

Example 1. Design a Turing machine that accepts $L = \{a^n b^n \mid n \geq 1\}$.

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$$

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_2, a) = (q_2, a, L)$$

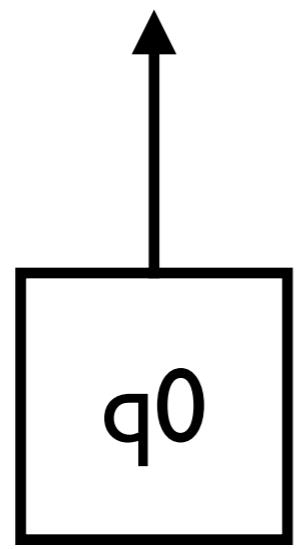
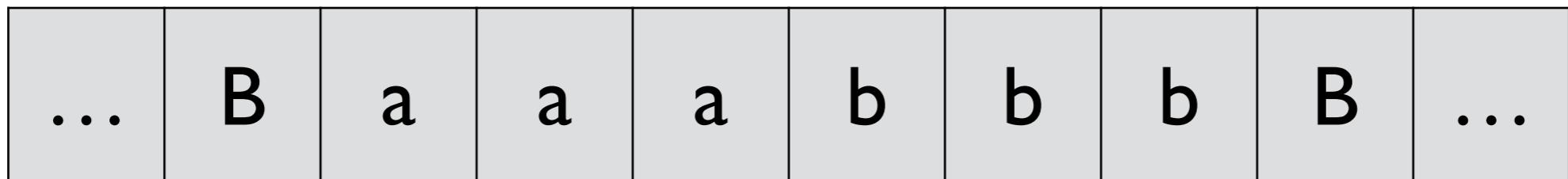
$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

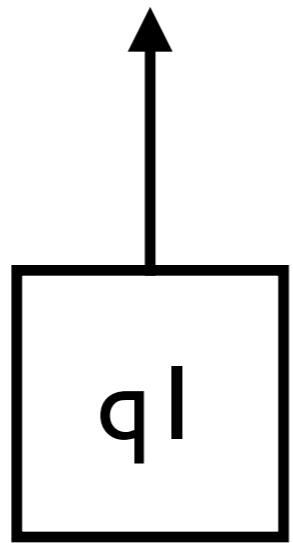
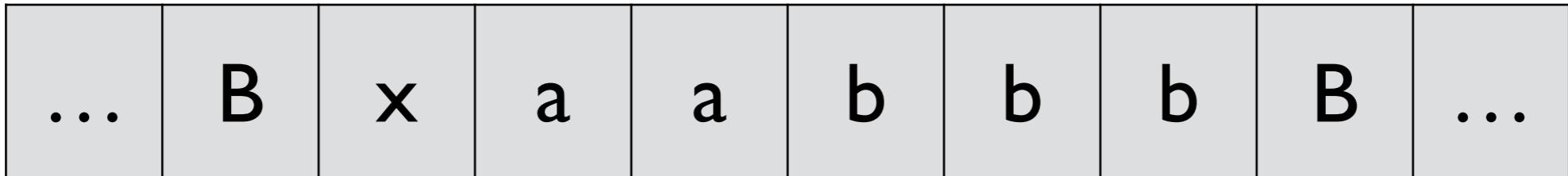
$$\delta(q_1, b) = (q_2, y, L)$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



(In qI , move right to search for the leftmost 'b')

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

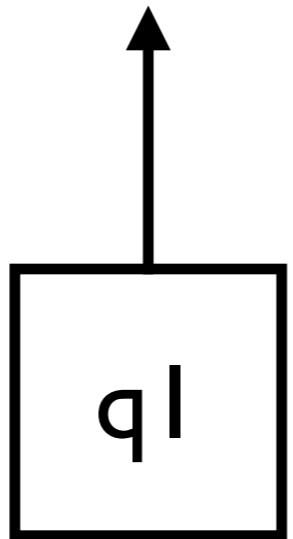
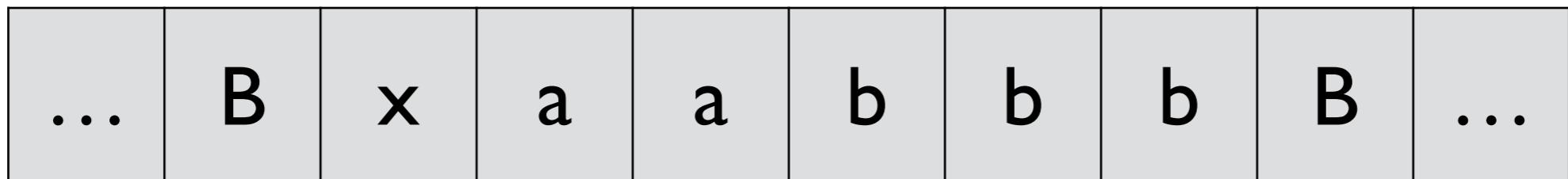
$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

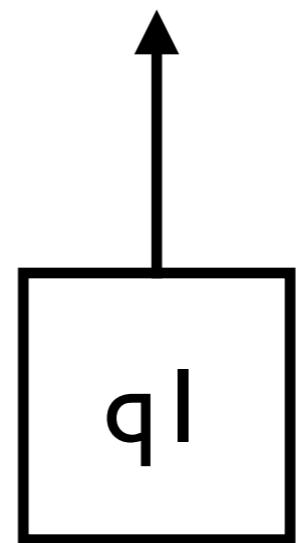
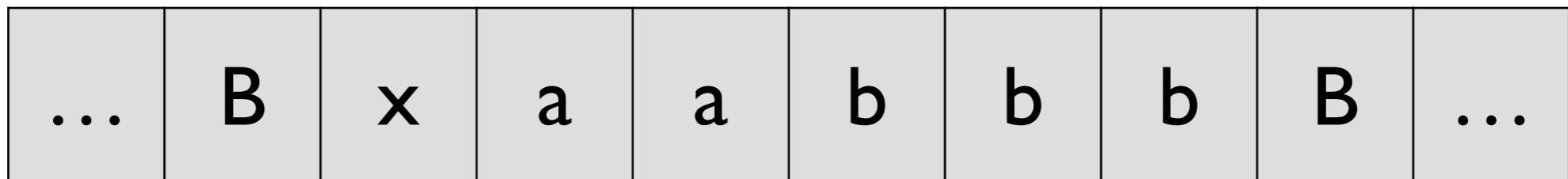
$$\delta(q_3, B) = (q_4, B, R)$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

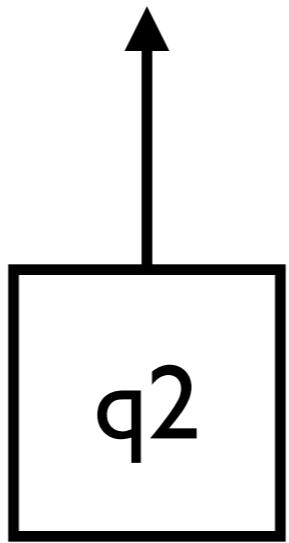


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	a	a	y	b	b	B	...
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(In q_2 , move left to search for the leftmost 'a')

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

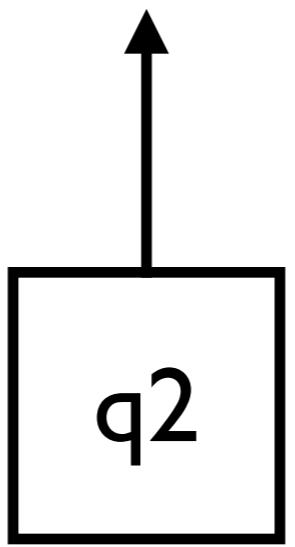
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

...	B	x	a	a	y	b	b	B	...
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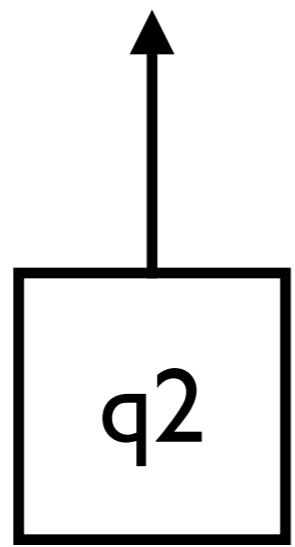


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

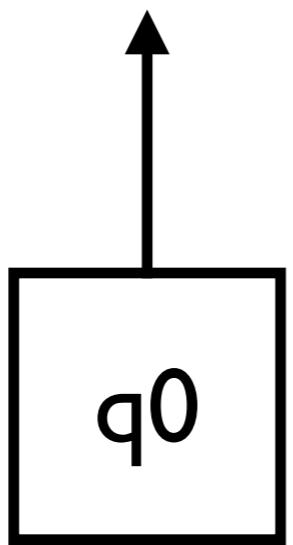
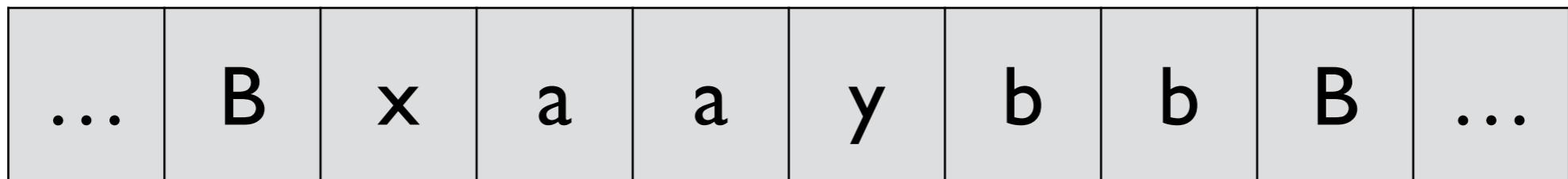
...	B	x	a	a	y	b	b	B	...
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$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

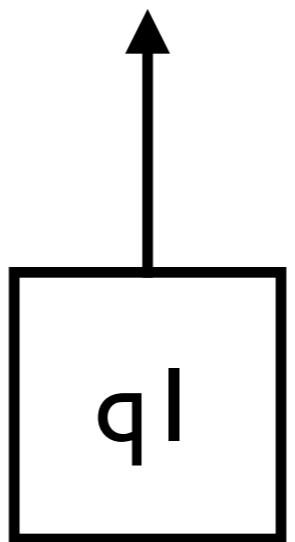
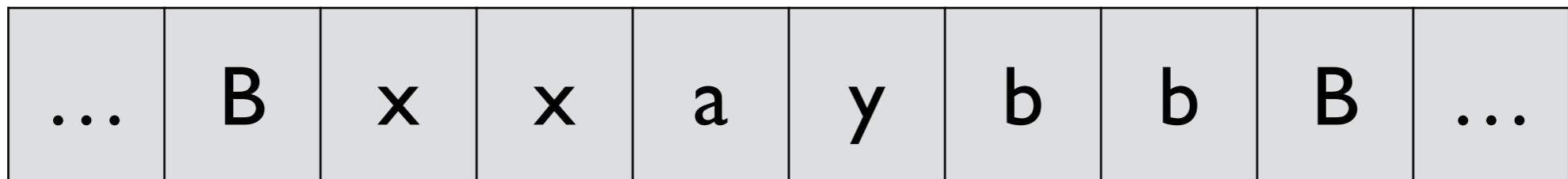
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

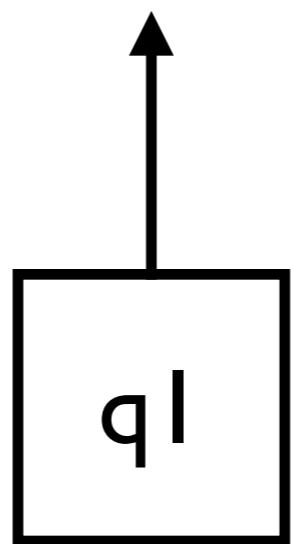
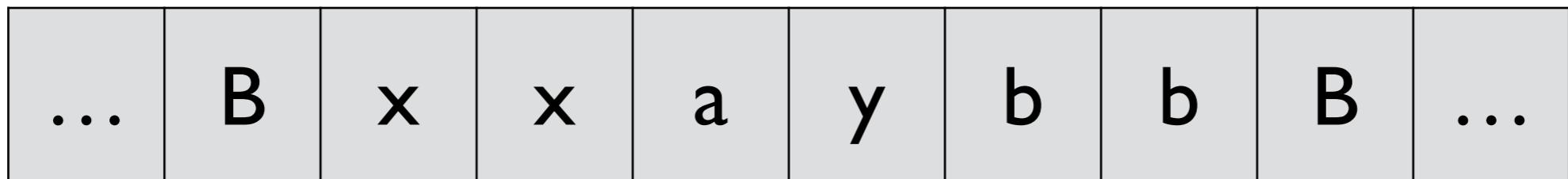
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

