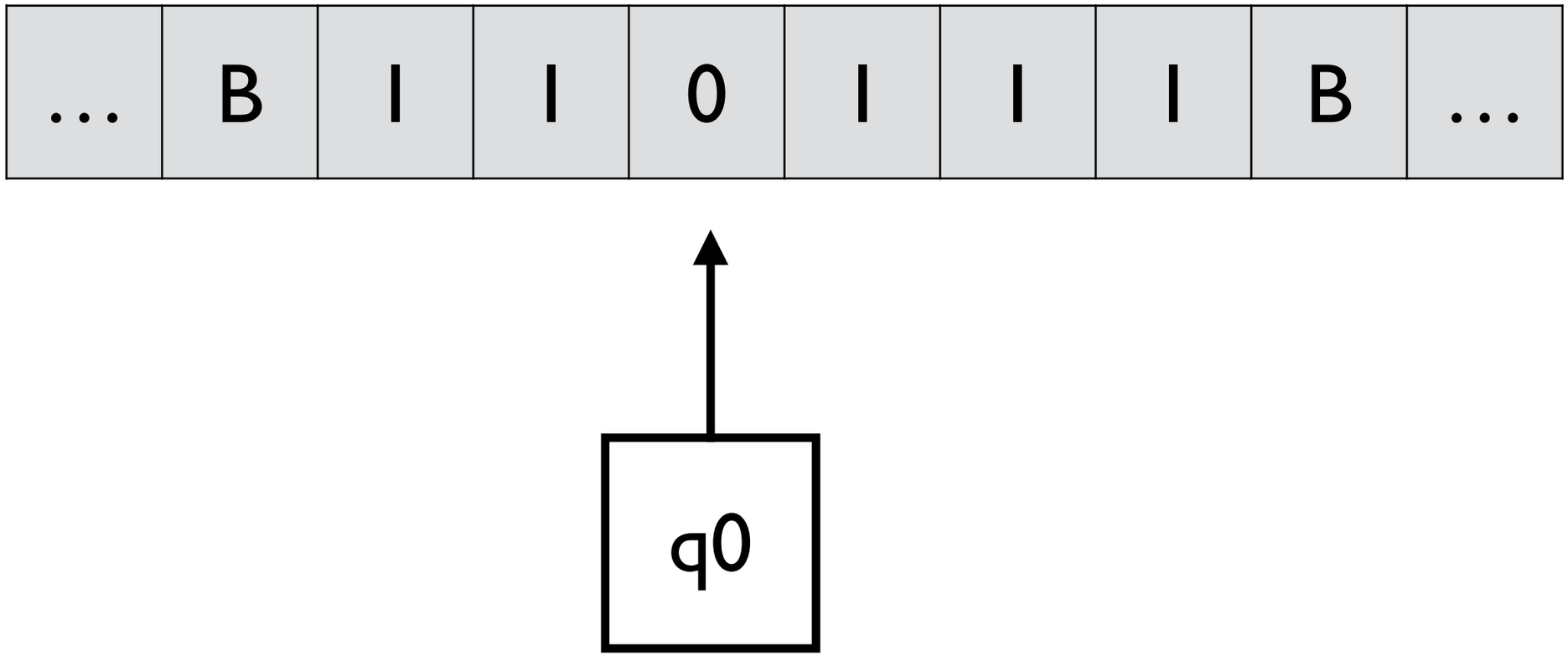
Addition in math is to move 0 to the right end



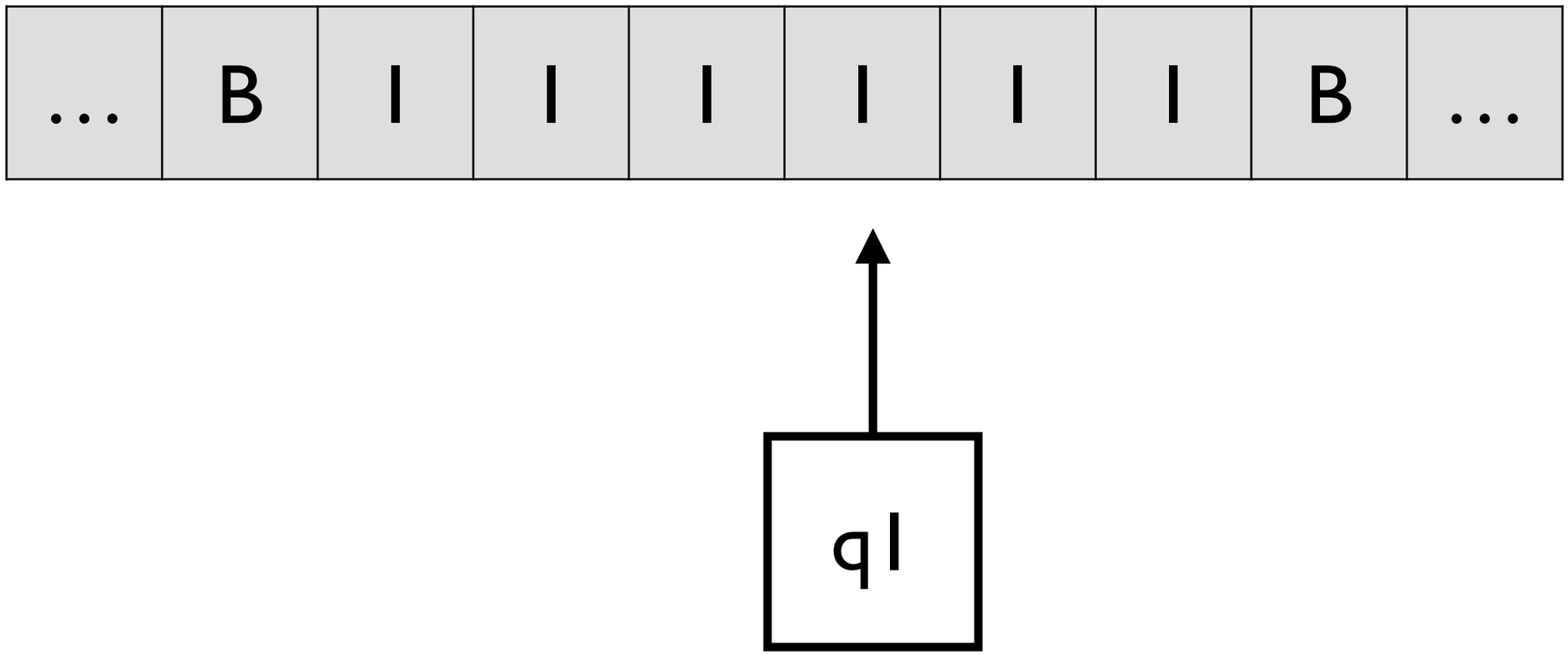
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$

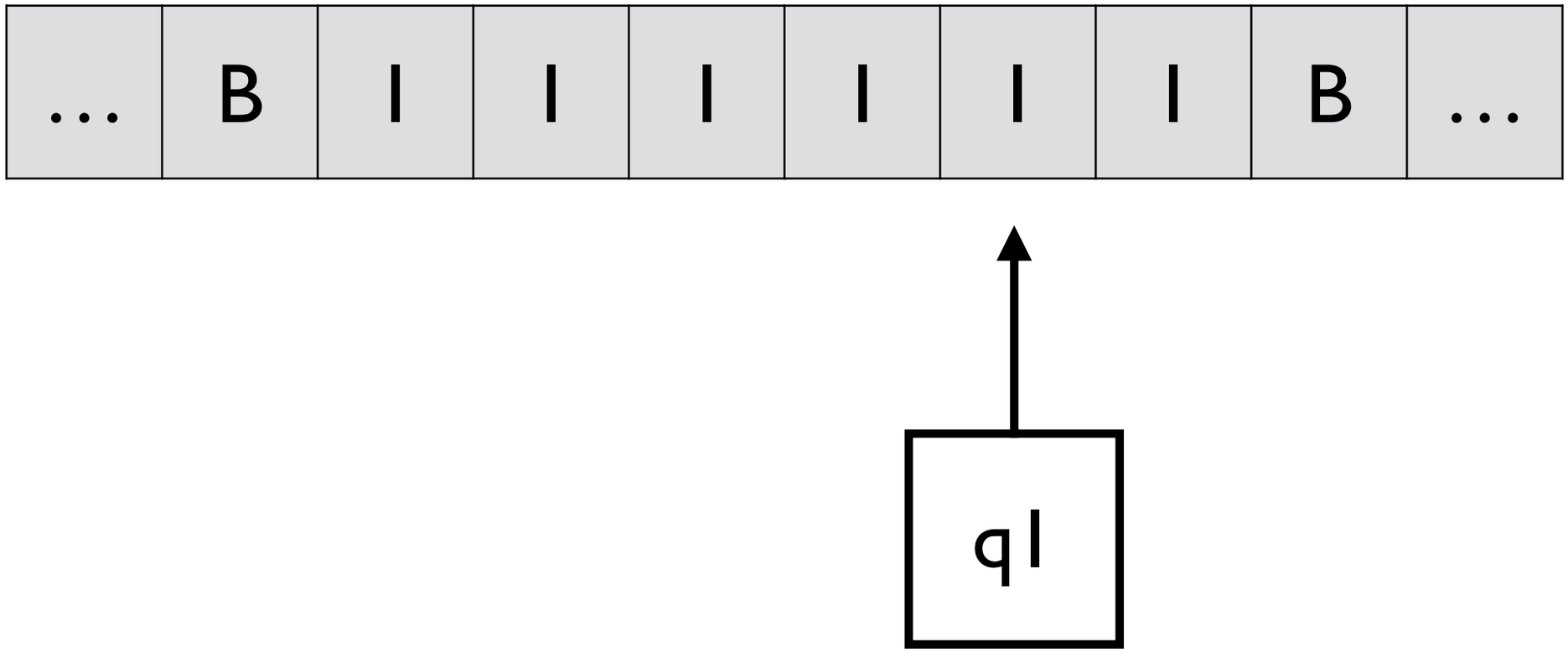


$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$

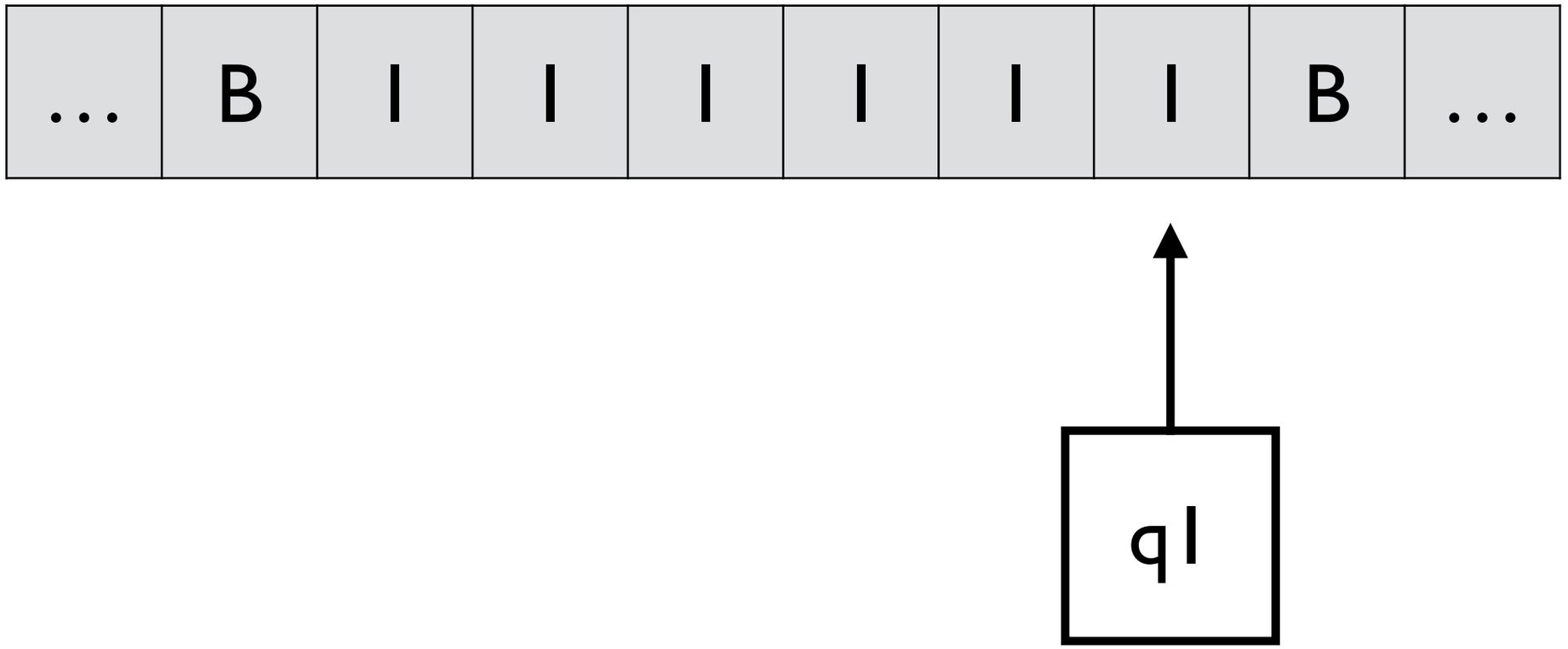


(In q1, search for the right end of y)

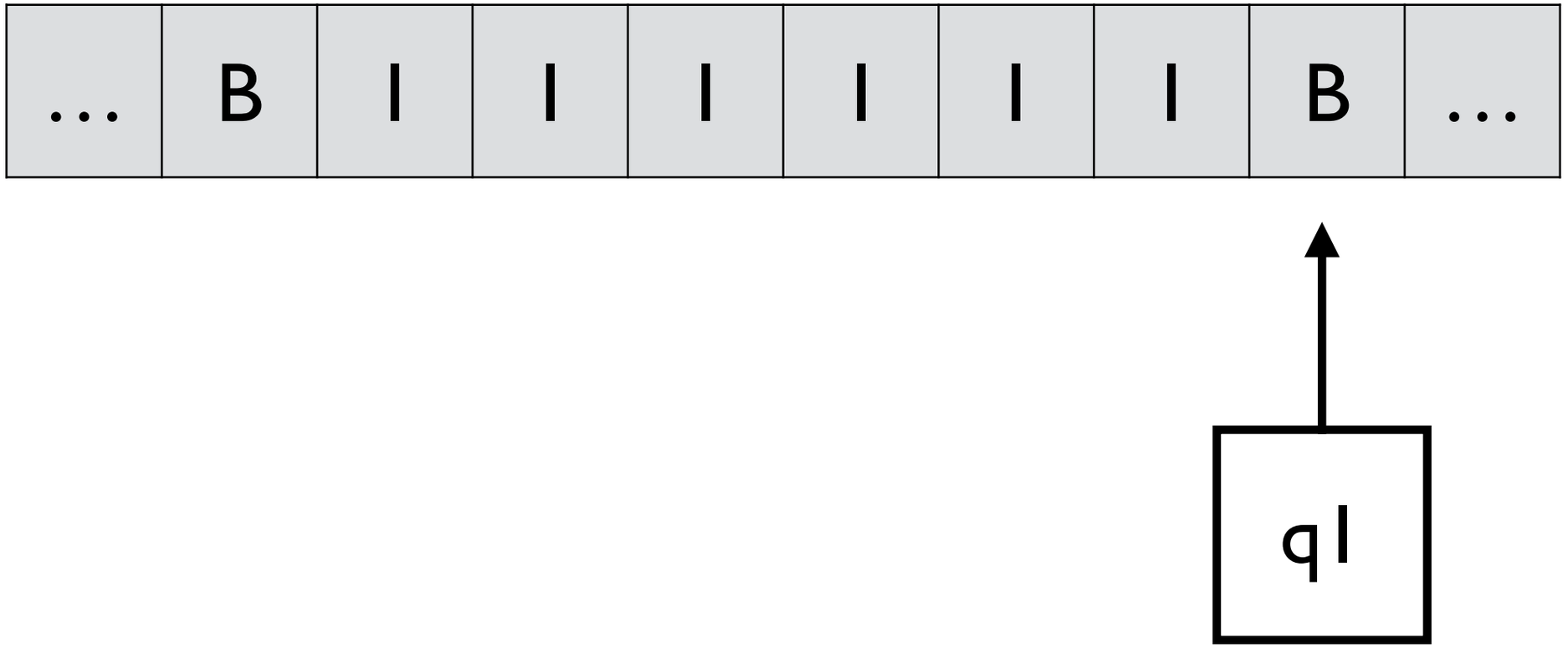
$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, B) = (q_2, B, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, B) = (q_4, B, R)$$