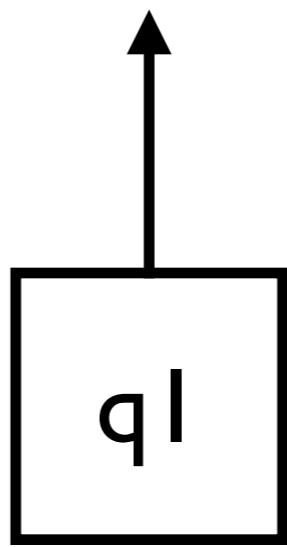


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	a	y	b	b	B	...
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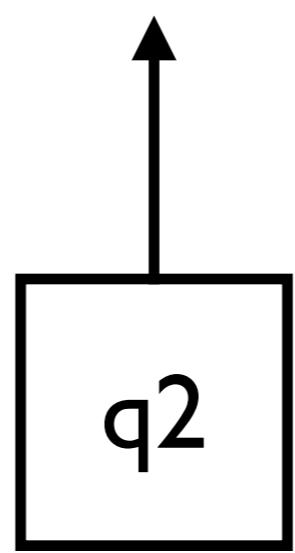


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	a	y	y	b	B	...
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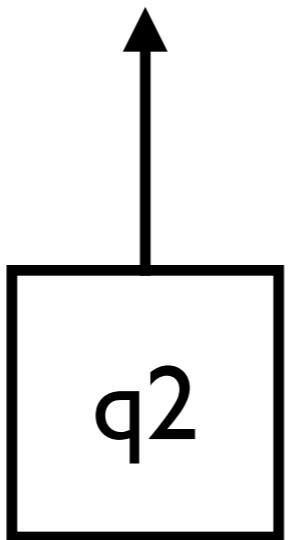


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

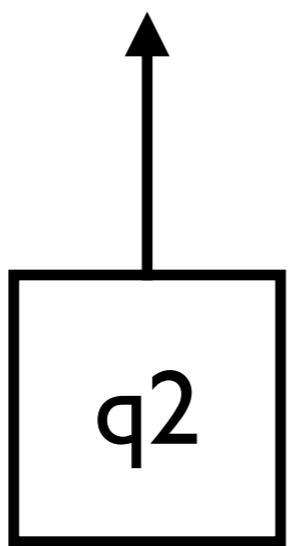
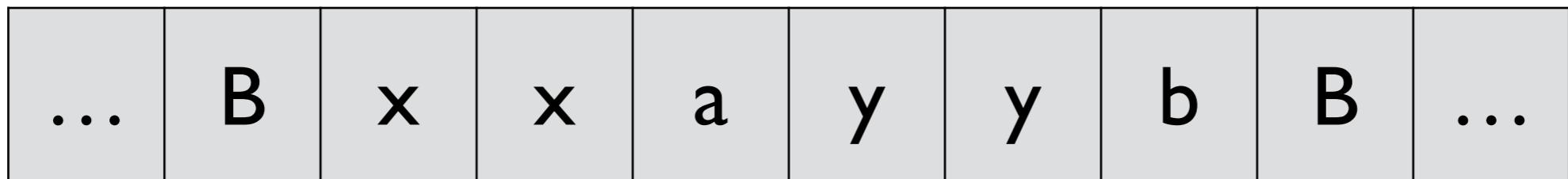
...	B	x	x	a	y	y	b	B	...
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$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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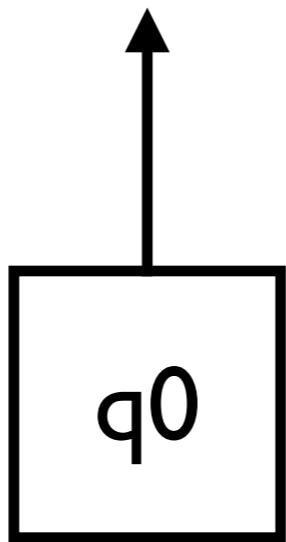
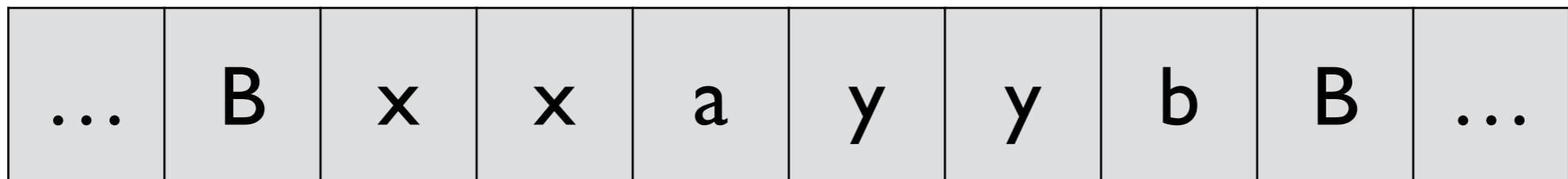
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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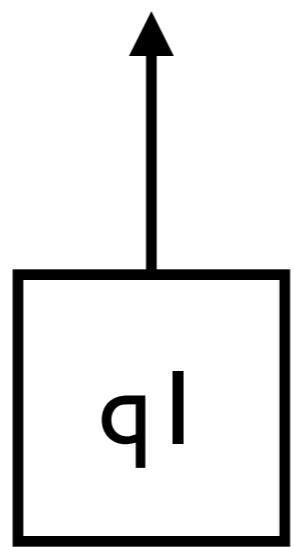


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	b	B	...
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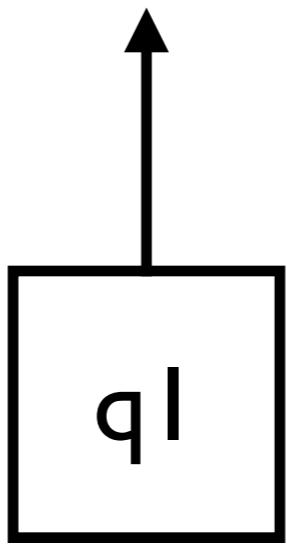


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	b	B	...
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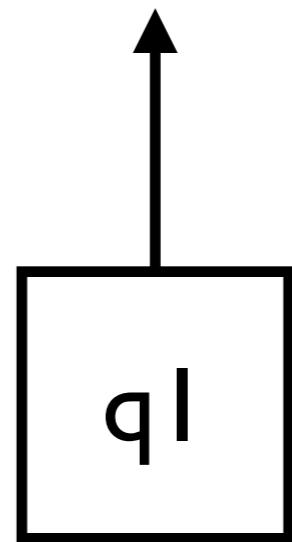


$$\begin{array}{l} \delta(q_0, a) = (q_1, x, R) \\ \delta(q_1, a) = (q_1, a, R) \\ \delta(q_1, y) = (q_1, y, R) \\ \delta(q_1, b) = (q_2, y, L) \end{array}$$

$$\begin{array}{l} \delta(q_2, y) = (q_2, y, L) \\ \delta(q_2, a) = (q_2, a, L) \\ \delta(q_2, x) = (q_0, x, R) \end{array}$$

$$\begin{array}{l} \delta(q_0, y) = (q_3, y, R) \\ \delta(q_3, y) = (q_3, y, R) \\ \delta(q_3, B) = (q_4, B, R) \end{array}$$

...	B	x	x	x	y	y	b	B	...
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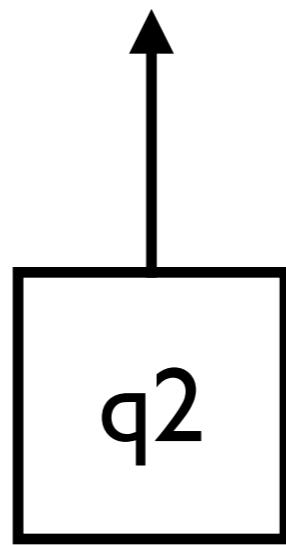


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	y	B	...
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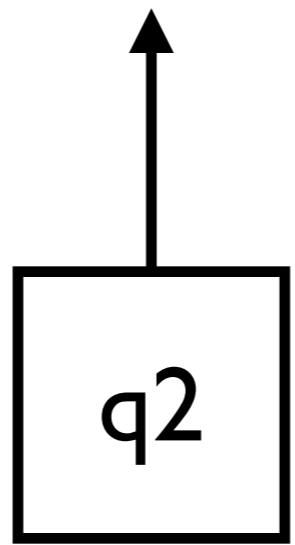


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	y	B	...
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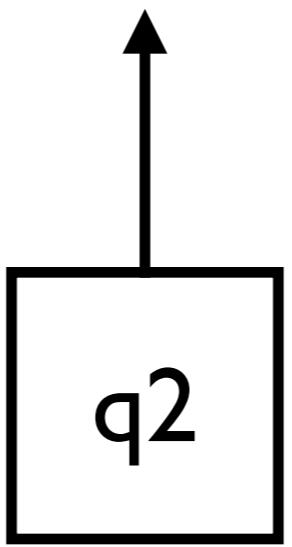


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

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$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	y	B	...
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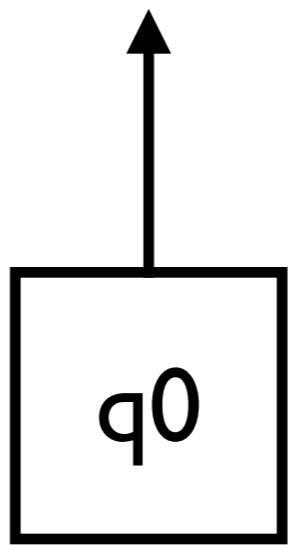


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	y	B	...
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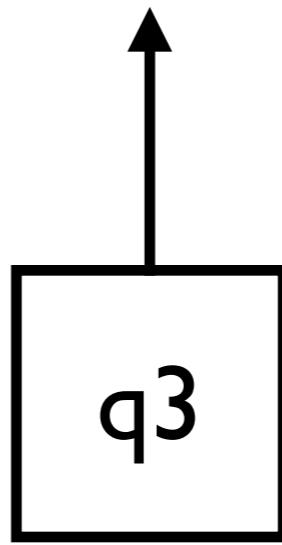


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	y	B	...
-----	---	---	---	---	---	---	---	---	-----



(In q3, move right to check that there are no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

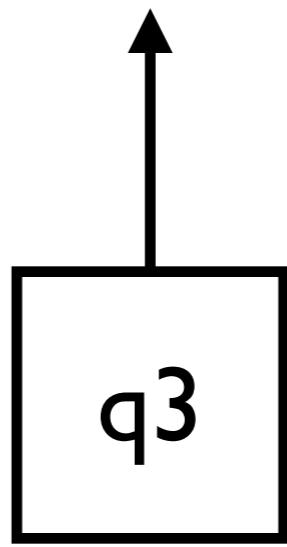
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

...	B	x	x	x	y	y	y	B	...
-----	----------	----------	----------	----------	----------	----------	----------	----------	-----

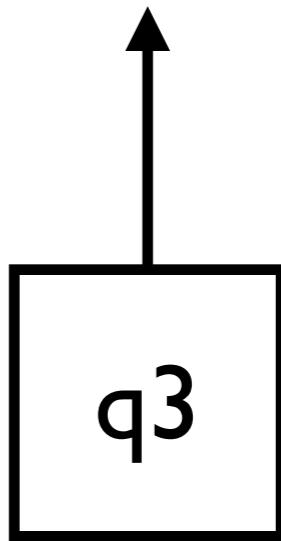


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	x	y	y	y	B	...
-----	----------	---	---	---	---	---	---	----------	-----



(no more b's)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

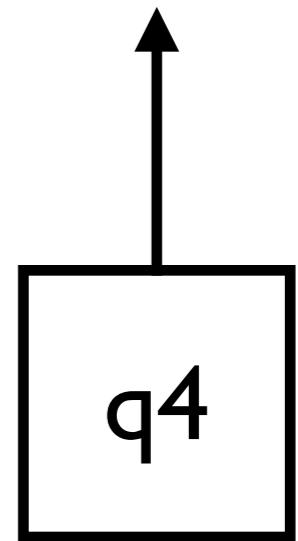
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

...	B	x	x	x	y	y	y	B	...
-----	----------	----------	----------	----------	----------	----------	----------	----------	-----



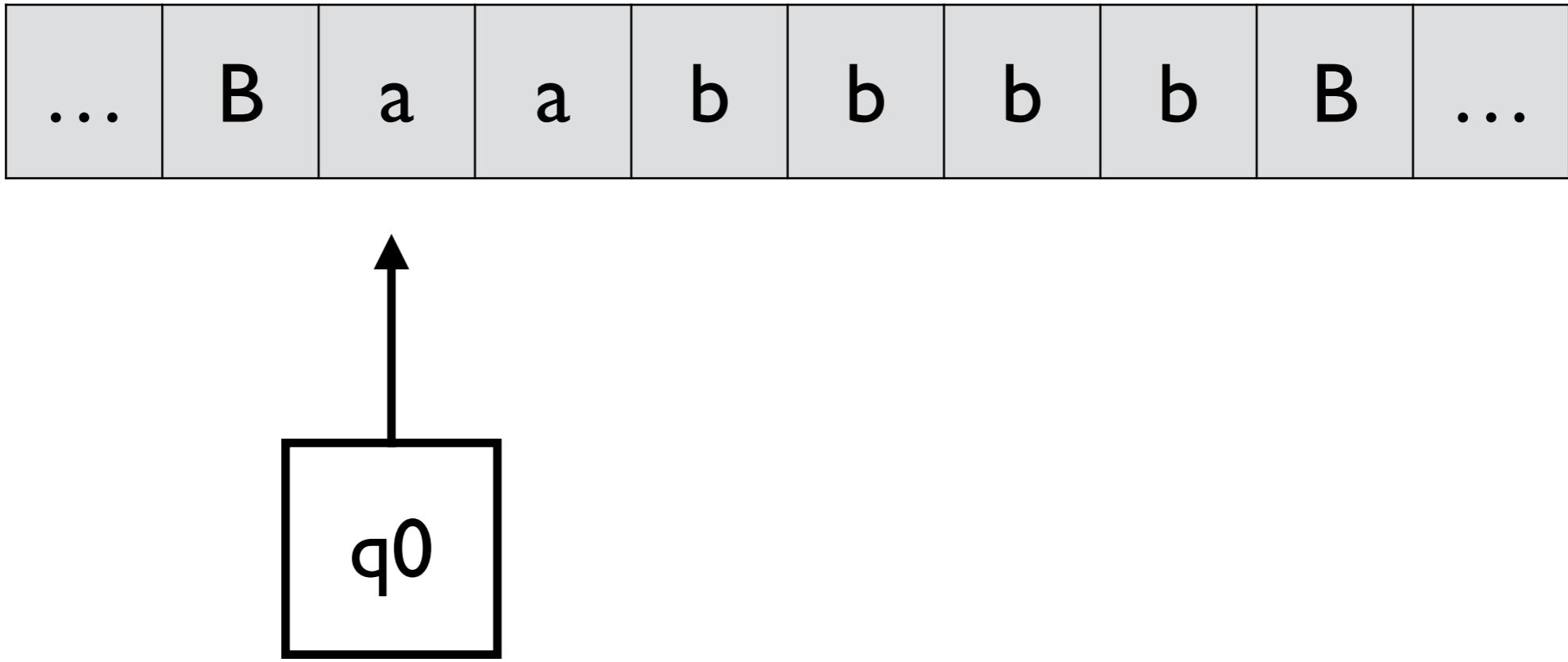
“final state”

$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

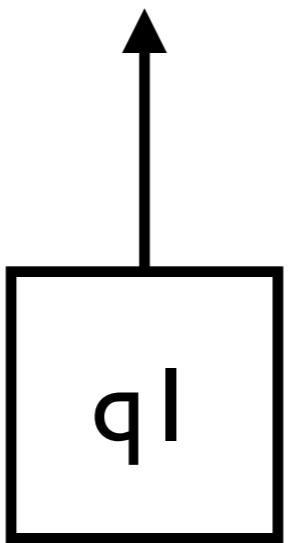
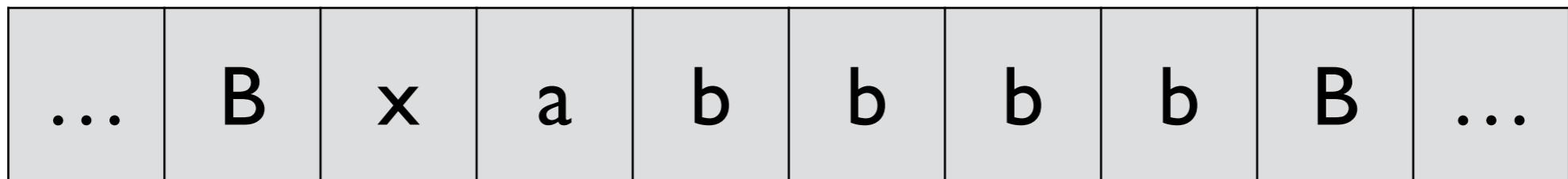
When the input string is not in the language:



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

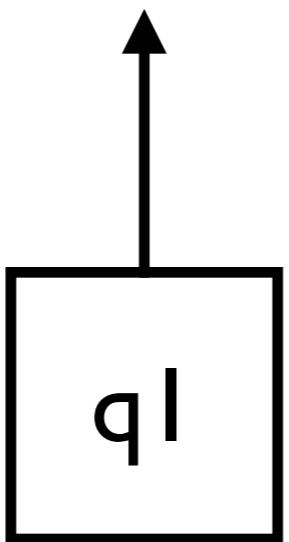
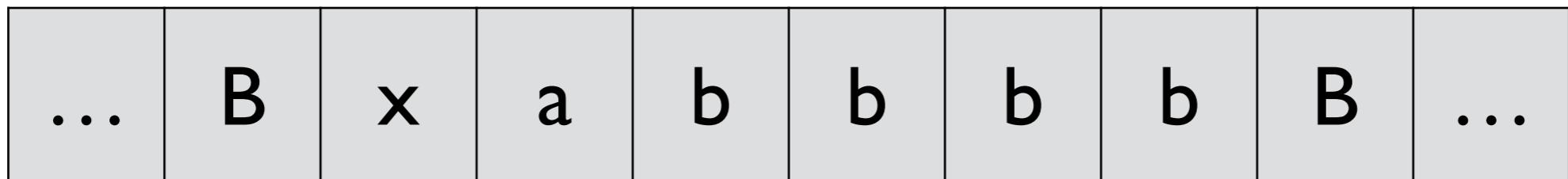
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

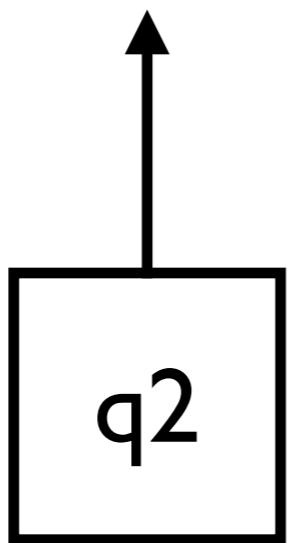
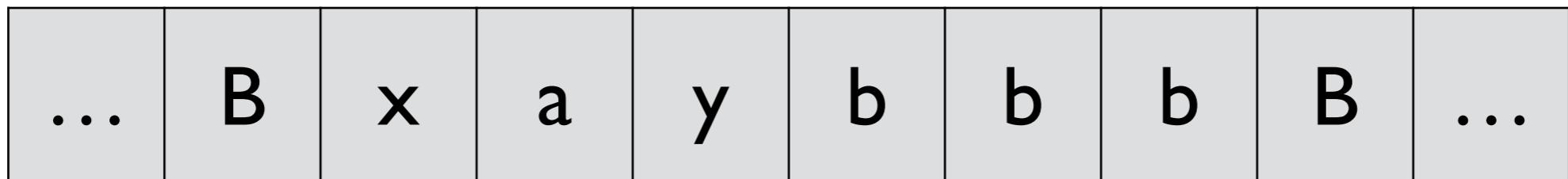
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

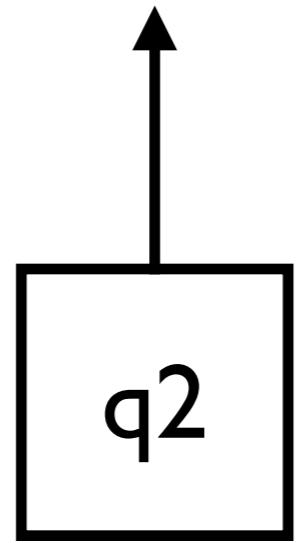
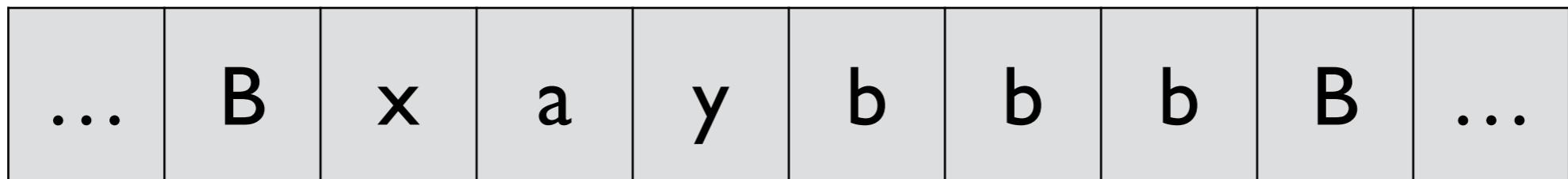
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

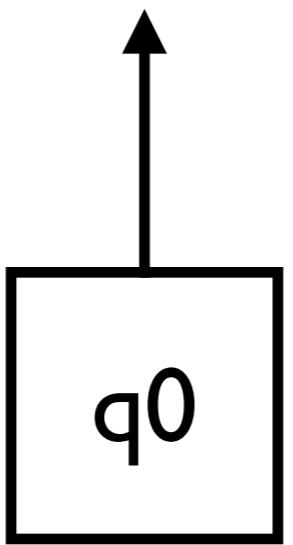
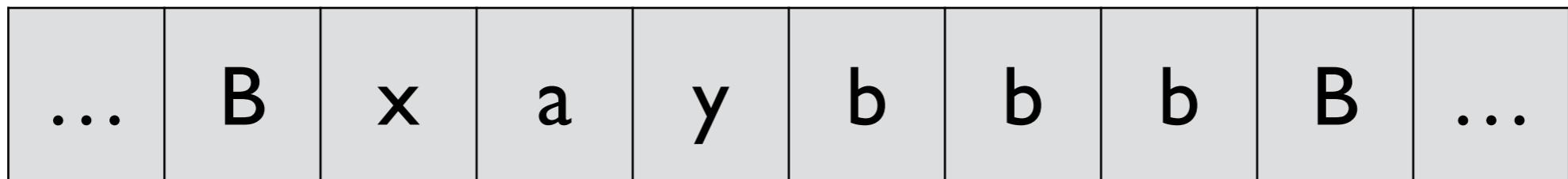
$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

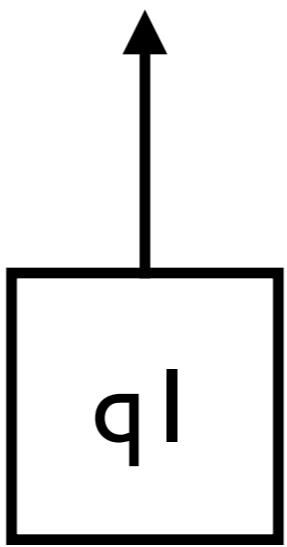


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

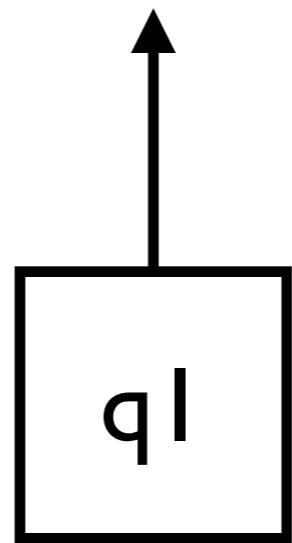
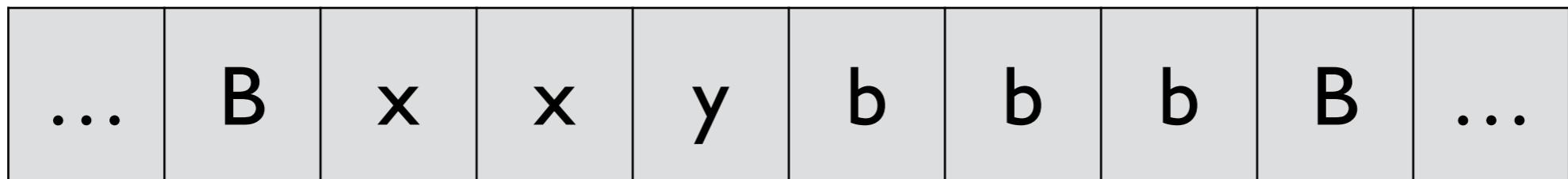
...	B	x	x	y	b	b	b	B	...
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$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