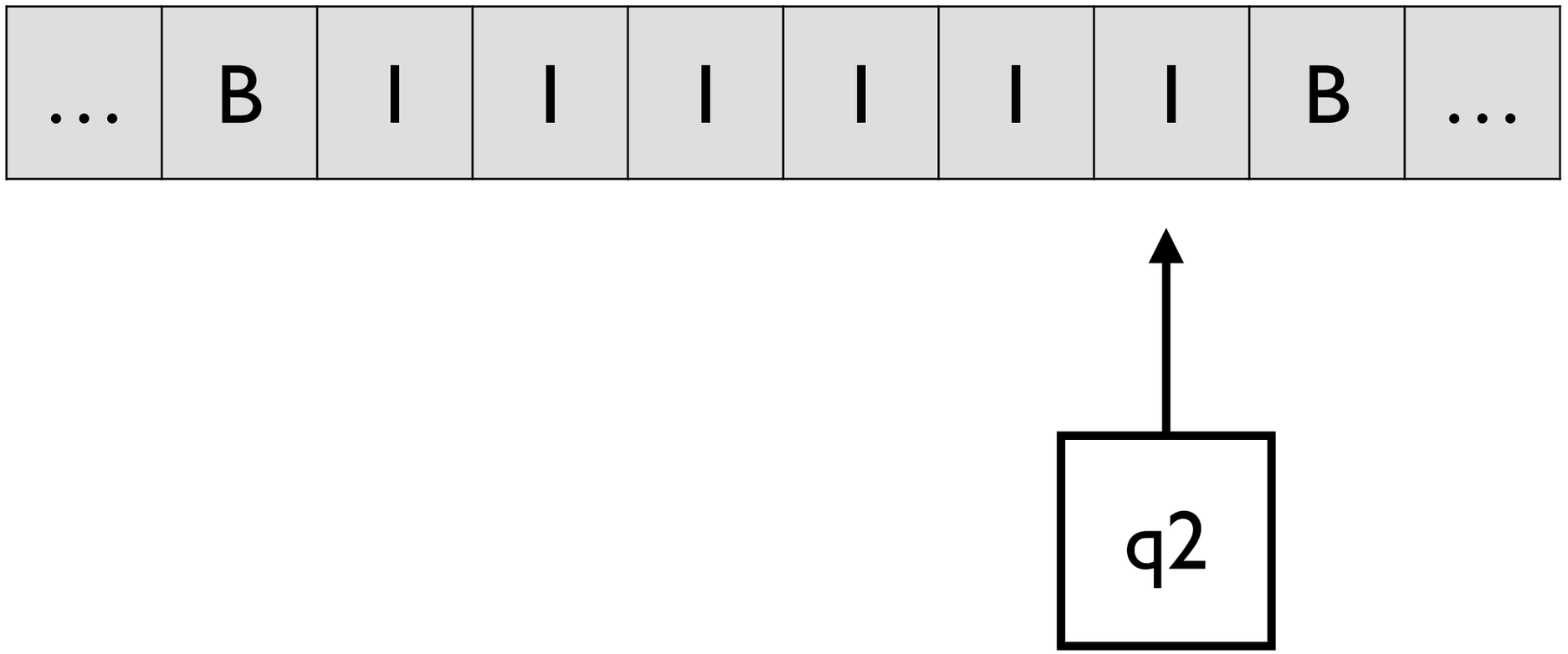
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$

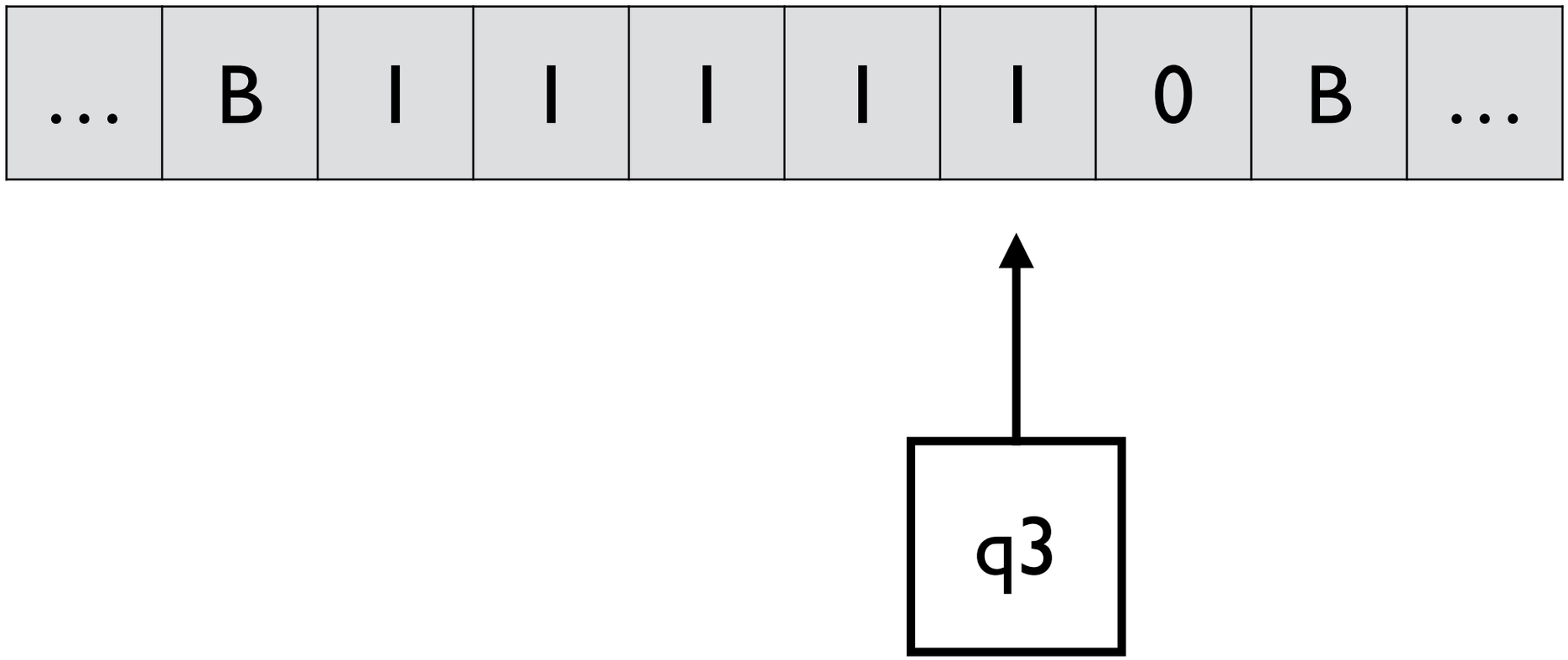


$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



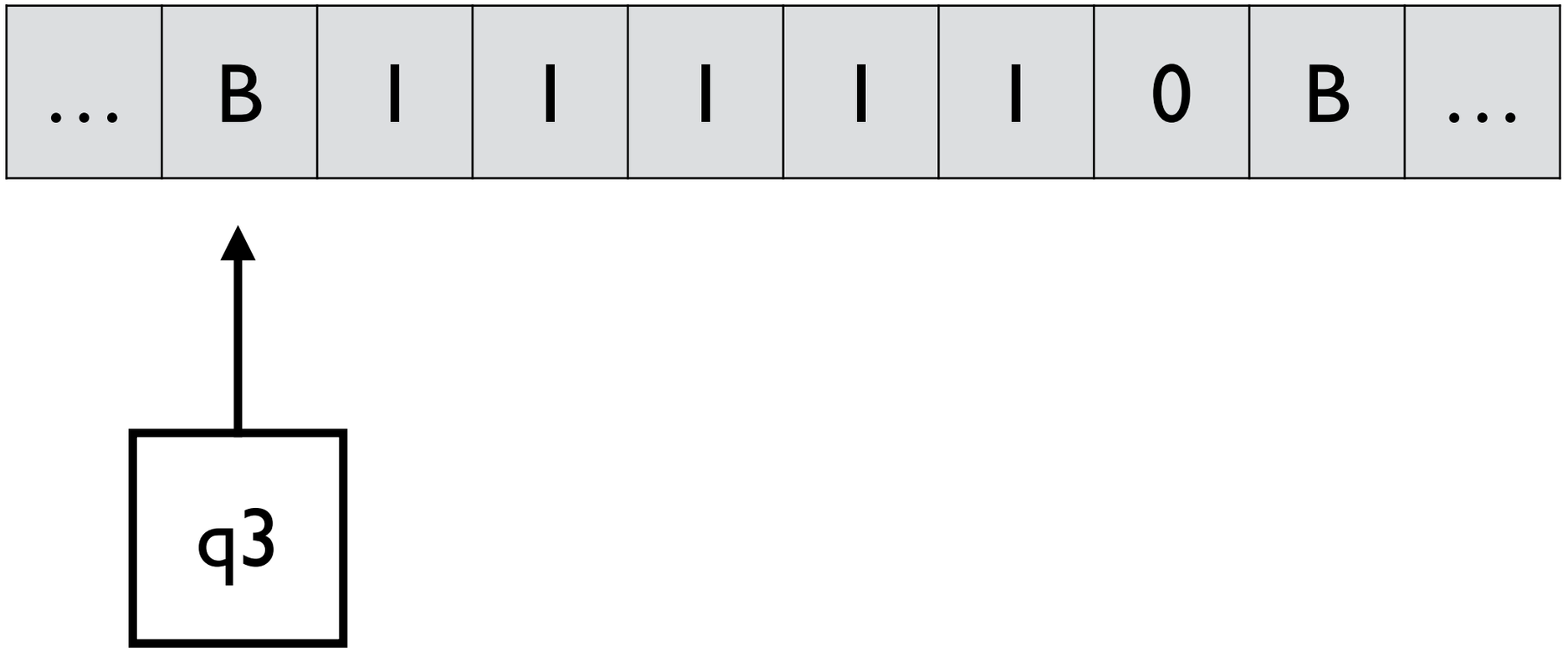
(In q2, replace the rightmost 1 by 0)

$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, B) = (q_2, B, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, B) = (q_4, B, R)$$

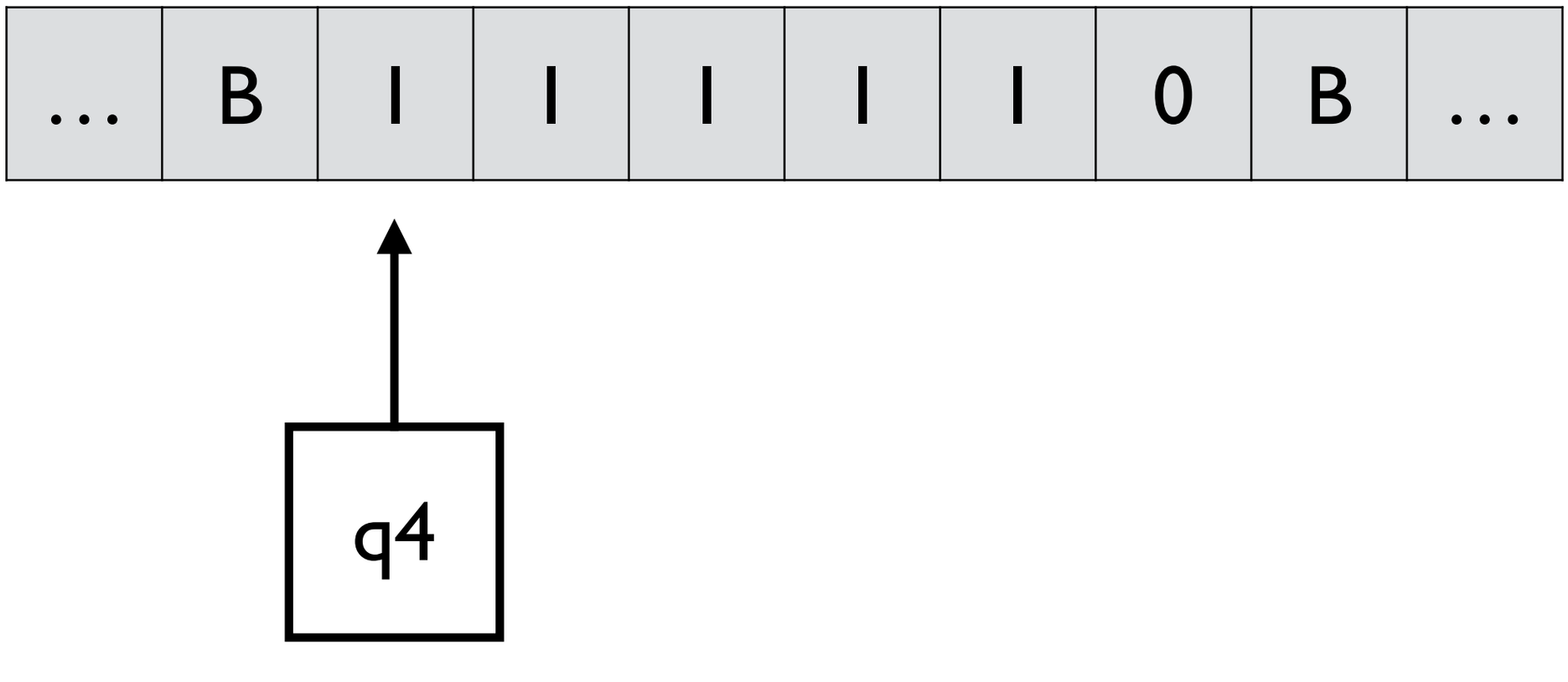


(In q3, look for the leftmost 1)

$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, B) = (q_2, B, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$
$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, B) = (q_4, B, R)$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}
$$



"final state"

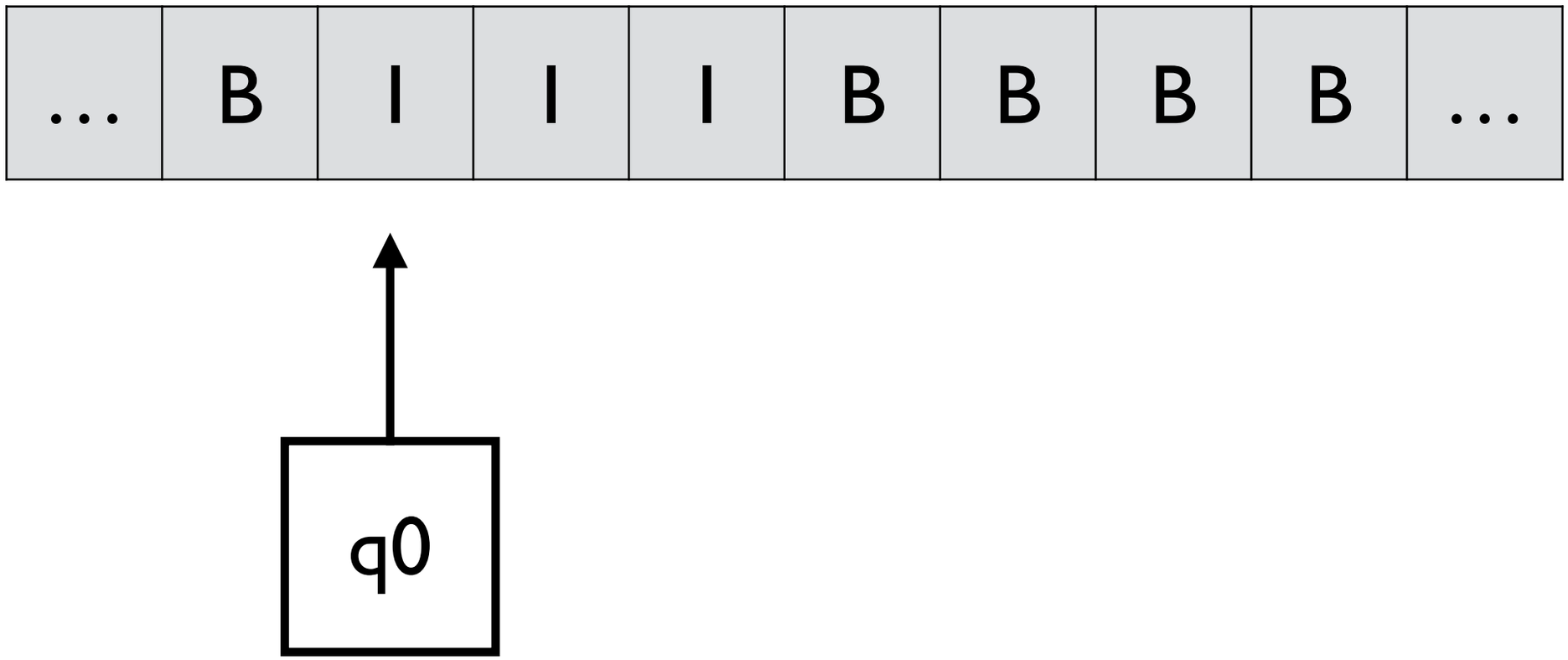
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, 1, R) \\
\delta(q_0, 0) &= (q_1, 1, R) \\
\delta(q_1, 1) &= (q_1, 1, R) \\
\delta(q_1, B) &= (q_2, B, L) \\
\delta(q_2, 1) &= (q_3, 0, L) \\
\delta(q_3, 1) &= (q_3, 1, L) \\
\delta(q_3, B) &= (q_4, B, R)
\end{aligned}$$

Example 3. Design a Turing machine that transforms w into ww.

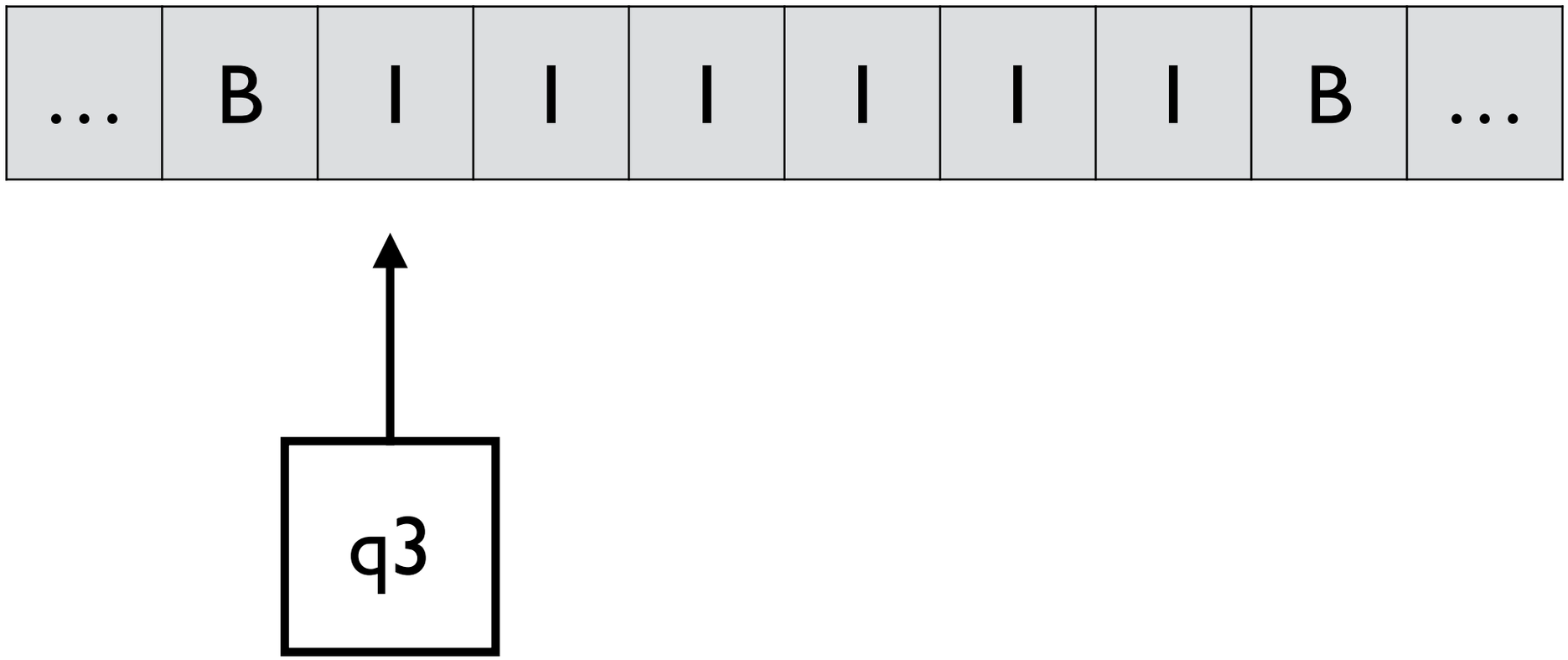
$$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$$

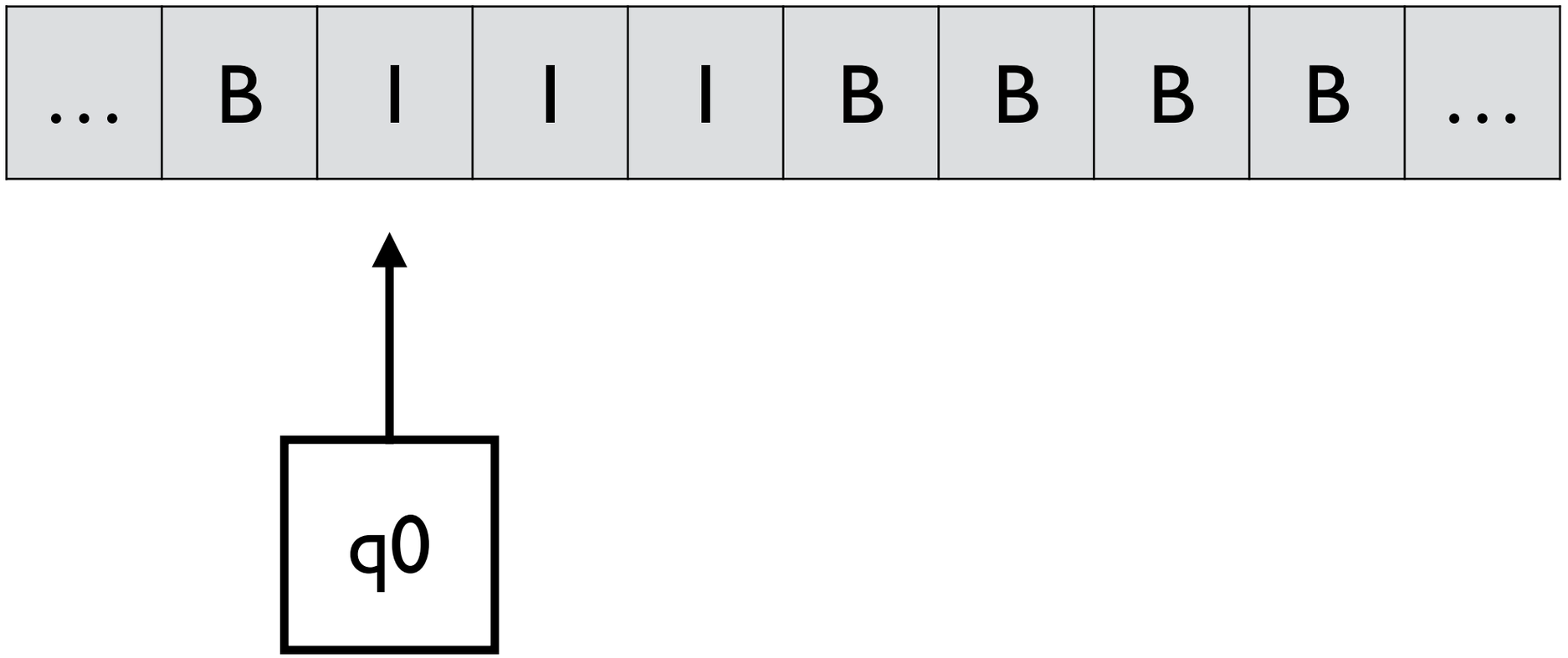
$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$

Initial machine configuration:

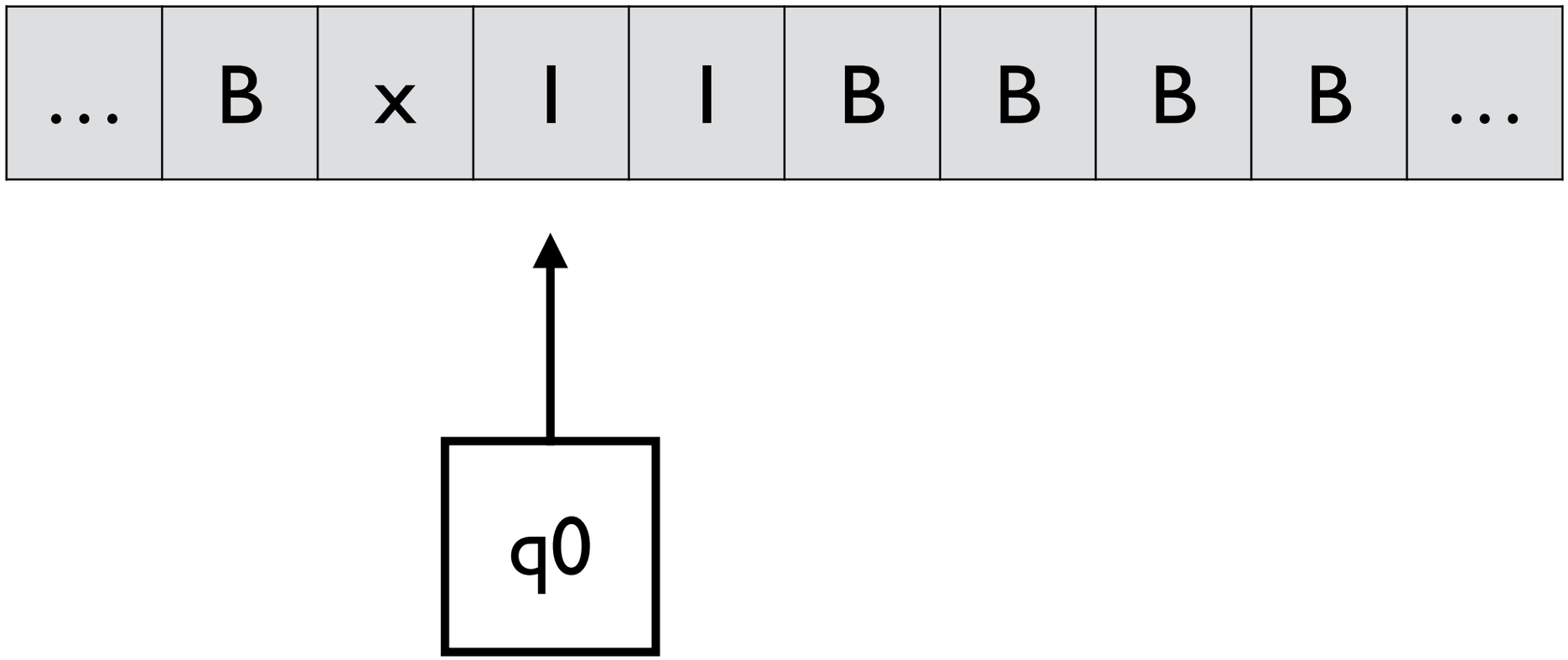


Final machine configuration:

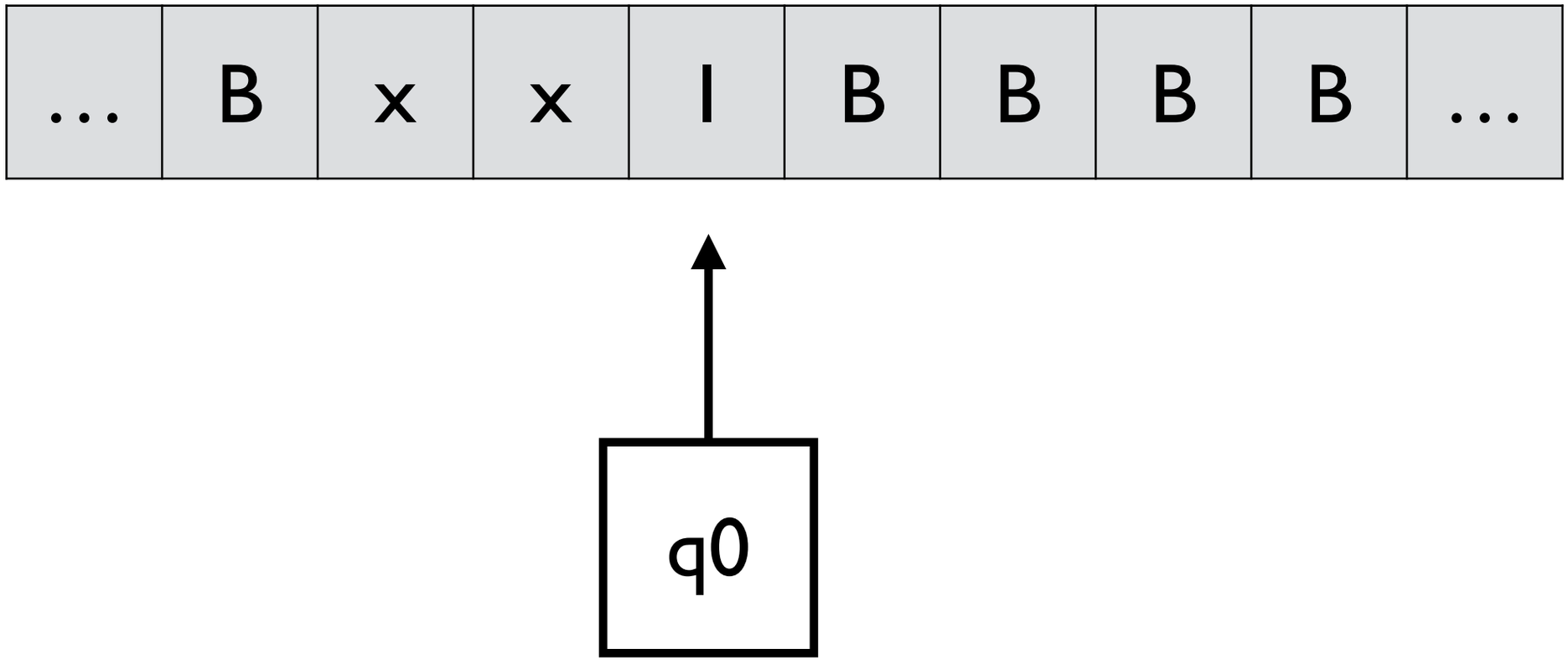




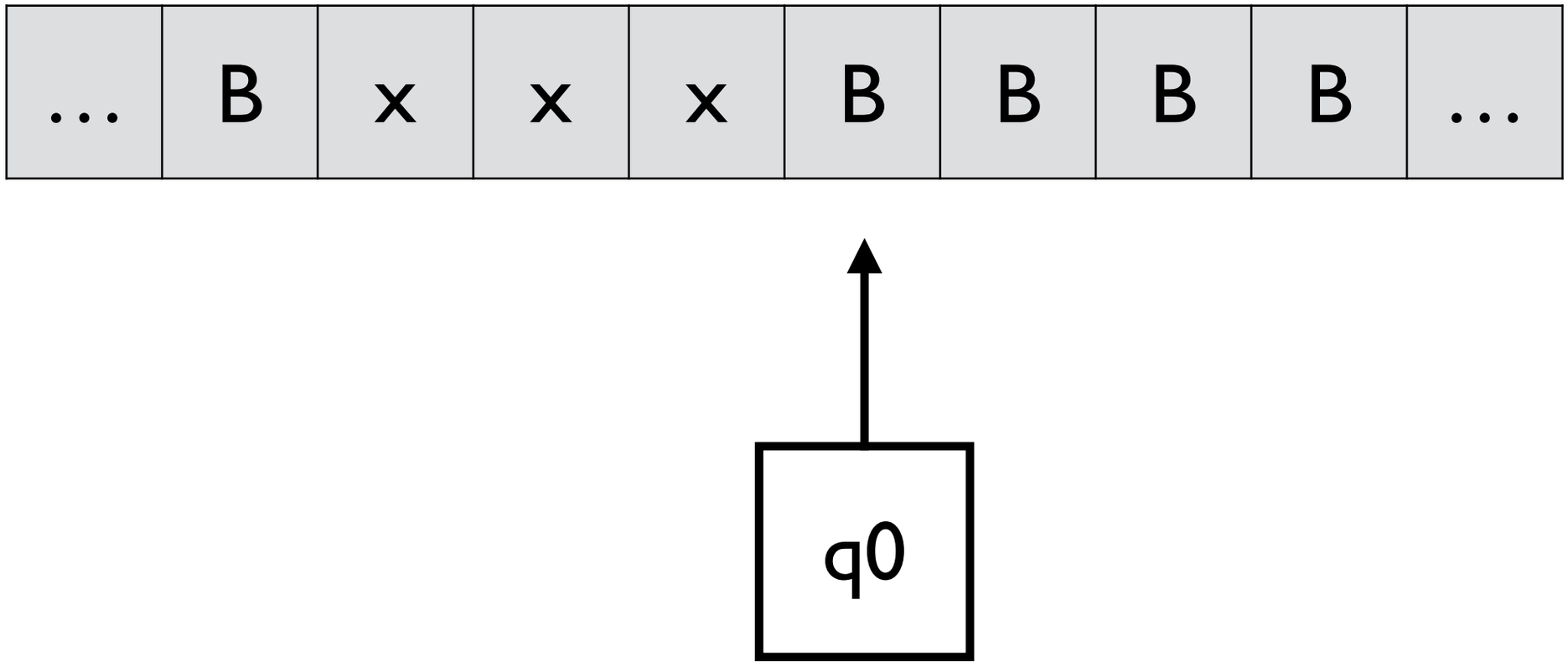
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$