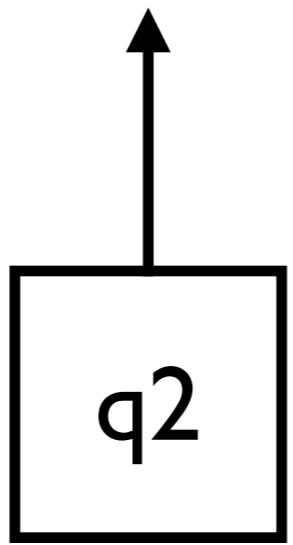


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	y	y	b	b	B	...
-----	----------	---	---	---	---	----------	---	----------	-----

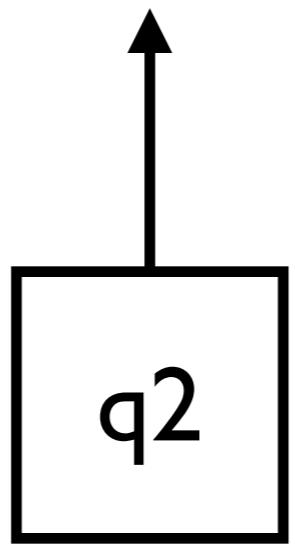


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	y	y	b	b	B	...
-----	----------	---	---	---	---	----------	---	----------	-----

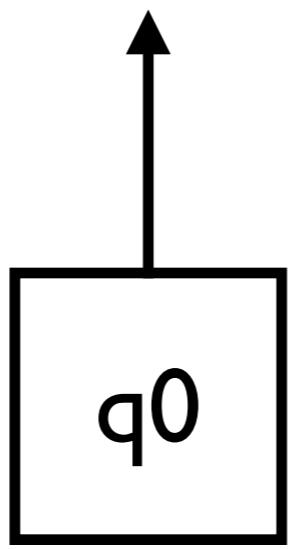


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	y	y	b	b	B	...
-----	----------	---	---	---	---	----------	---	----------	-----

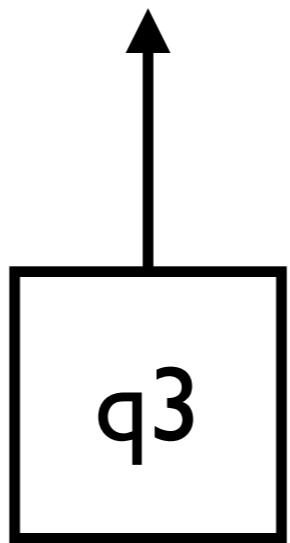


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	y	y	b	b	B	...
-----	----------	---	---	---	---	----------	---	----------	-----

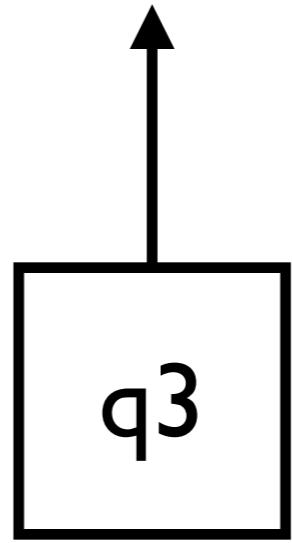


$$\begin{aligned}\delta(q_0, a) &= (q_1, x, R) \\ \delta(q_1, a) &= (q_1, a, R) \\ \delta(q_1, y) &= (q_1, y, R) \\ \delta(q_1, b) &= (q_2, y, L)\end{aligned}$$

$$\begin{aligned}\delta(q_2, y) &= (q_2, y, L) \\ \delta(q_2, a) &= (q_2, a, L) \\ \delta(q_2, x) &= (q_0, x, R)\end{aligned}$$

$$\begin{aligned}\delta(q_0, y) &= (q_3, y, R) \\ \delta(q_3, y) &= (q_3, y, R) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B	x	x	y	y	b	b	B	...
-----	----------	---	---	---	---	----------	---	----------	-----



(undefined, halt)

$$\delta(q_0, a) = (q_1, x, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, y) = (q_1, y, R)$$

$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$

$$\delta(q_2, a) = (q_2, a, L)$$

$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$

$$\delta(q_3, y) = (q_3, y, R)$$

$$\delta(q_3, B) = (q_4, B, R)$$

Example 2. Given x and y , design a Turing machine that computes $x + y$.

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_4\})$$

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

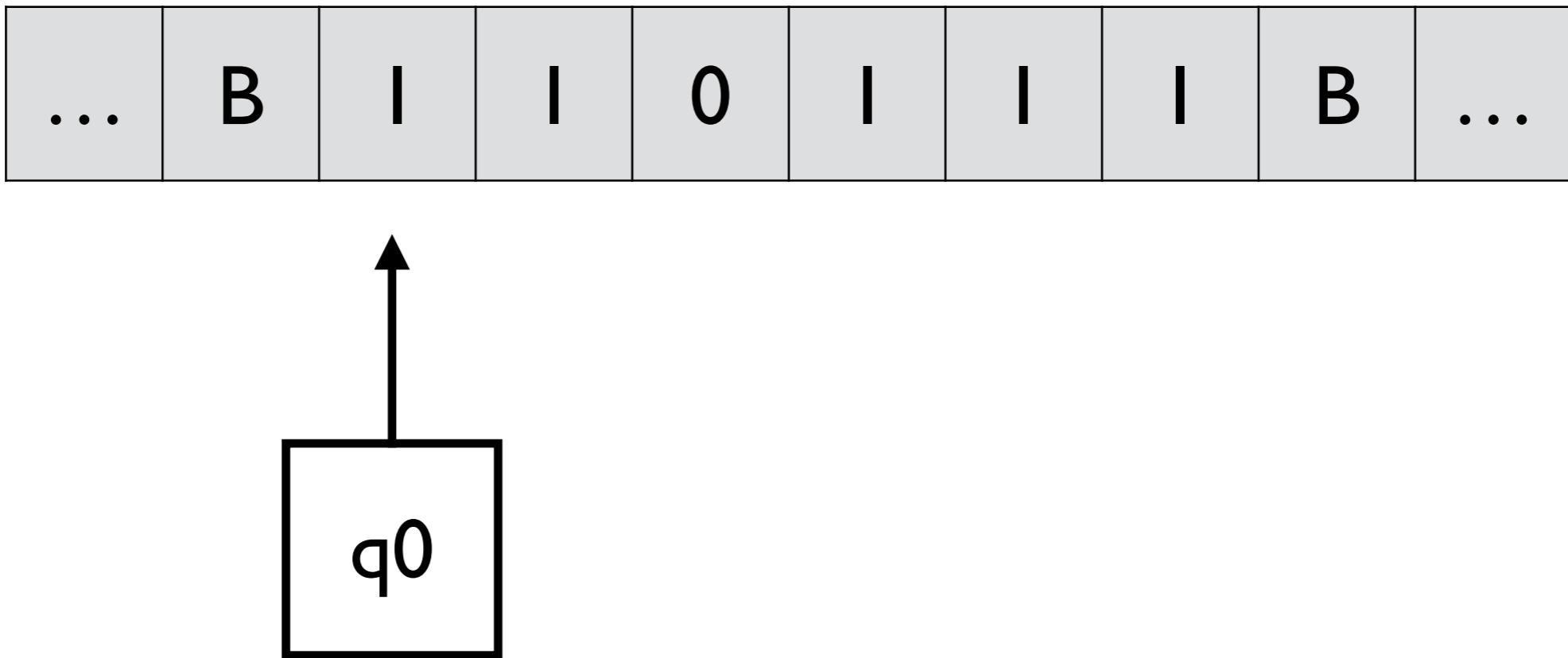
$$\delta(q_1, B) = (q_2, B, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

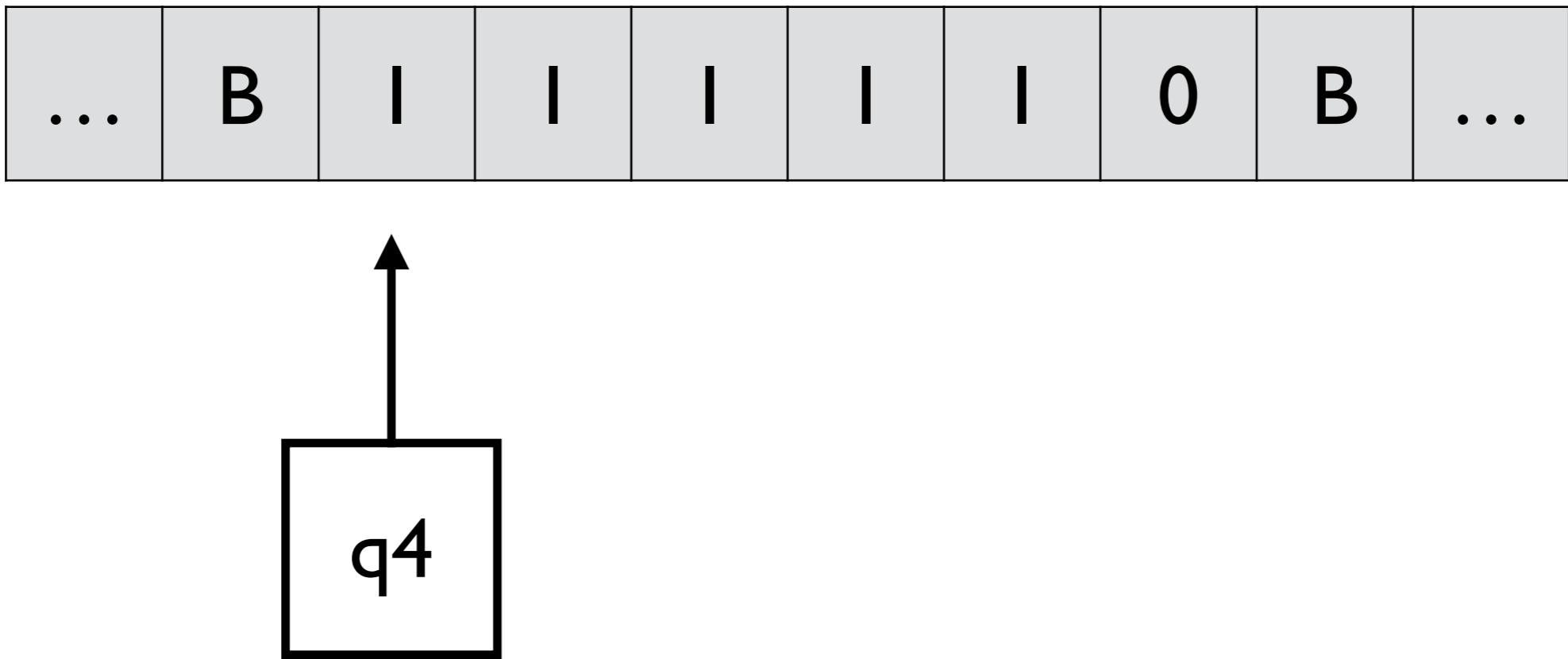
$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, B) = (q_4, B, R)$$

Initial machine configuration:

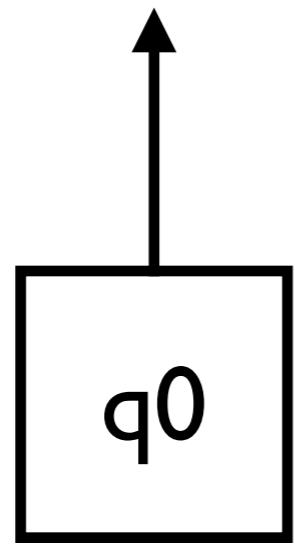


Final machine configuration:

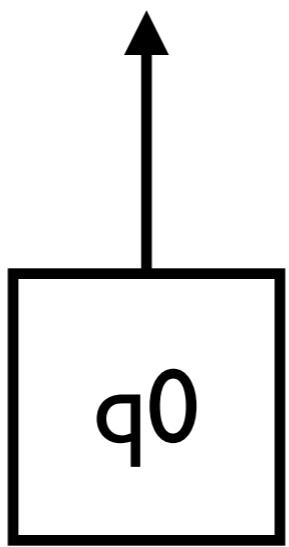
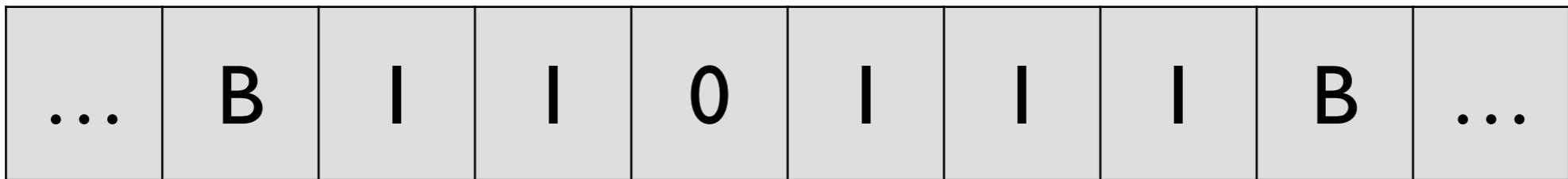


Addition in math is to move 0 to the right end

...	B			0				B	...
-----	---	--	--	---	--	--	--	---	-----

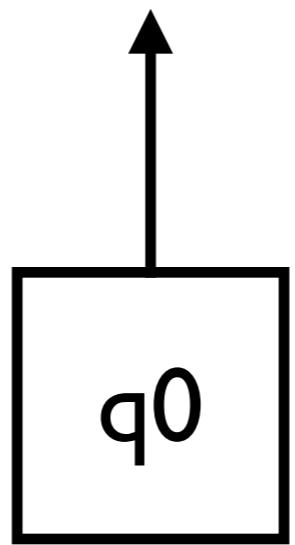


$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

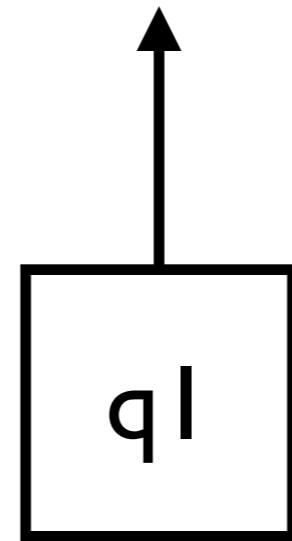
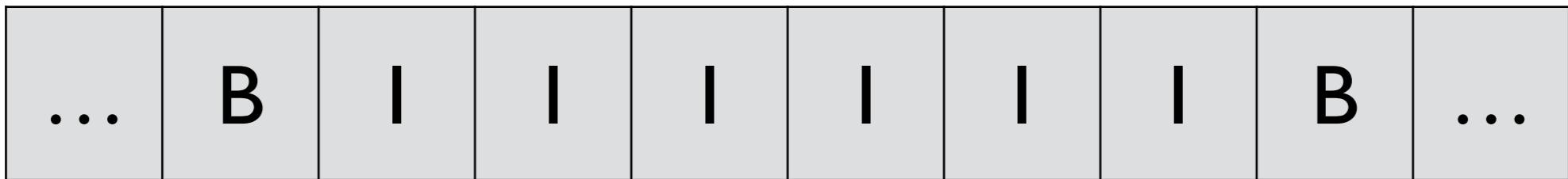


$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B			0				B	...
-----	---	--	--	---	--	--	--	---	-----

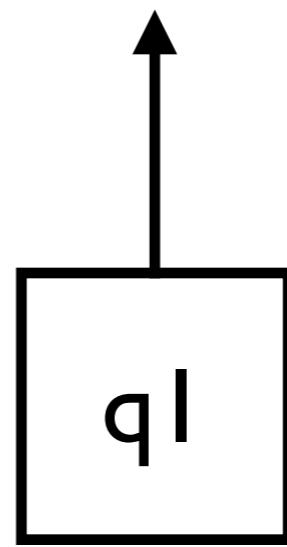
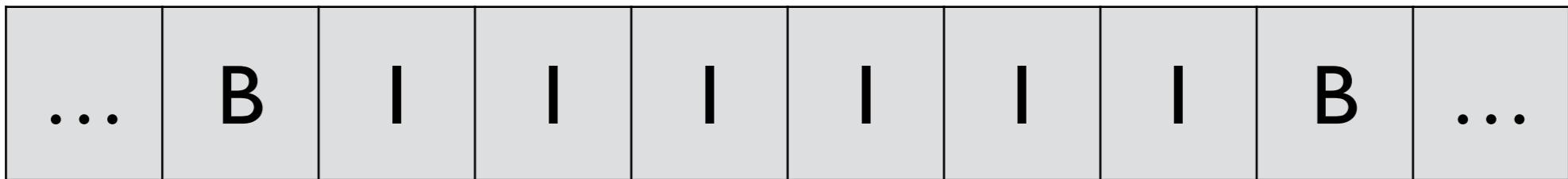


$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

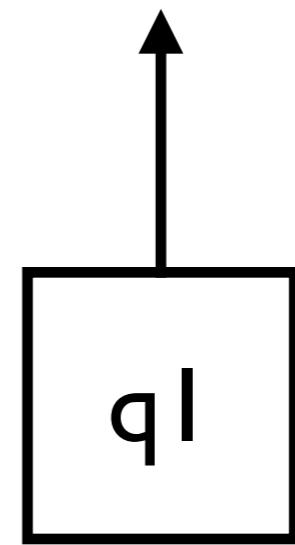
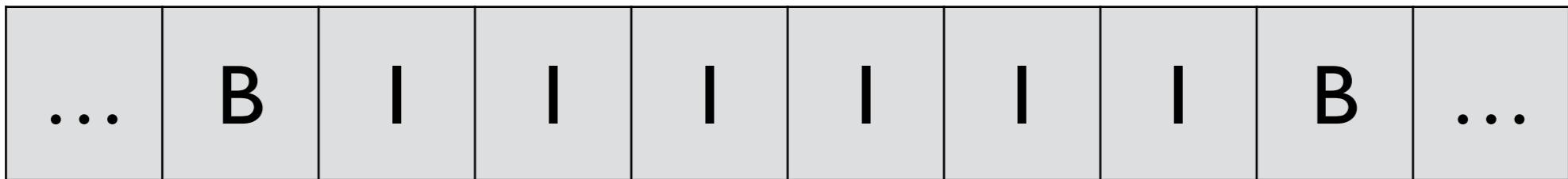


(In ql, search for the right end of y)

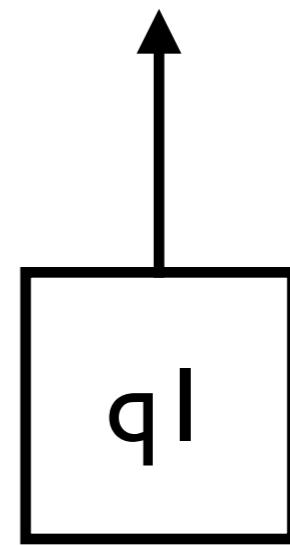
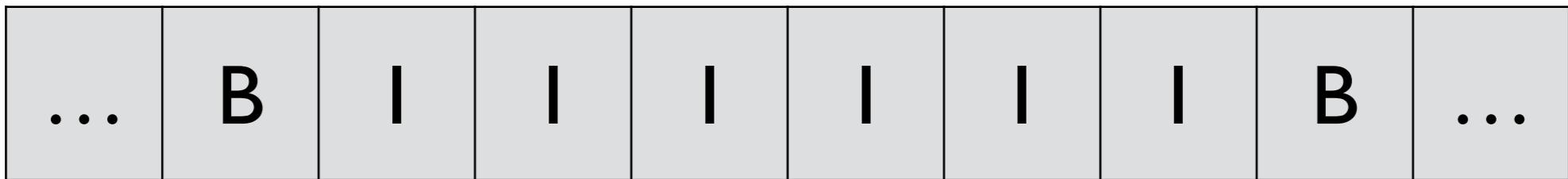
$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, 1, R) \\
 \delta(q_0, 0) &= (q_1, 1, R) \\
 \delta(q_1, 1) &= (q_1, 1, R) \\
 \delta(q_1, B) &= (q_2, B, L) \\
 \delta(q_2, 1) &= (q_3, 0, L) \\
 \delta(q_3, 1) &= (q_3, 1, L) \\
 \delta(q_3, B) &= (q_4, B, R)
 \end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

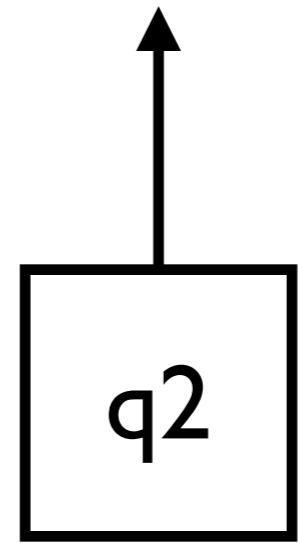


$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

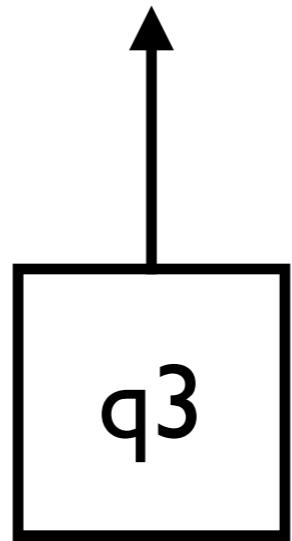
...	B							B	...
-----	---	--	--	--	--	--	--	---	-----



(In q_2 , replace the rightmost $|$ by 0)

$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, 1, R) \\
 \delta(q_0, 0) &= (q_1, 1, R) \\
 \delta(q_1, 1) &= (q_1, 1, R) \\
 \delta(q_1, B) &= (q_2, B, L) \\
 \delta(q_2, 1) &= (q_3, 0, L) \\
 \delta(q_3, 1) &= (q_3, 1, L) \\
 \delta(q_3, B) &= (q_4, B, R)
 \end{aligned}$$

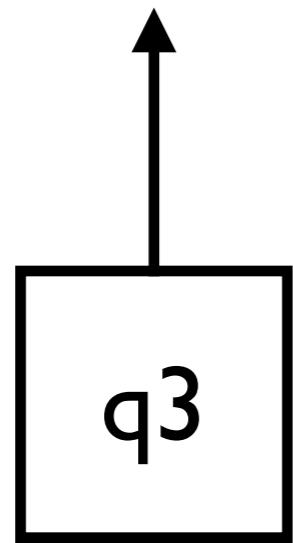
...	B						0	B	...
-----	---	--	--	--	--	--	---	---	-----



(In q_3 , look for the leftmost |)

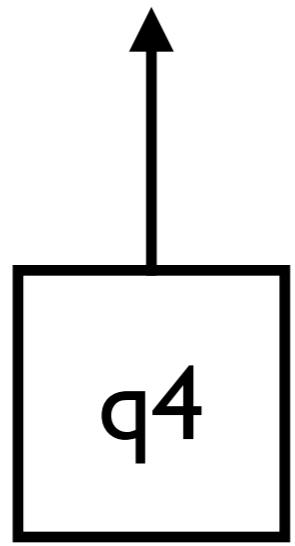
$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, 1, R) \\
 \delta(q_0, 0) &= (q_1, 1, R) \\
 \delta(q_1, 1) &= (q_1, 1, R) \\
 \delta(q_1, B) &= (q_2, B, L) \\
 \delta(q_2, 1) &= (q_3, 0, L) \\
 \delta(q_3, 1) &= (q_3, 1, L) \\
 \delta(q_3, B) &= (q_4, B, R)
 \end{aligned}$$

...	B						0	B	...
-----	---	--	--	--	--	--	---	---	-----



$$\begin{aligned}\delta(q_0, 1) &= (q_0, 1, R) \\ \delta(q_0, 0) &= (q_1, 1, R) \\ \delta(q_1, 1) &= (q_1, 1, R) \\ \delta(q_1, B) &= (q_2, B, L) \\ \delta(q_2, 1) &= (q_3, 0, L) \\ \delta(q_3, 1) &= (q_3, 1, L) \\ \delta(q_3, B) &= (q_4, B, R)\end{aligned}$$

...	B						0	B	...
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“final state”

$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, 1, R) \\
 \delta(q_0, 0) &= (q_1, 1, R) \\
 \delta(q_1, 1) &= (q_1, 1, R) \\
 \delta(q_1, B) &= (q_2, B, L) \\
 \delta(q_2, 1) &= (q_3, 0, L) \\
 \delta(q_3, 1) &= (q_3, 1, L) \\
 \delta(q_3, B) &= (q_4, B, R)
 \end{aligned}$$