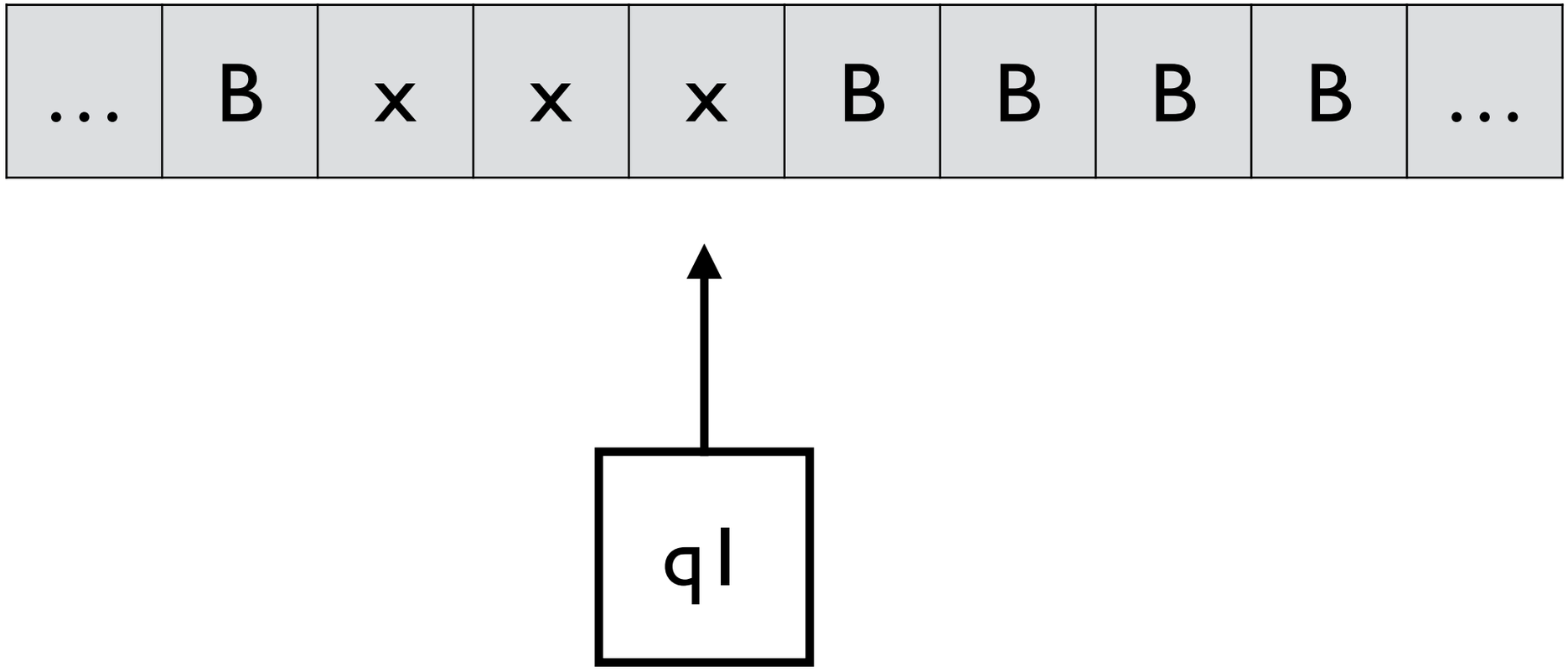


$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



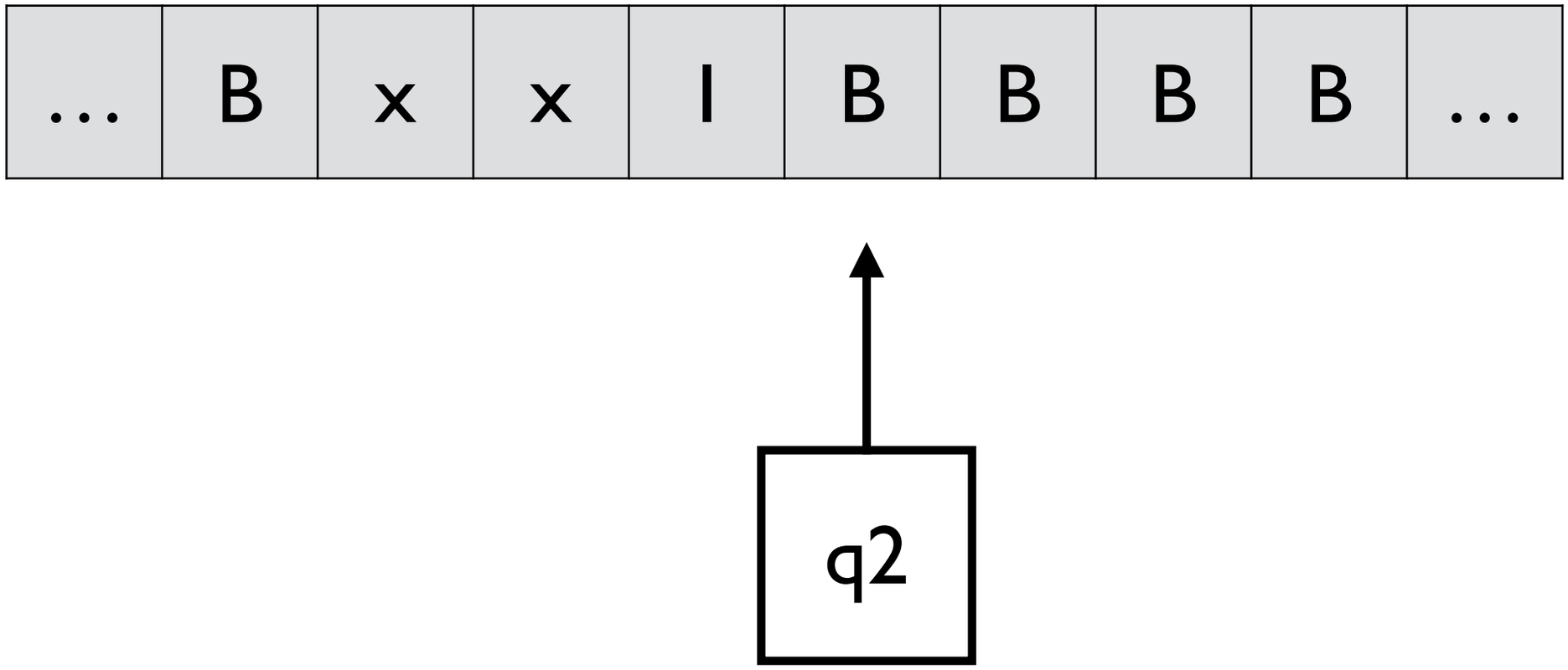
Initially, replace every 1 by x

$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$



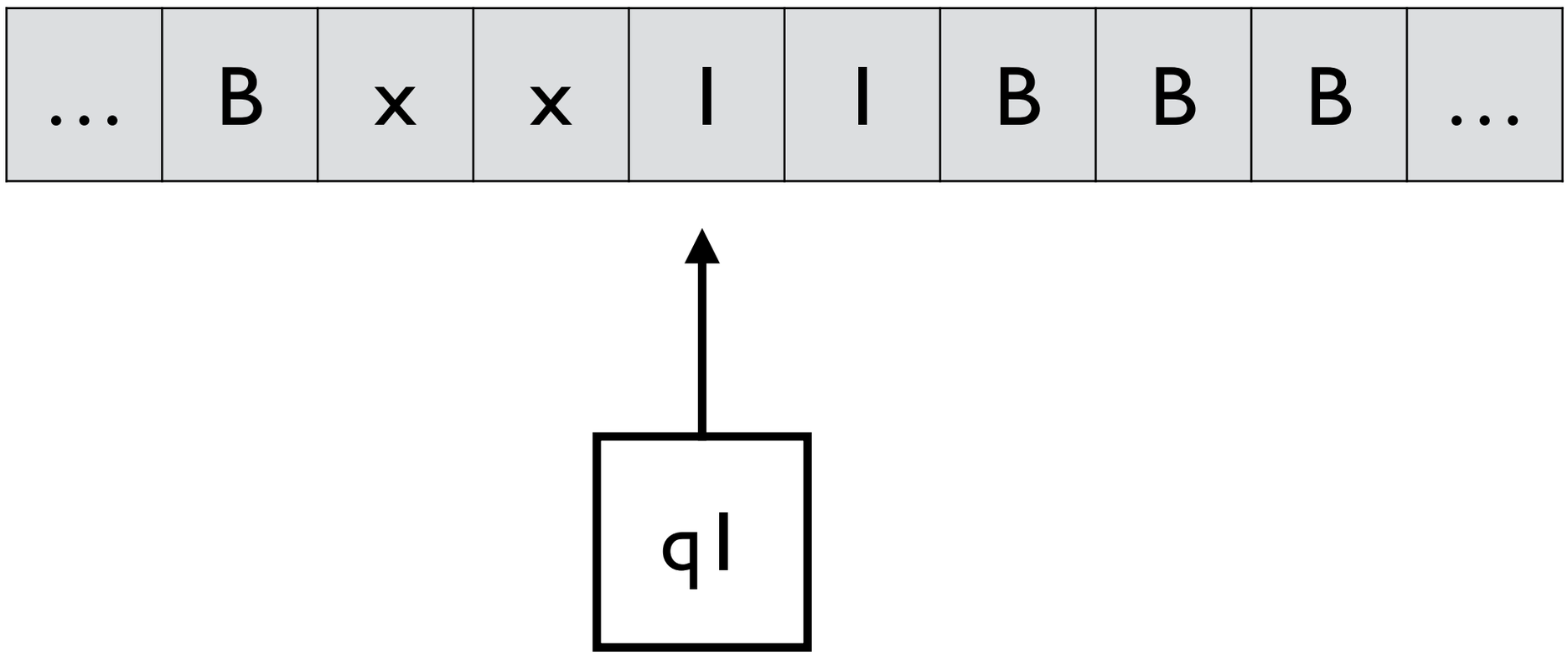
In q1, look for the rightmost x

$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$

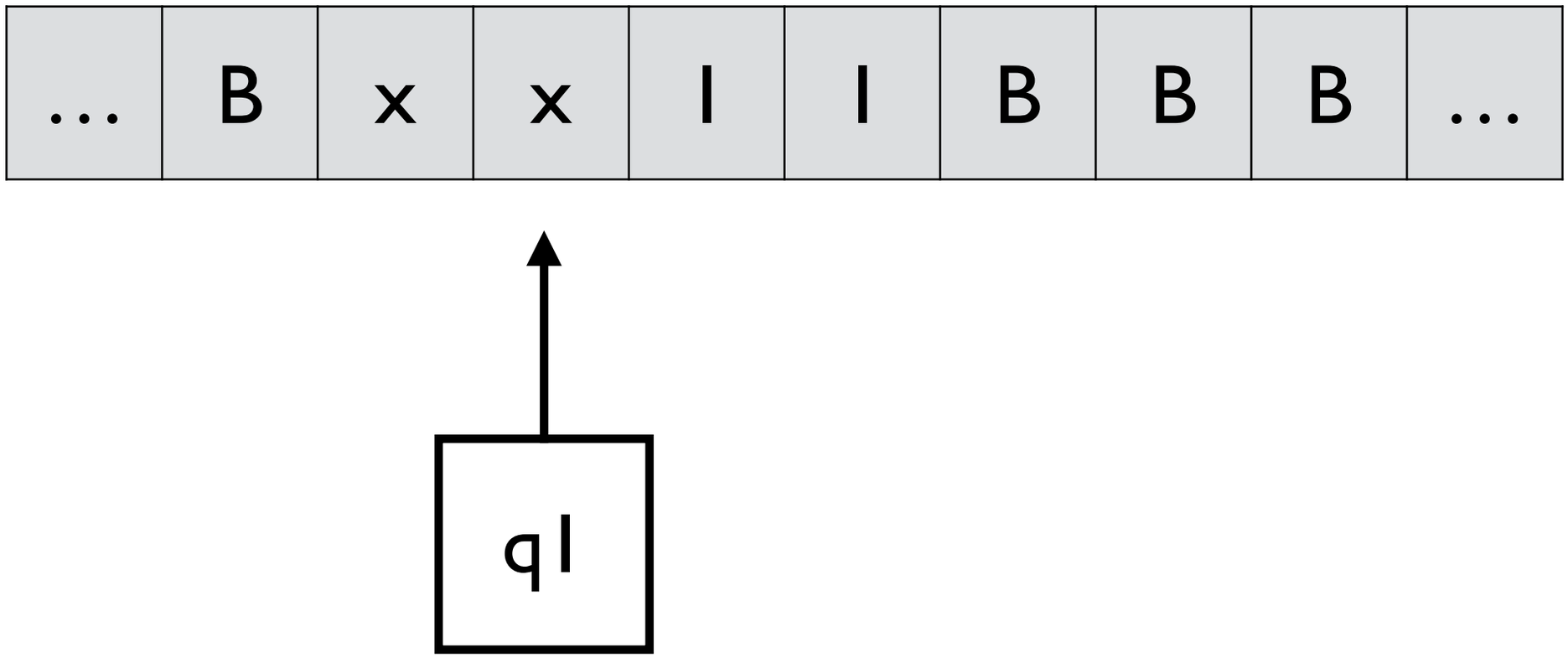


In q2, look for the first blank and write 1

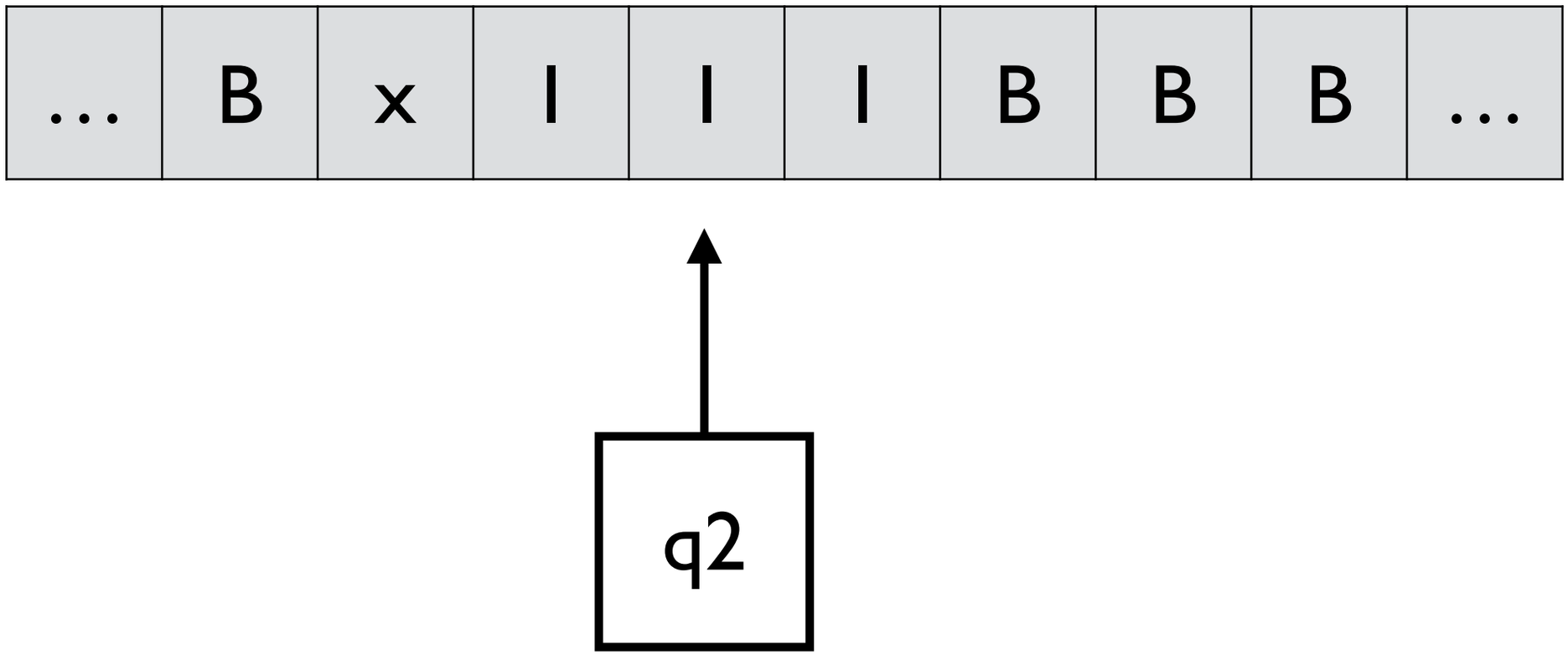
$$\delta(q_0, 1) = (q_0, x, R)$$
$$\delta(q_0, B) = (q_1, B, L)$$
$$\delta(q_1, 1) = (q_1, 1, L)$$
$$\delta(q_1, x) = (q_2, 1, R)$$
$$\delta(q_2, 1) = (q_2, 1, R)$$
$$\delta(q_2, B) = (q_1, 1, L)$$
$$\delta(q_1, B) = (q_3, B, R)$$



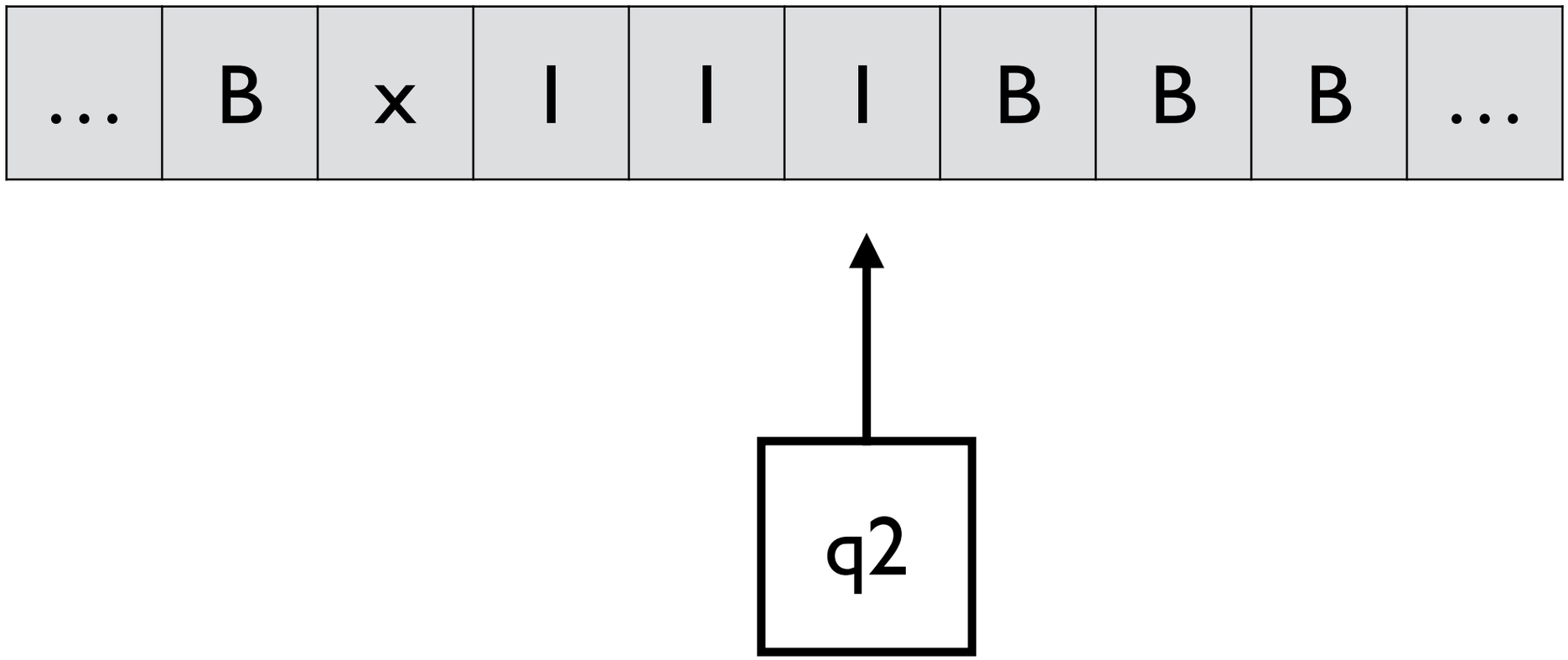
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



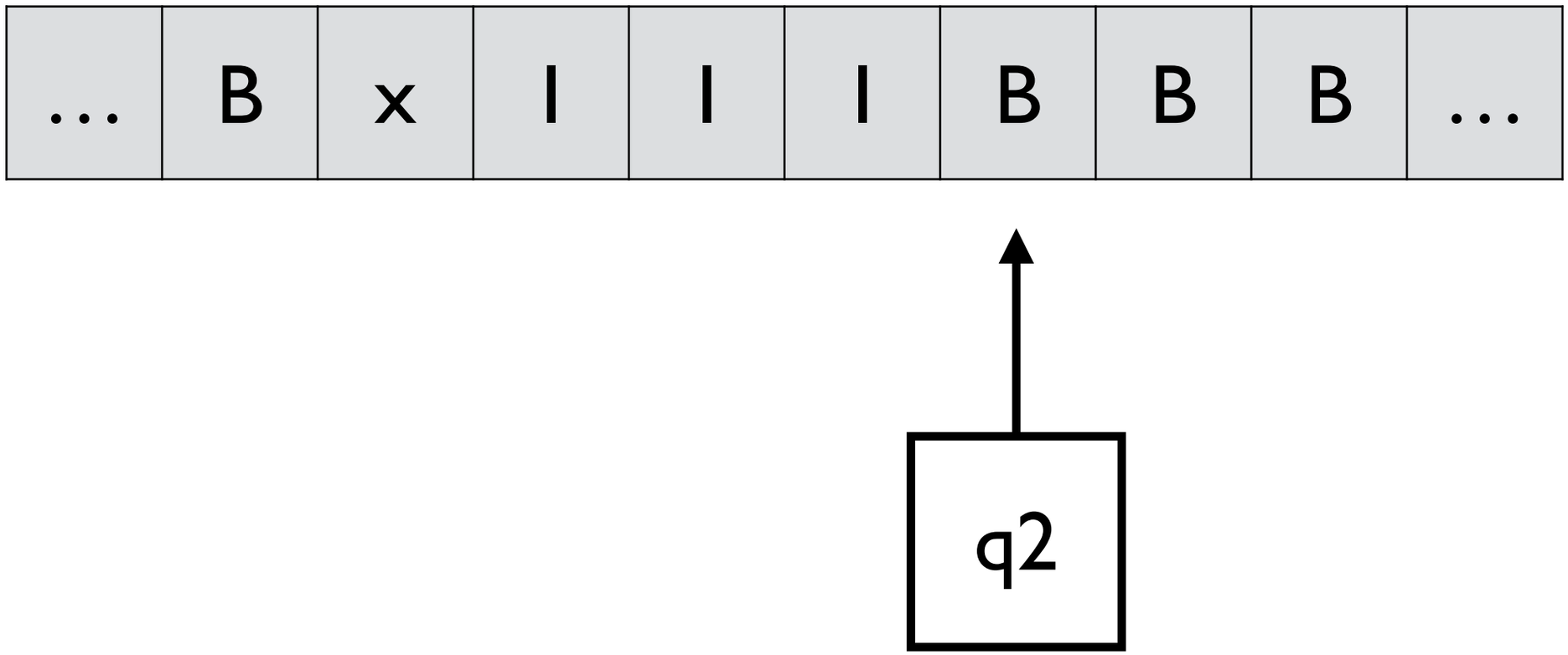
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



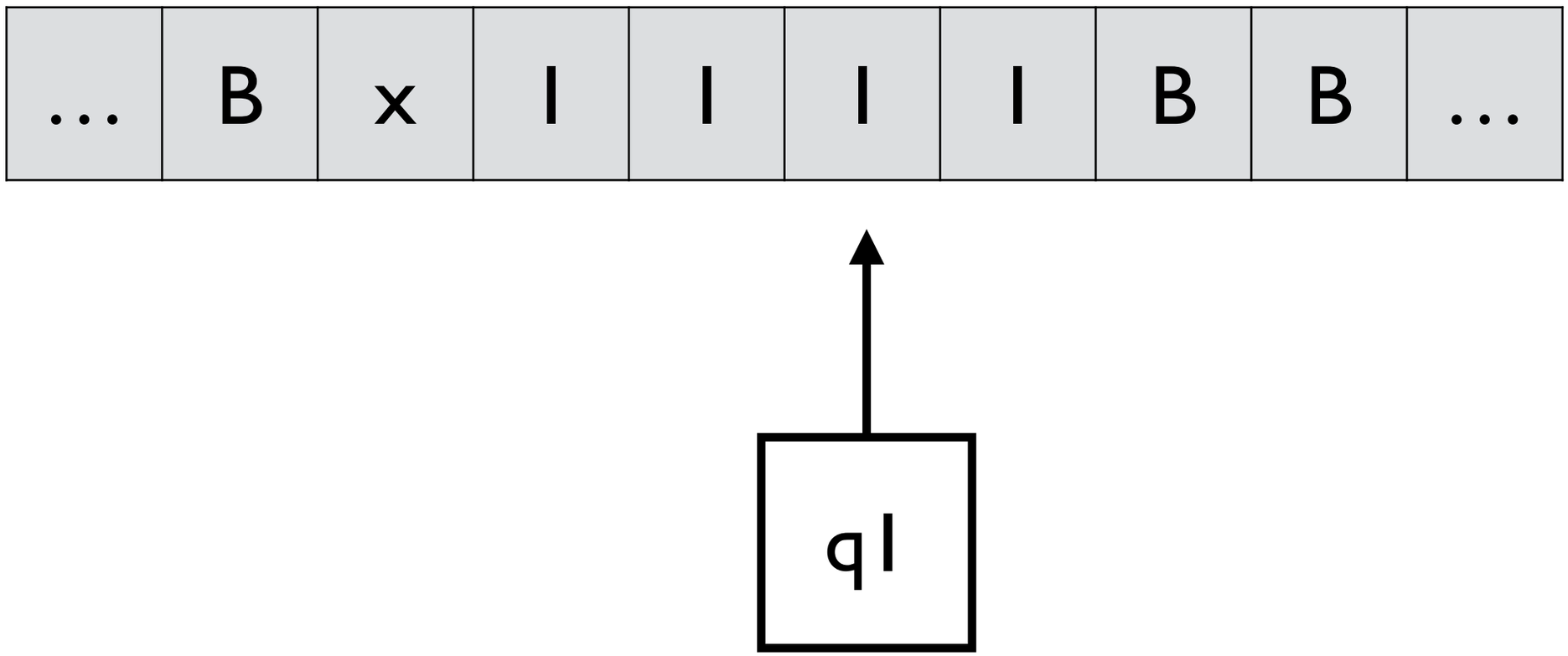
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



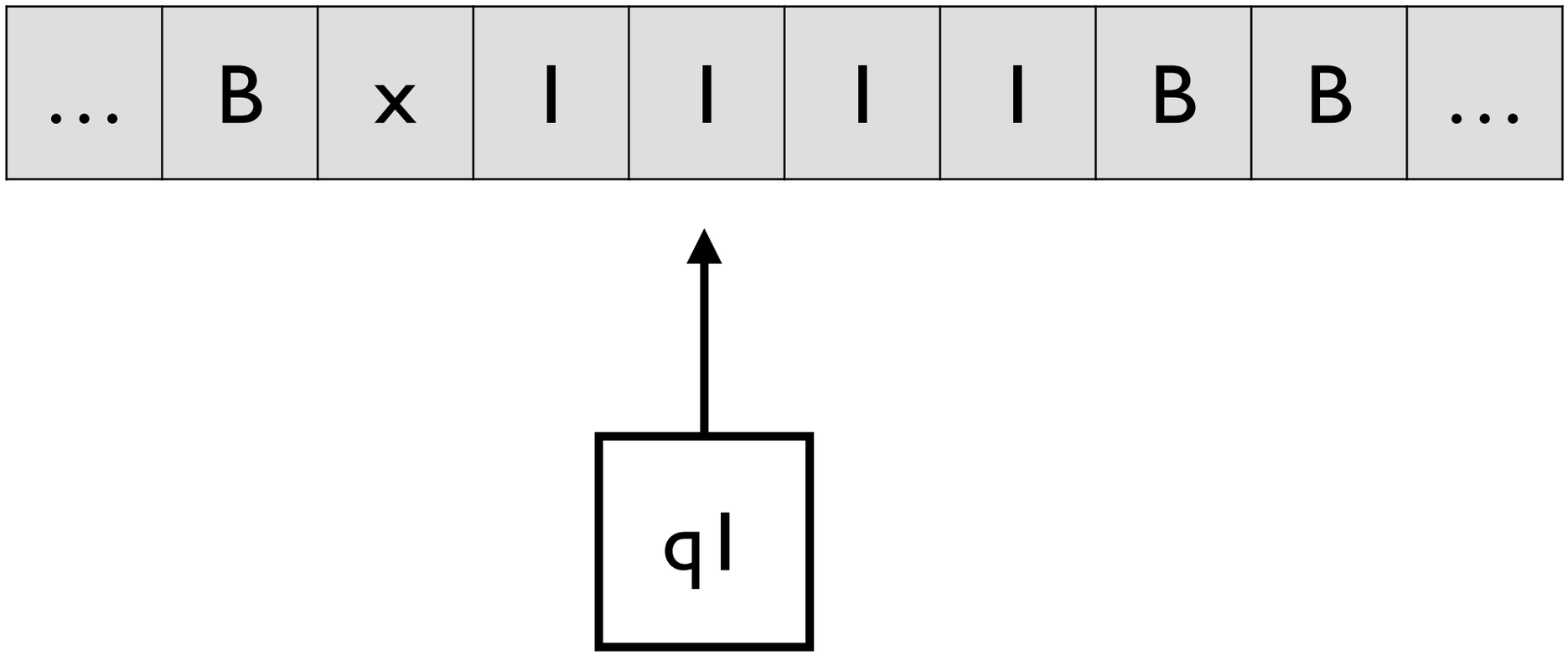
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



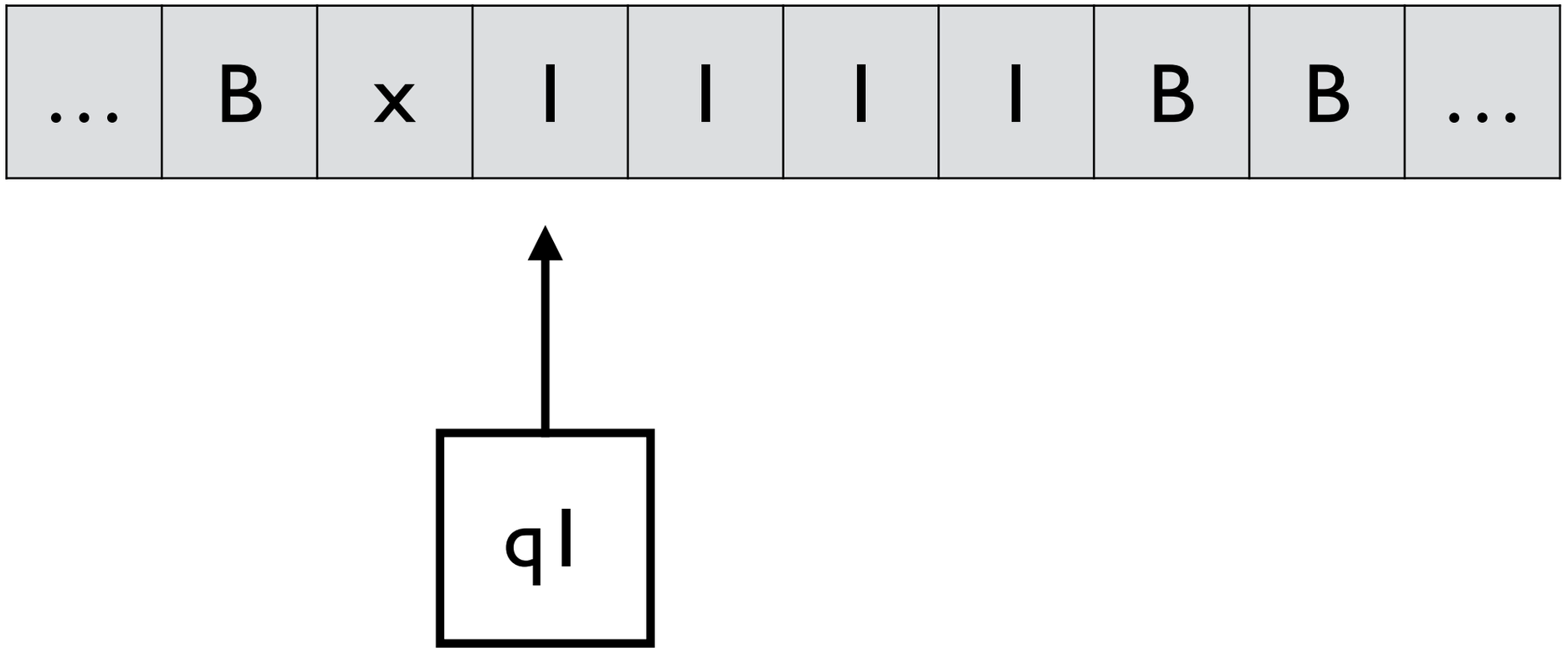
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



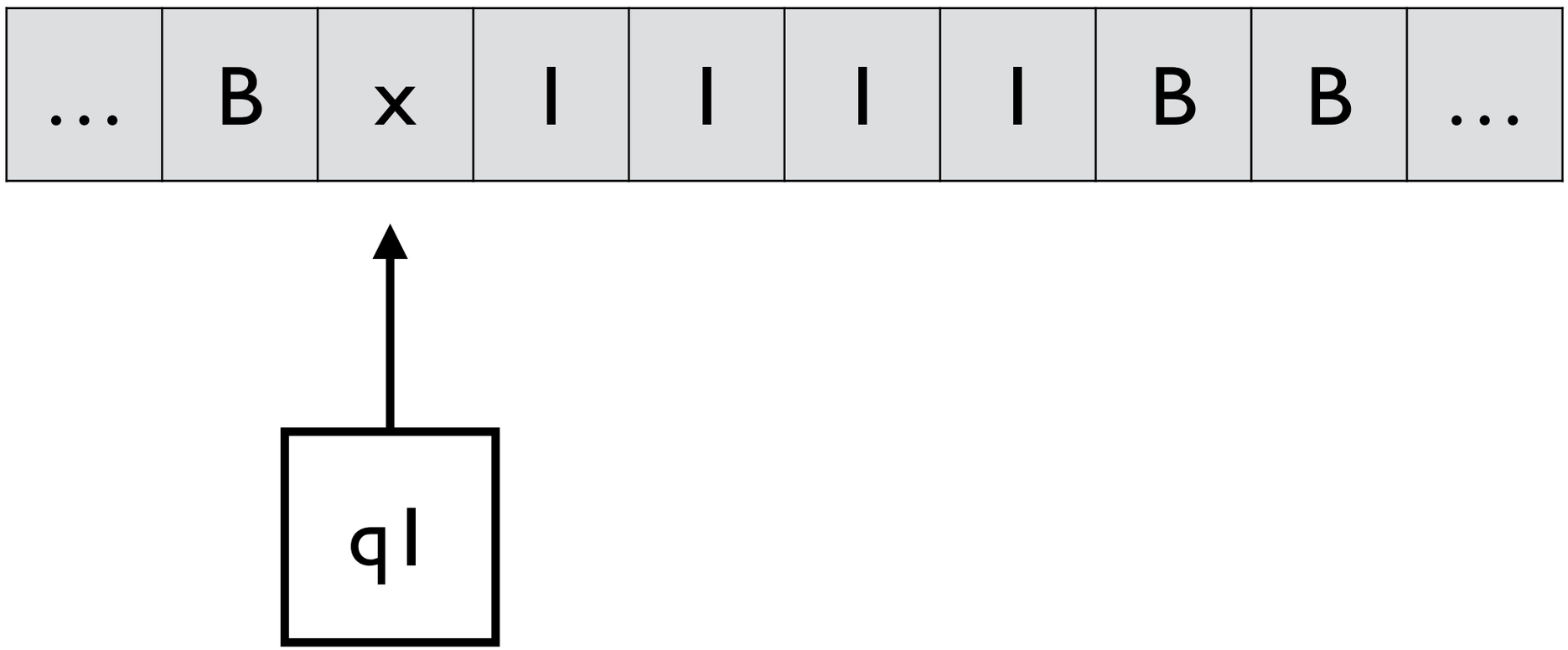
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



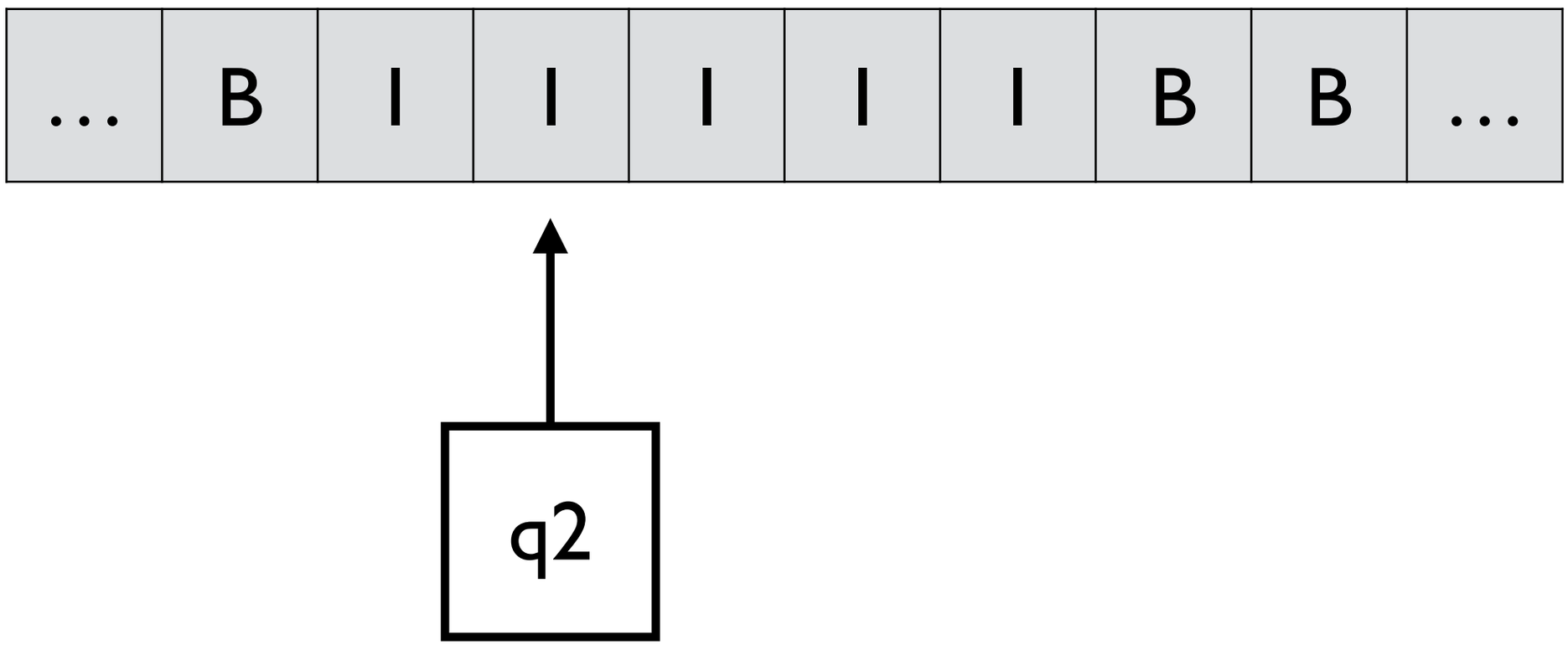
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



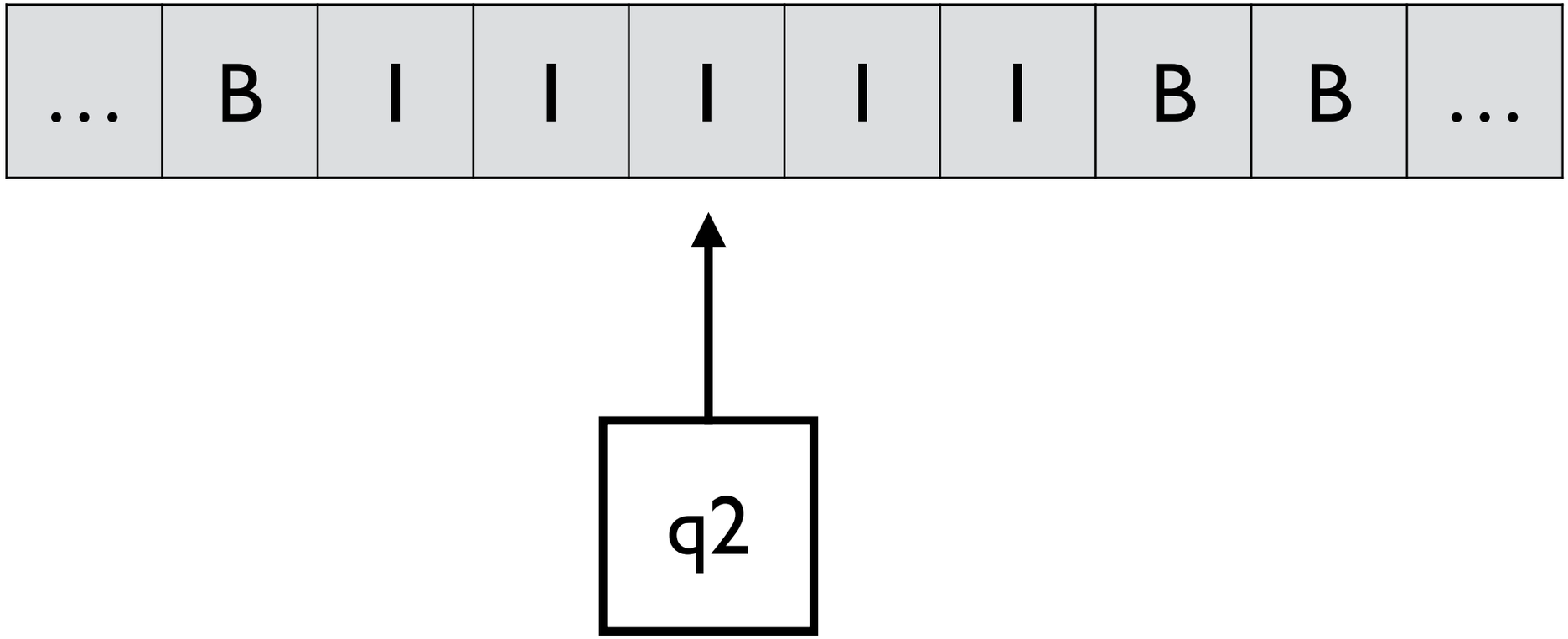
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