Example 3. Design a Turing machine that transforms w into ww .

$$M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$$

$$\delta(q_0, 1) = (q_0, x, R)$$

$$\delta(q_0, B) = (q_1, B, L)$$

$$\delta(q_1, 1) = (q_1, 1, L)$$

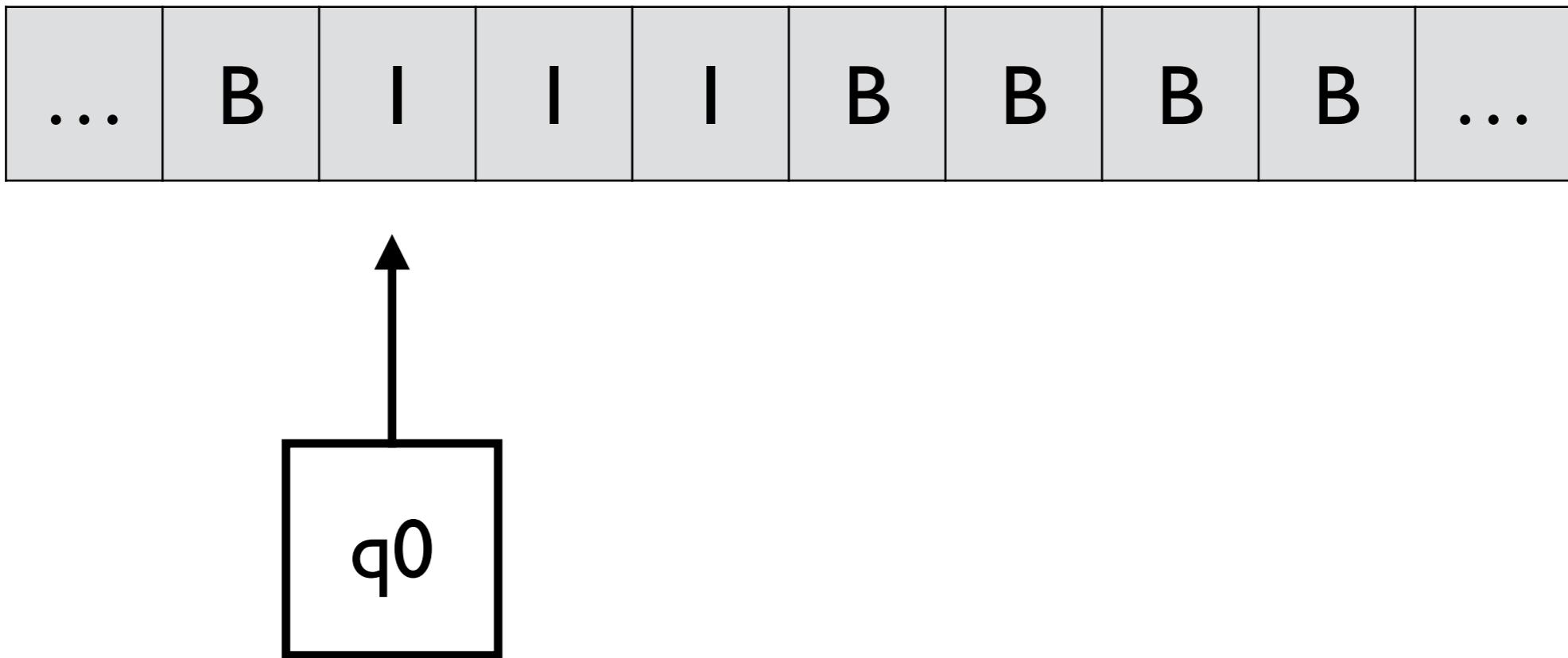
$$\delta(q_1, x) = (q_2, 1, R)$$

$$\delta(q_2, 1) = (q_2, 1, R)$$

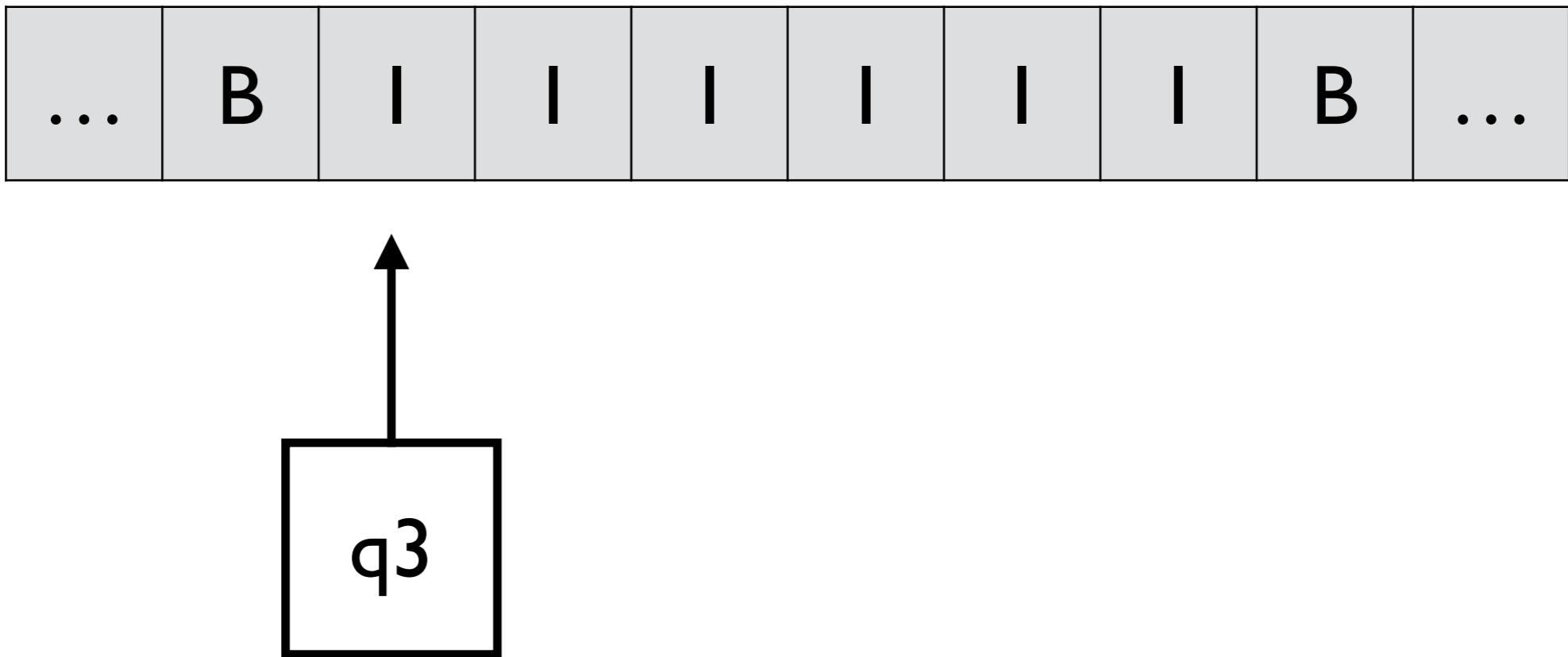
$$\delta(q_2, B) = (q_1, 1, L)$$

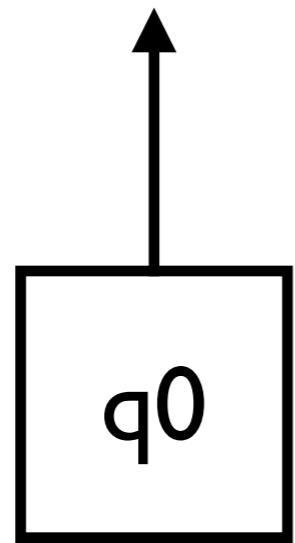
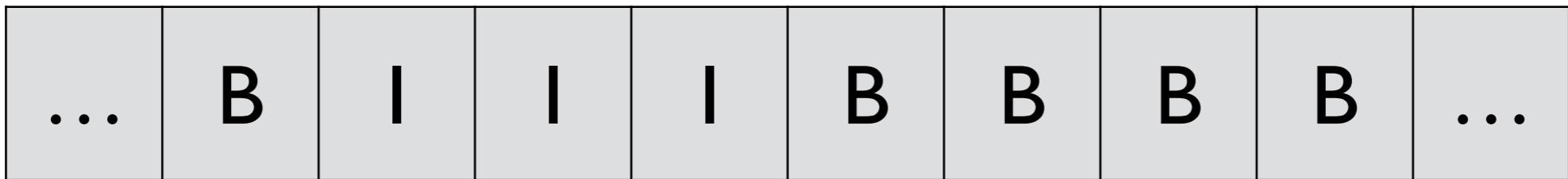
$$\delta(q_1, B) = (q_3, B, R)$$

Initial machine configuration:



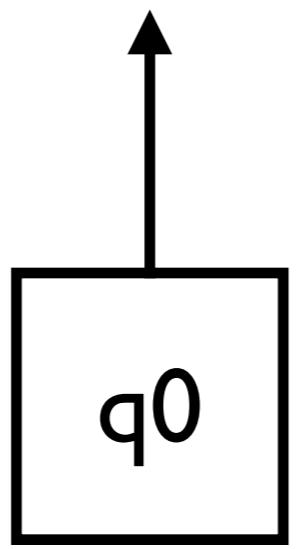
Final machine configuration:





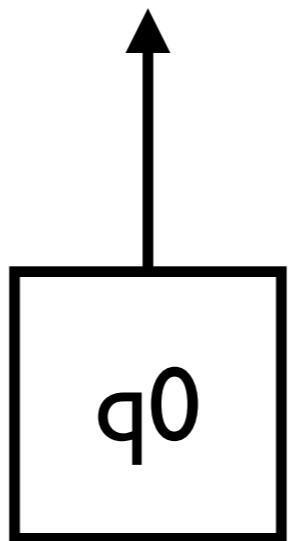
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x			B	B	B	B	...
-----	---	---	--	--	---	---	---	---	-----



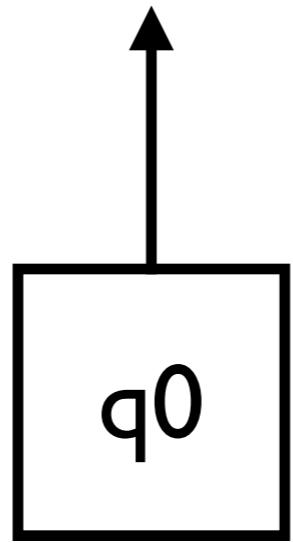
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x	x		B	B	B	B	...
-----	---	---	---	--	---	---	---	---	-----



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

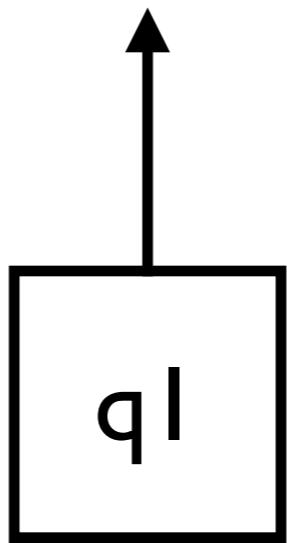
...	B	x	x	x	B	B	B	B	...
-----	---	---	---	---	---	---	---	---	-----



Initially, replace every 1 by x

$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, x, R) \\
 \delta(q_0, B) &= (q_1, B, L) \\
 \delta(q_1, 1) &= (q_1, 1, L) \\
 \delta(q_1, x) &= (q_2, 1, R) \\
 \delta(q_2, 1) &= (q_2, 1, R) \\
 \delta(q_2, B) &= (q_1, 1, L) \\
 \delta(q_1, B) &= (q_3, B, R)
 \end{aligned}$$

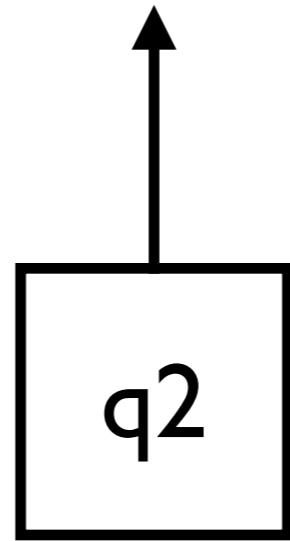
...	B	x	x	x	B	B	B	B	...
-----	---	---	---	---	---	---	---	---	-----