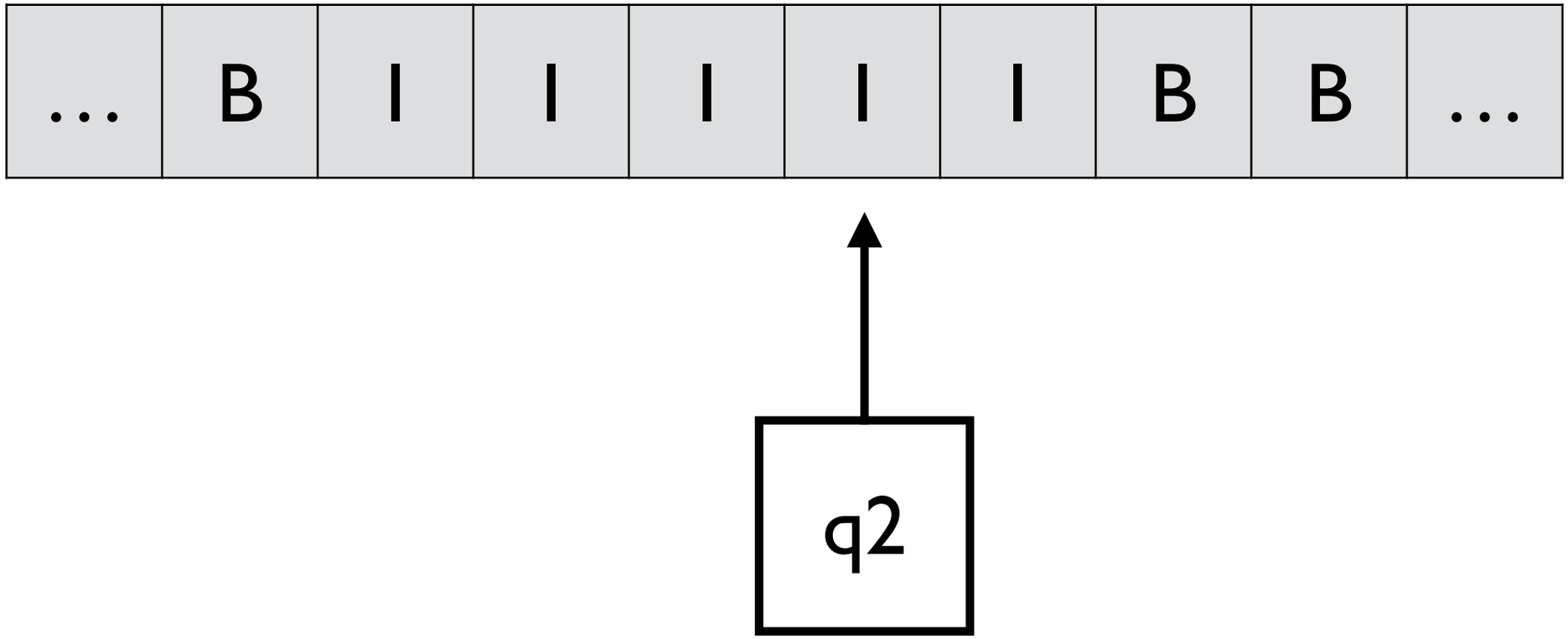
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



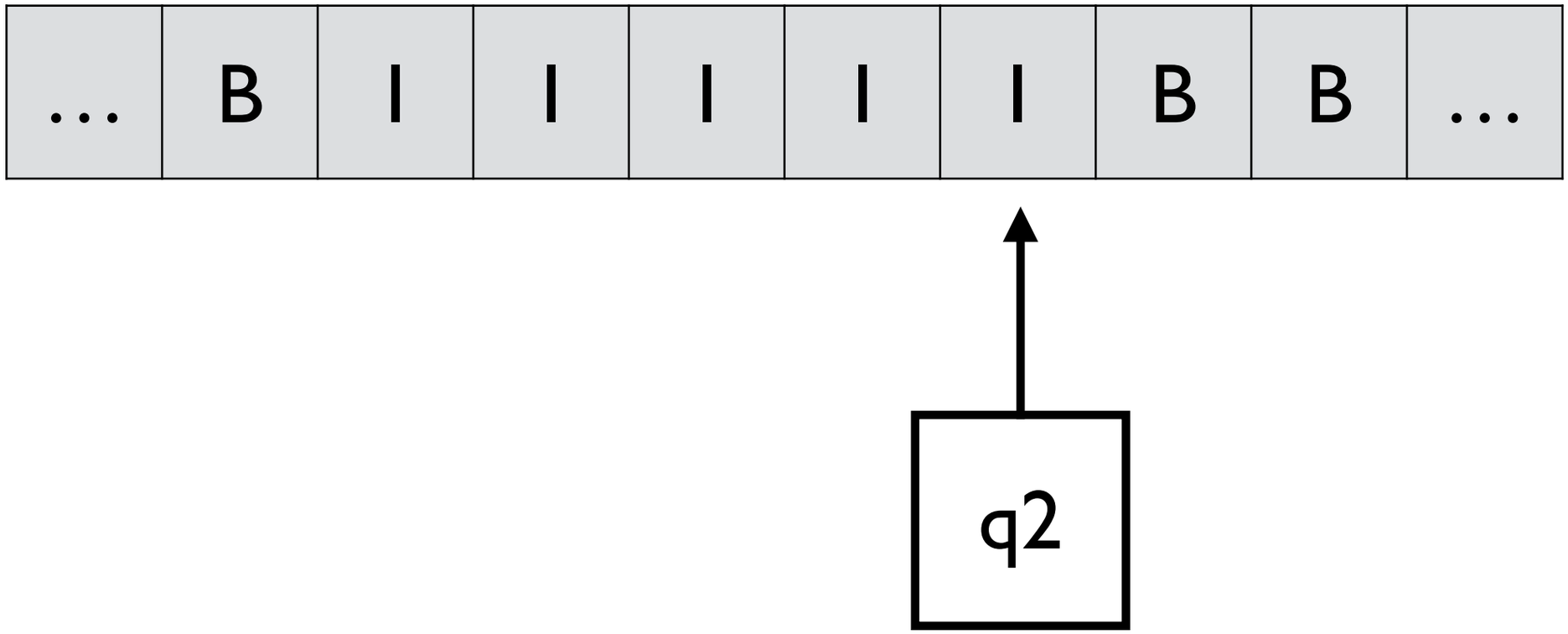
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



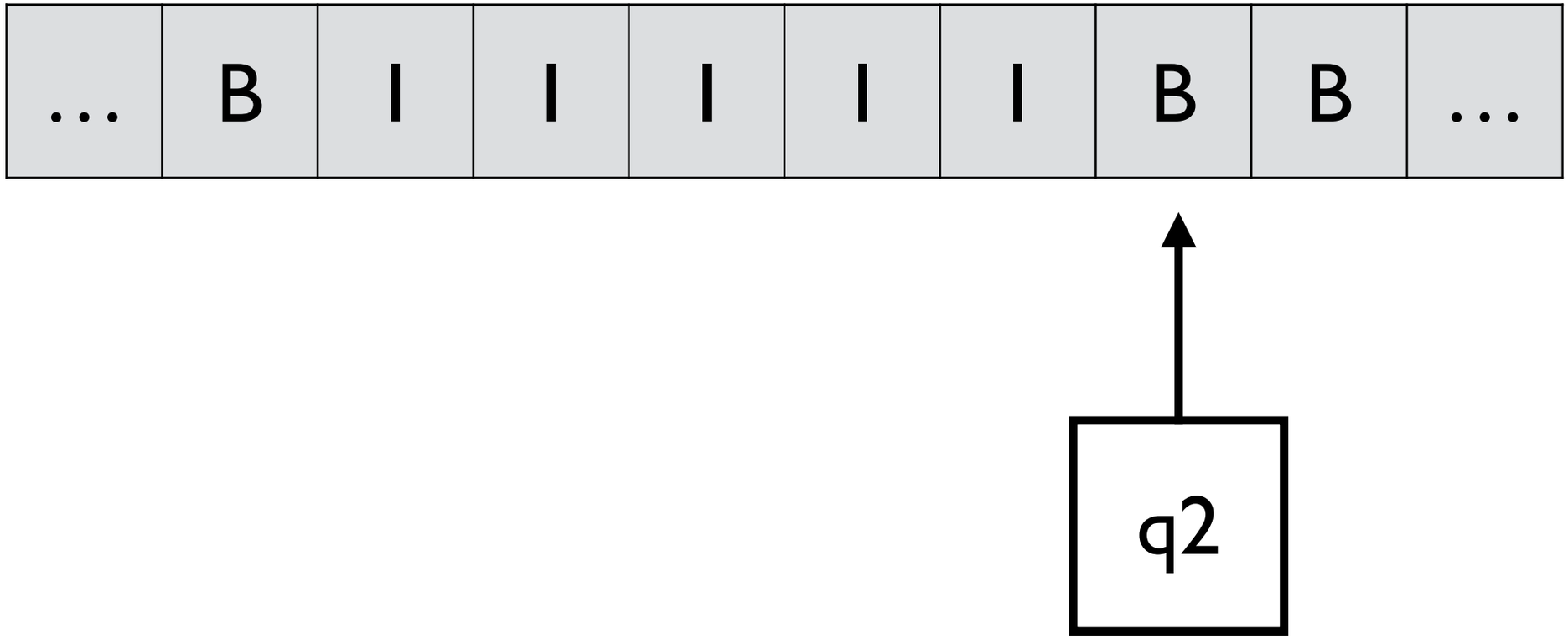
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



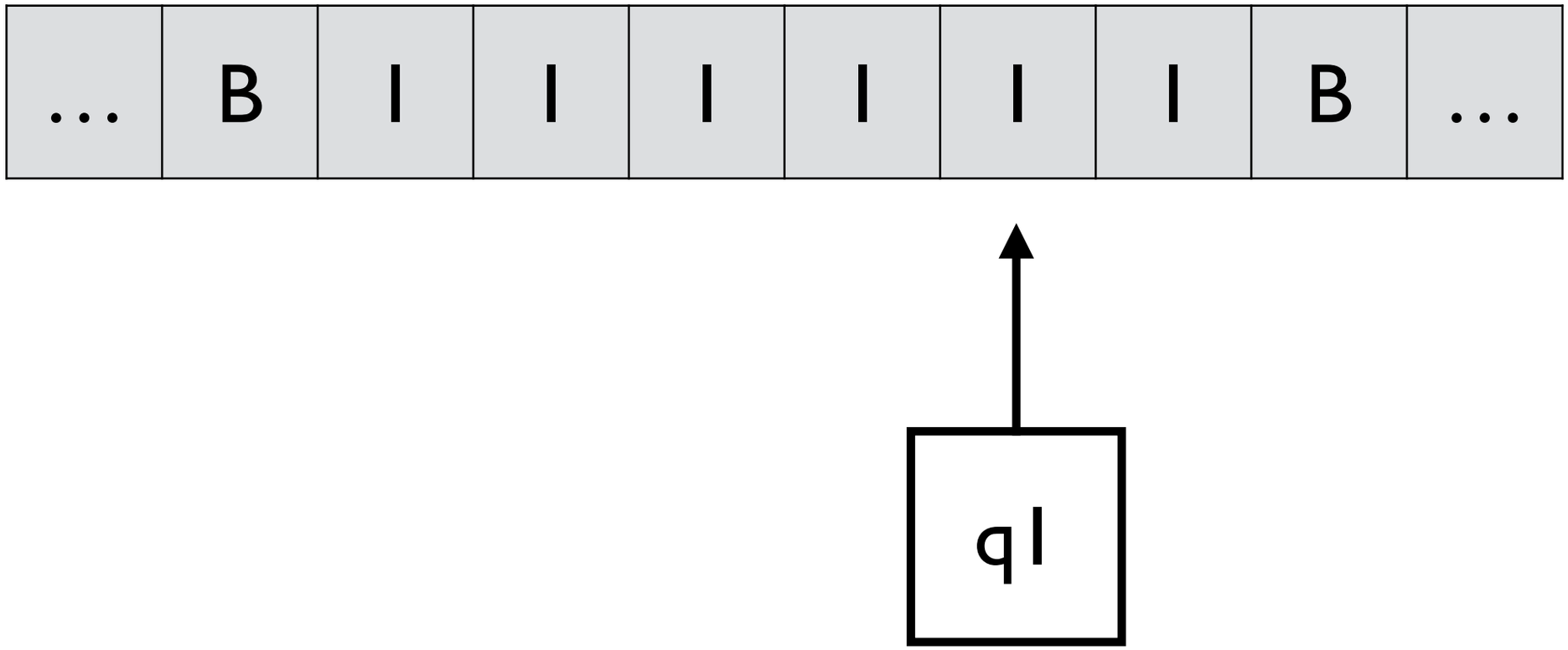
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



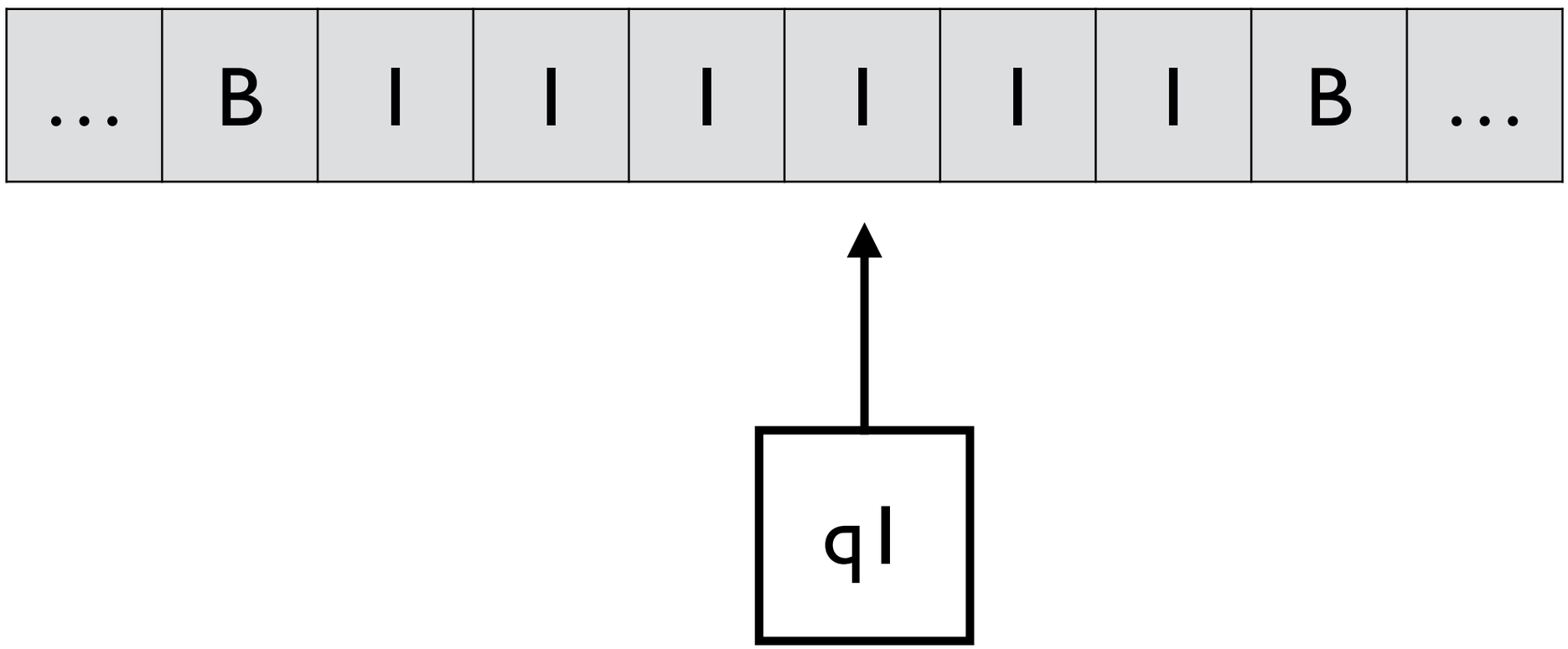
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



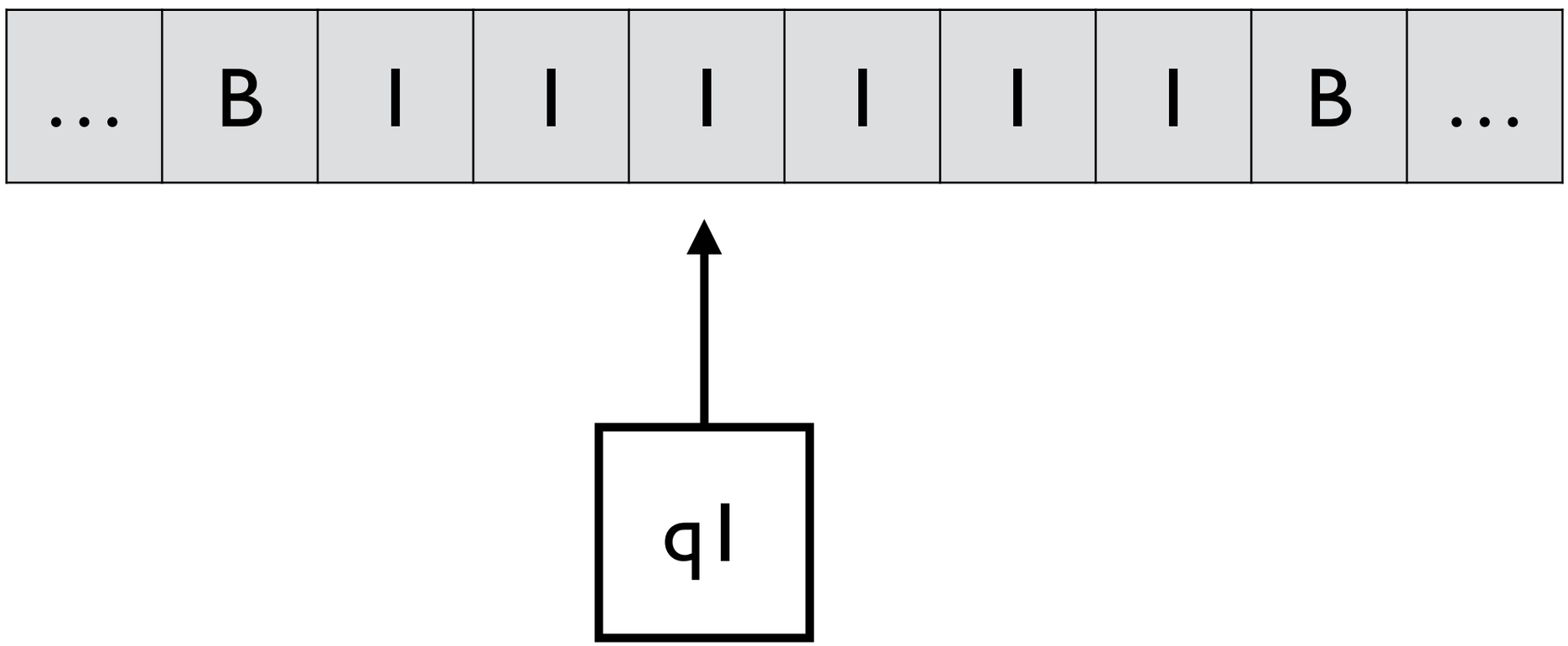
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



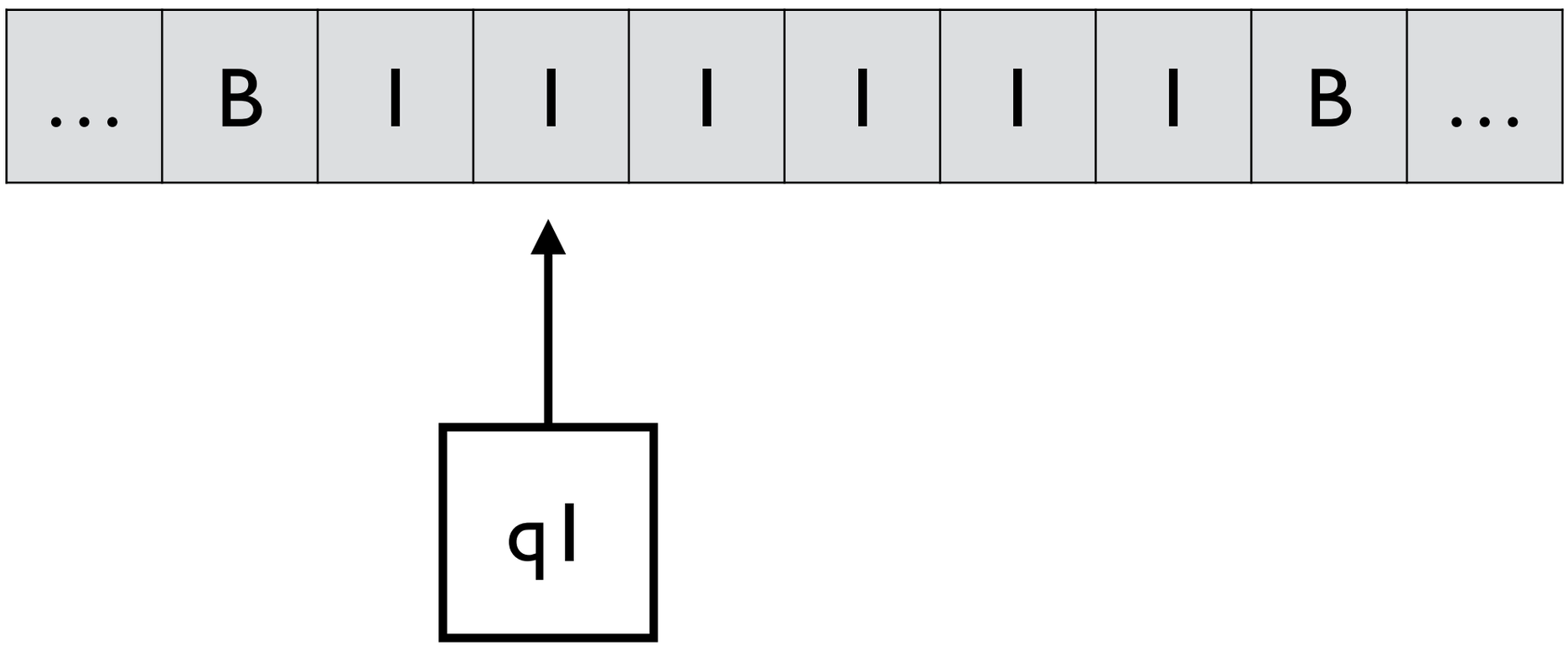
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



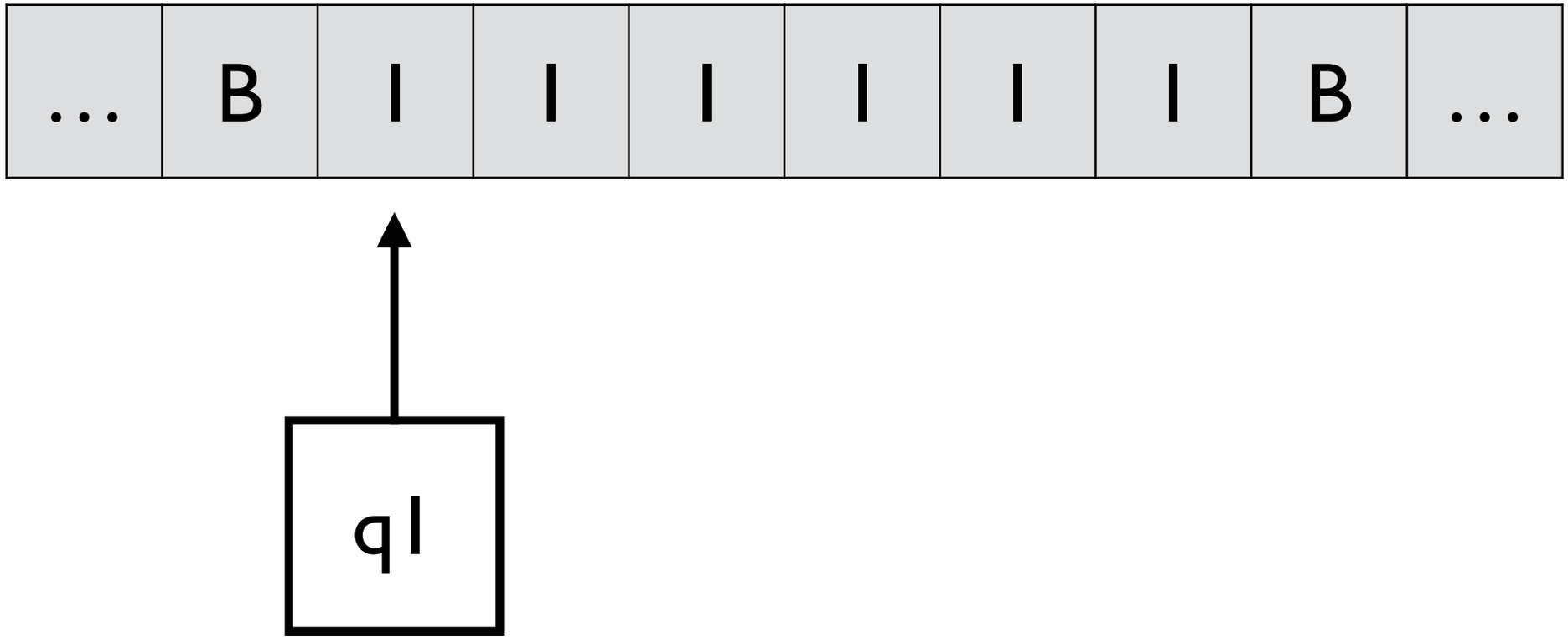
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



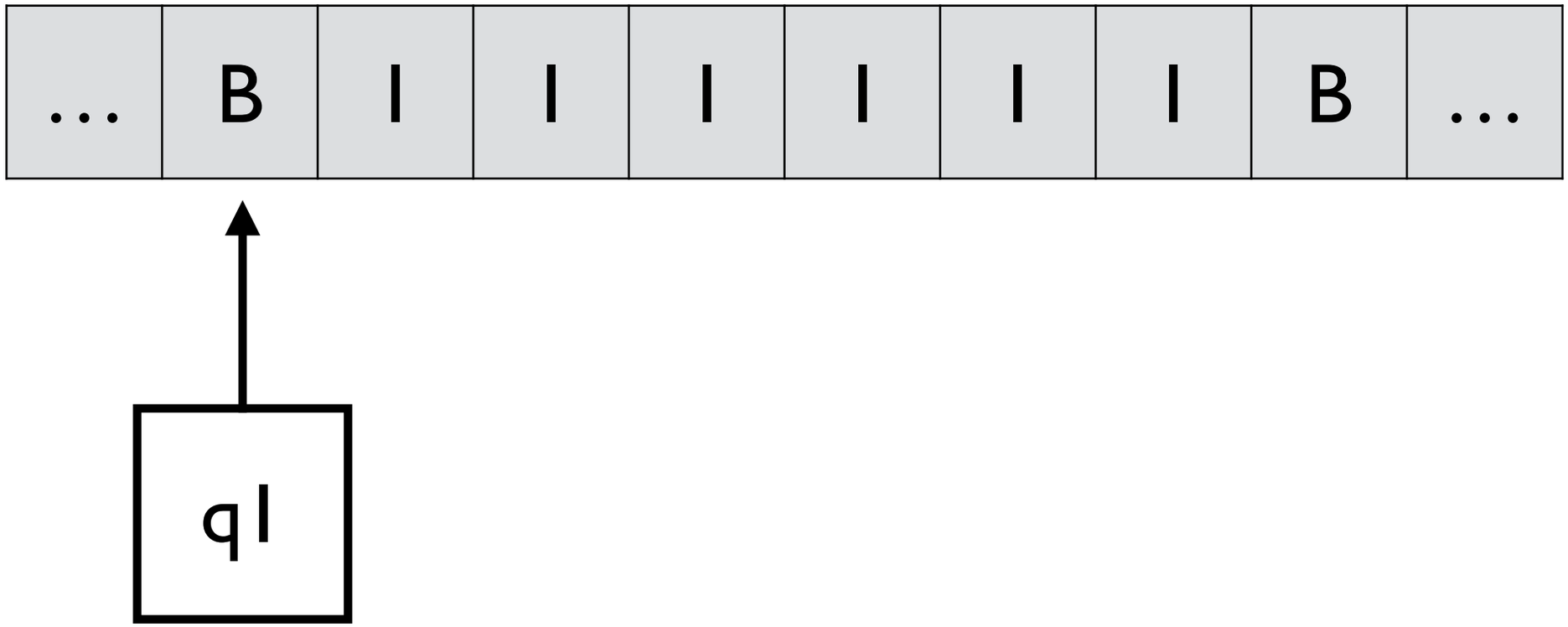
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



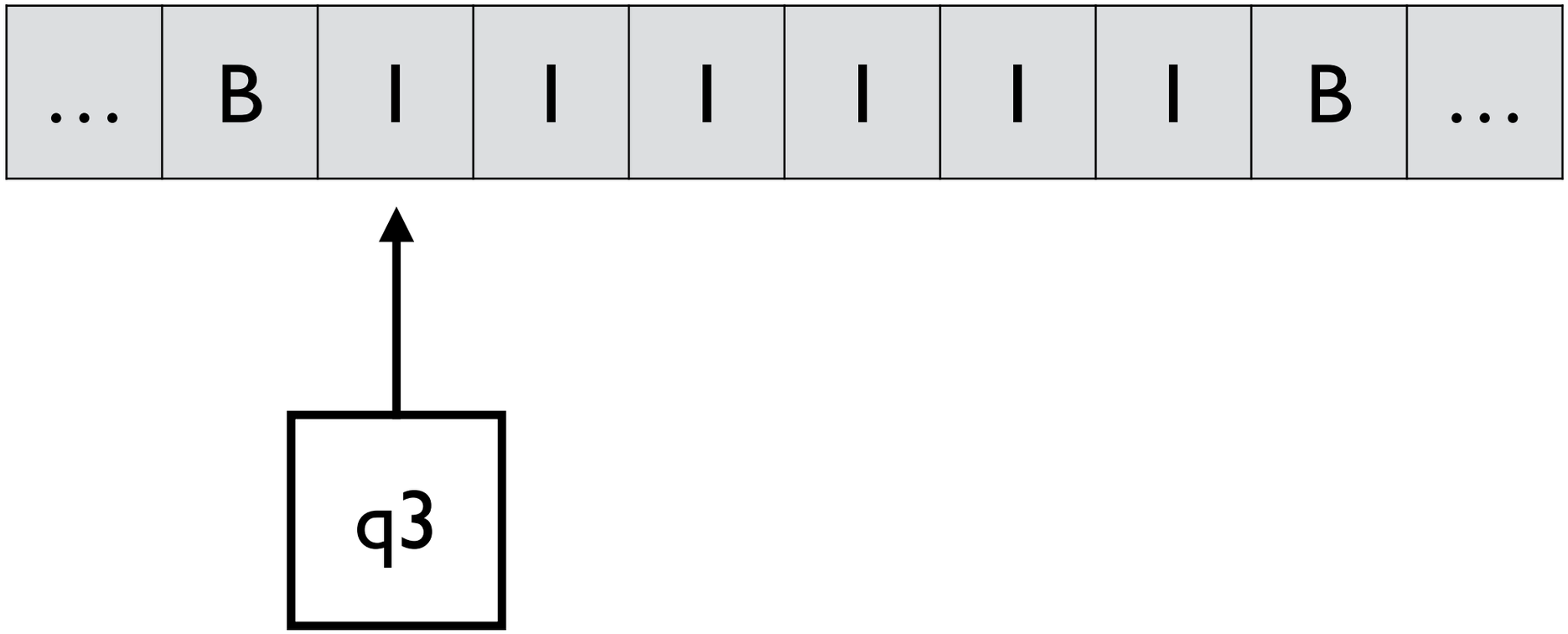
$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



$$
\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}
$$



"final state"

$$\begin{aligned}
\delta(q_0, 1) &= (q_0, x, R) \\
\delta(q_0, B) &= (q_1, B, L) \\
\delta(q_1, 1) &= (q_1, 1, L) \\
\delta(q_1, x) &= (q_2, 1, R) \\
\delta(q_2, 1) &= (q_2, 1, R) \\
\delta(q_2, B) &= (q_1, 1, L) \\
\delta(q_1, B) &= (q_3, B, R)
\end{aligned}$$