In q_l , look for the rightmost x

$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, x, R) \\
 \delta(q_0, B) &= (q_1, B, L) \\
 \delta(q_1, 1) &= (q_1, 1, L) \\
 \delta(q_1, x) &= (q_2, 1, R) \\
 \delta(q_2, 1) &= (q_2, 1, R) \\
 \delta(q_2, B) &= (q_1, 1, L) \\
 \delta(q_1, B) &= (q_3, B, R)
 \end{aligned}$$

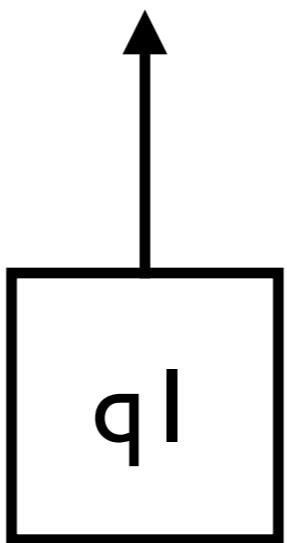
...	B	x	x		B	B	B	B	...
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In q2, look for the first blank and write |

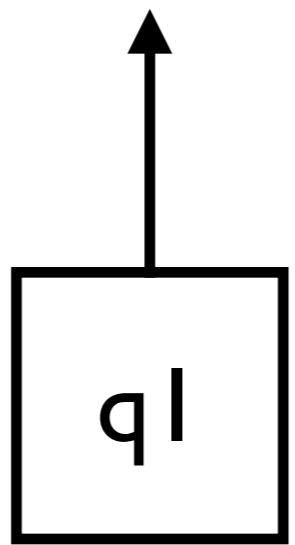
$$\begin{aligned}
 \delta(q_0, 1) &= (q_0, x, R) \\
 \delta(q_0, B) &= (q_1, B, L) \\
 \delta(q_1, 1) &= (q_1, 1, L) \\
 \delta(q_1, x) &= (q_2, 1, R) \\
 \delta(q_2, 1) &= (q_2, 1, R) \\
 \delta(q_2, B) &= (q_1, 1, L) \\
 \delta(q_1, B) &= (q_3, B, R)
 \end{aligned}$$

...	B	x	x			B	B	B	...
-----	---	---	---	--	--	---	---	---	-----



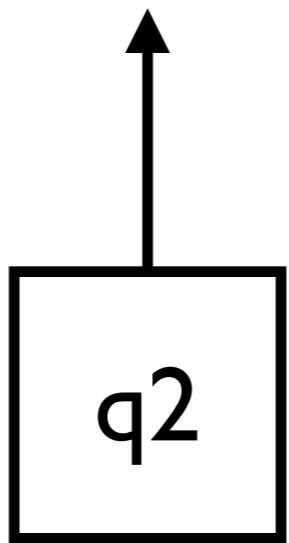
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x	x			B	B	B	...
-----	---	---	---	--	--	---	---	---	-----



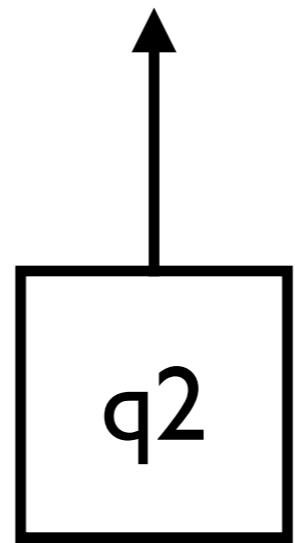
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x				B	B	B	...
-----	---	---	--	--	--	---	---	---	-----



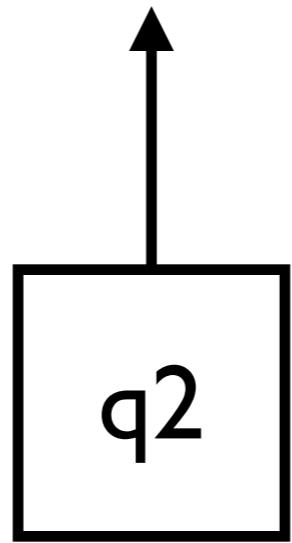
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x				B	B	B	...
-----	---	---	--	--	--	---	---	---	-----



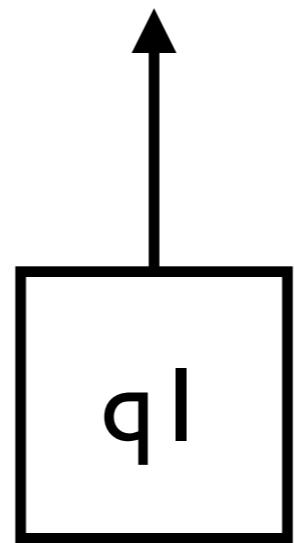
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x				B	B	B	...
-----	---	---	--	--	--	---	---	---	-----



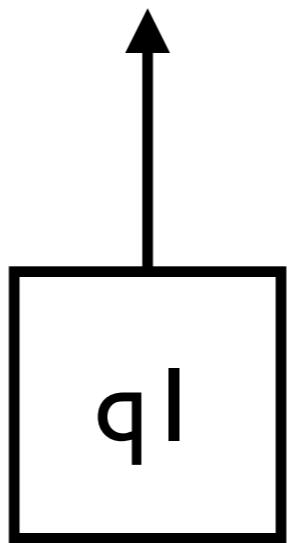
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x					B	B	...
-----	---	---	--	--	--	--	---	---	-----



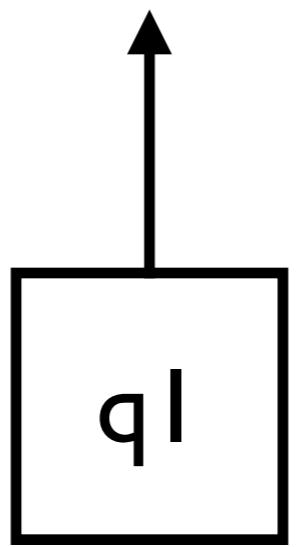
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x					B	B	...
-----	----------	----------	--	--	--	--	----------	----------	-----



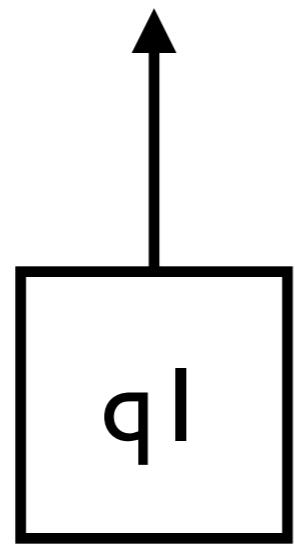
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

...	B	x					B	B	...
-----	----------	----------	--	--	--	--	----------	----------	-----

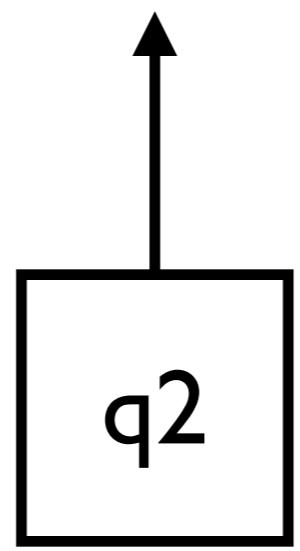
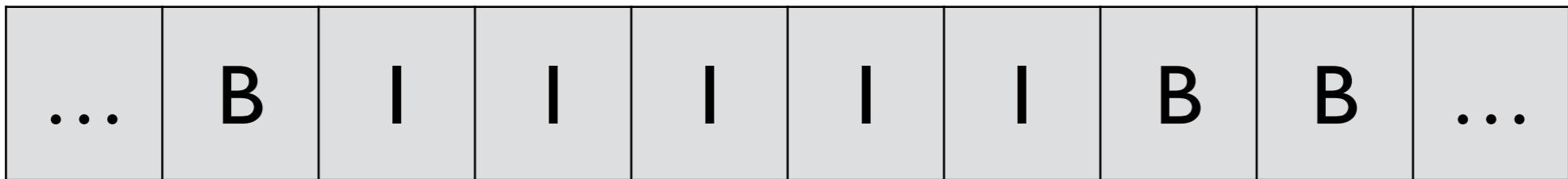


$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$

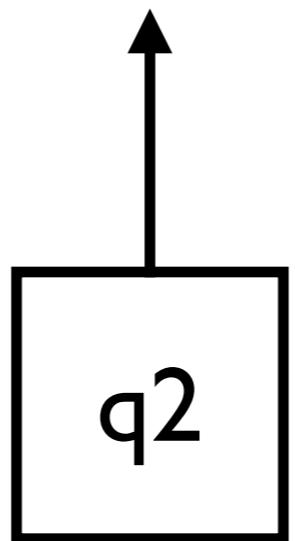
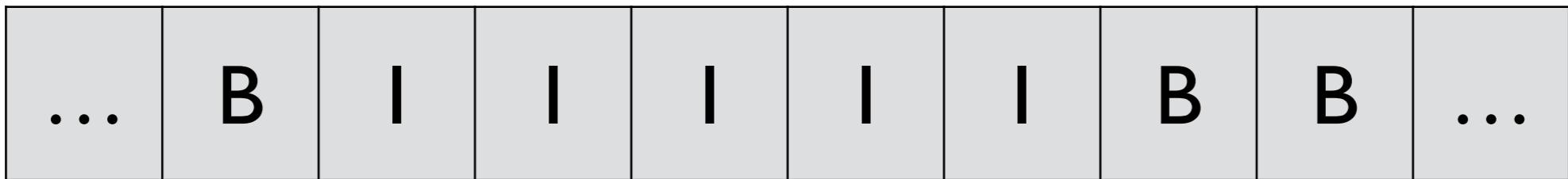
...	B	x					B	B	...
-----	---	---	--	--	--	--	---	---	-----



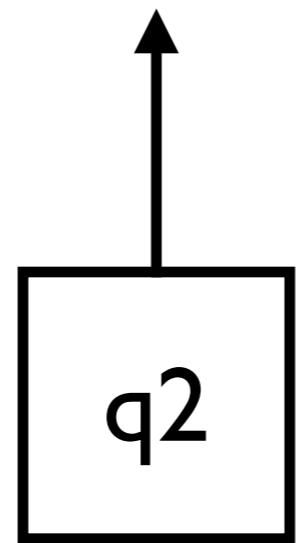
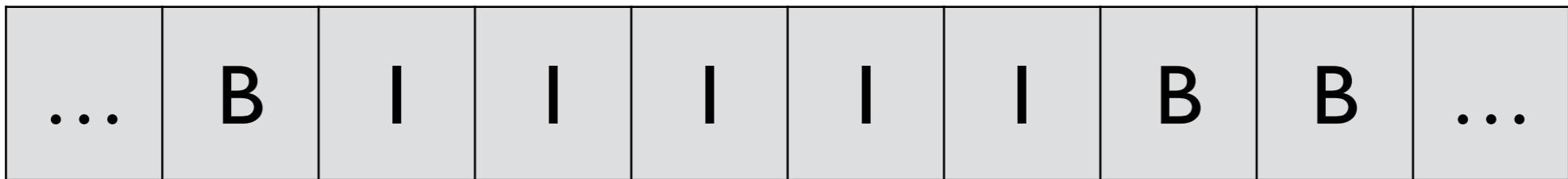
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



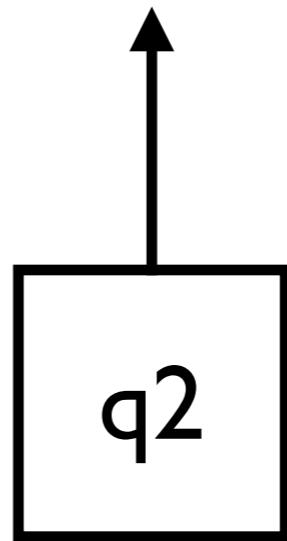
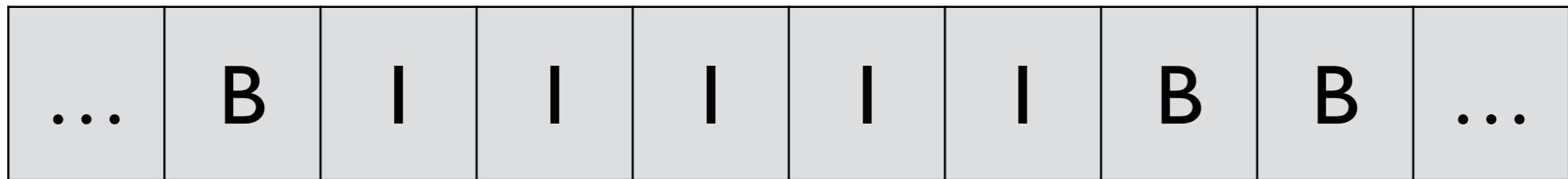
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



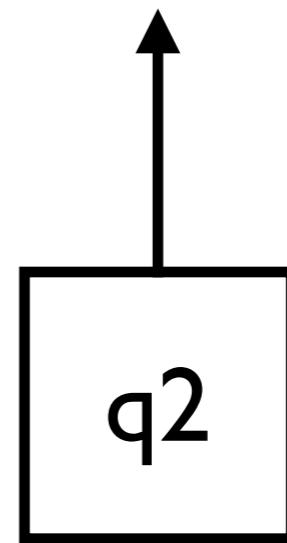
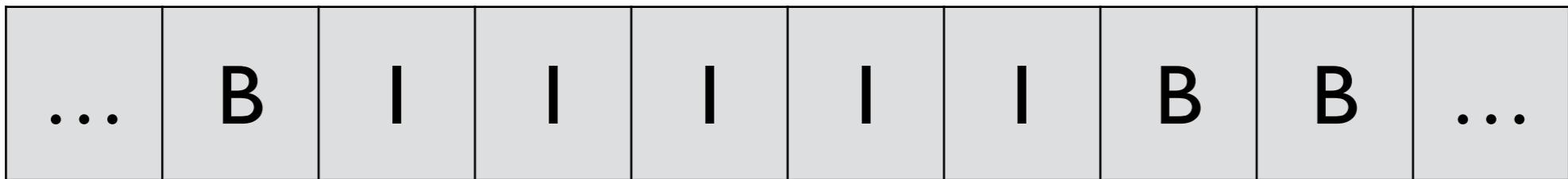
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



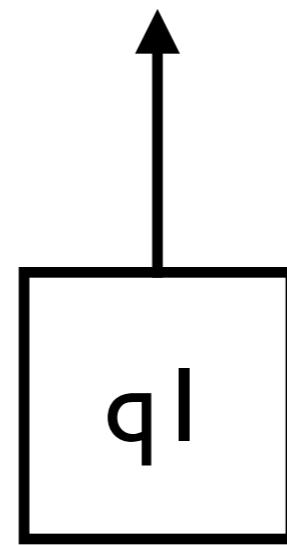
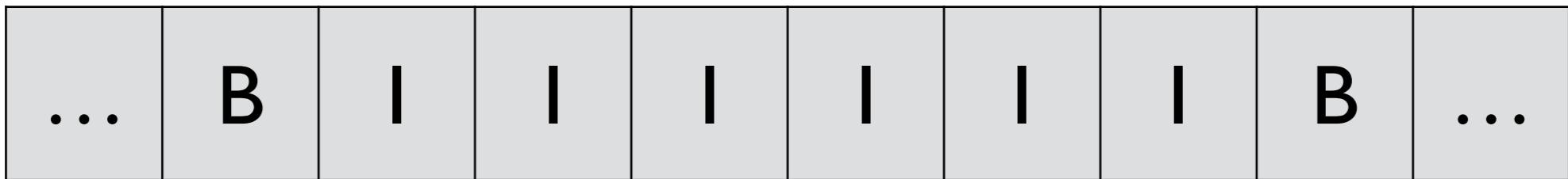
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



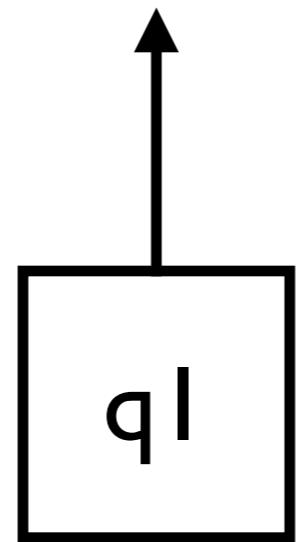
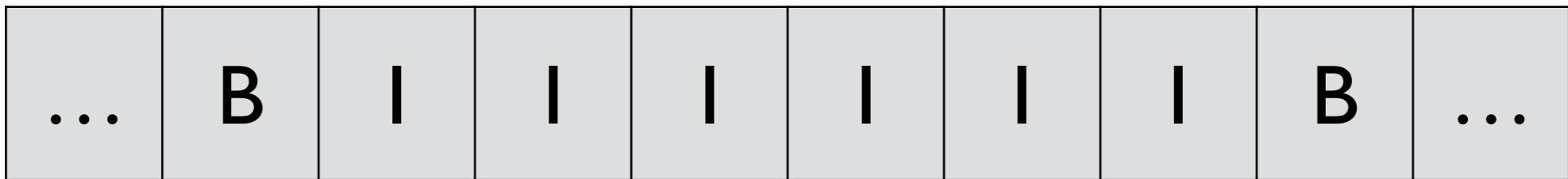
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



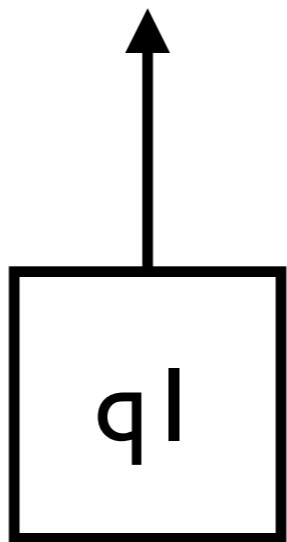
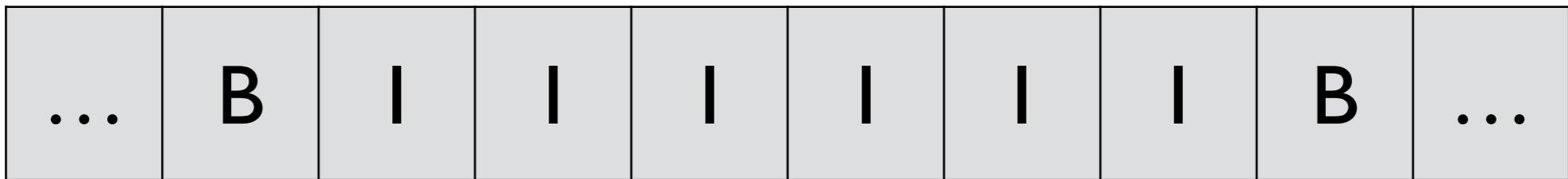
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



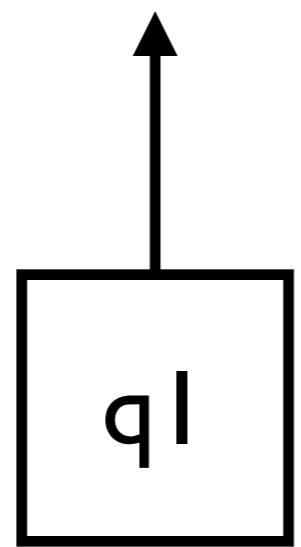
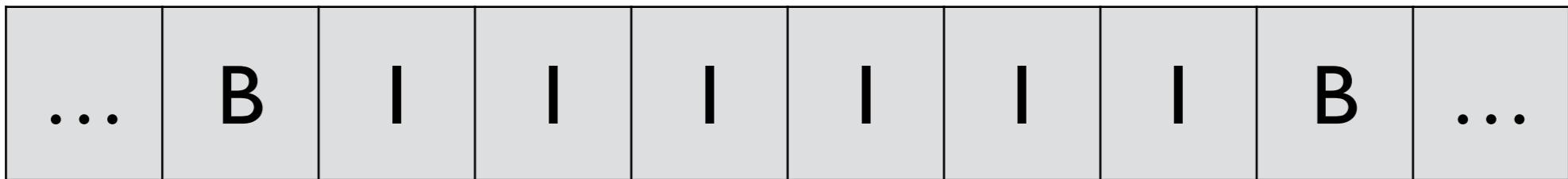
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



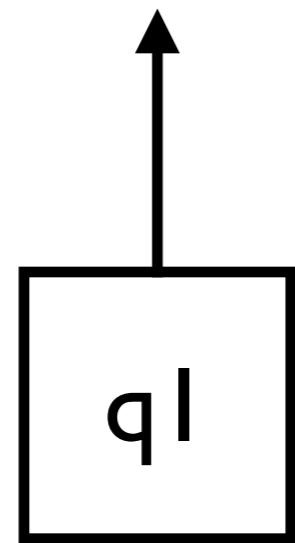
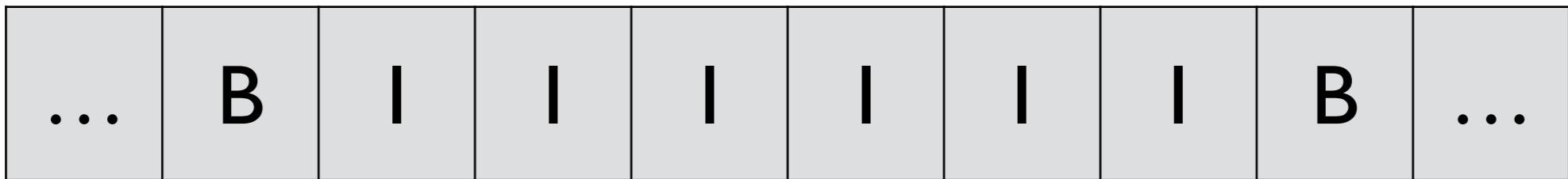
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



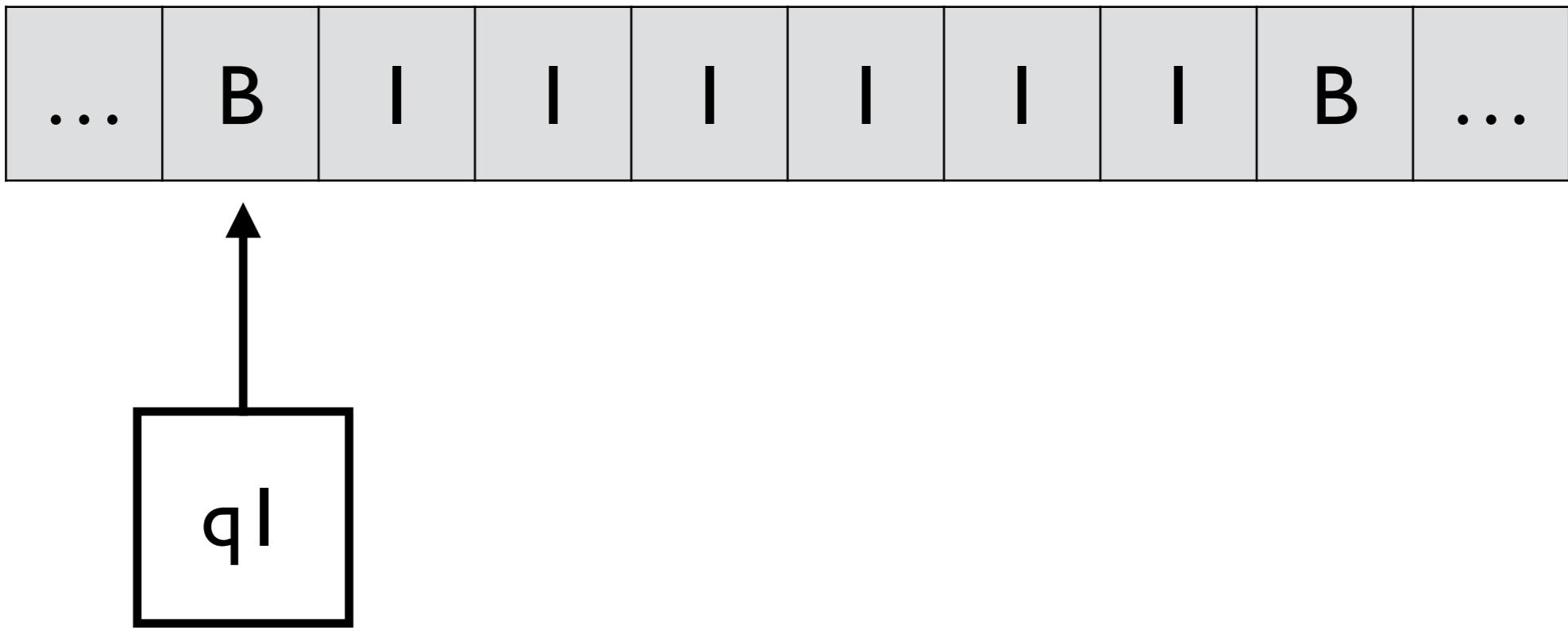
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



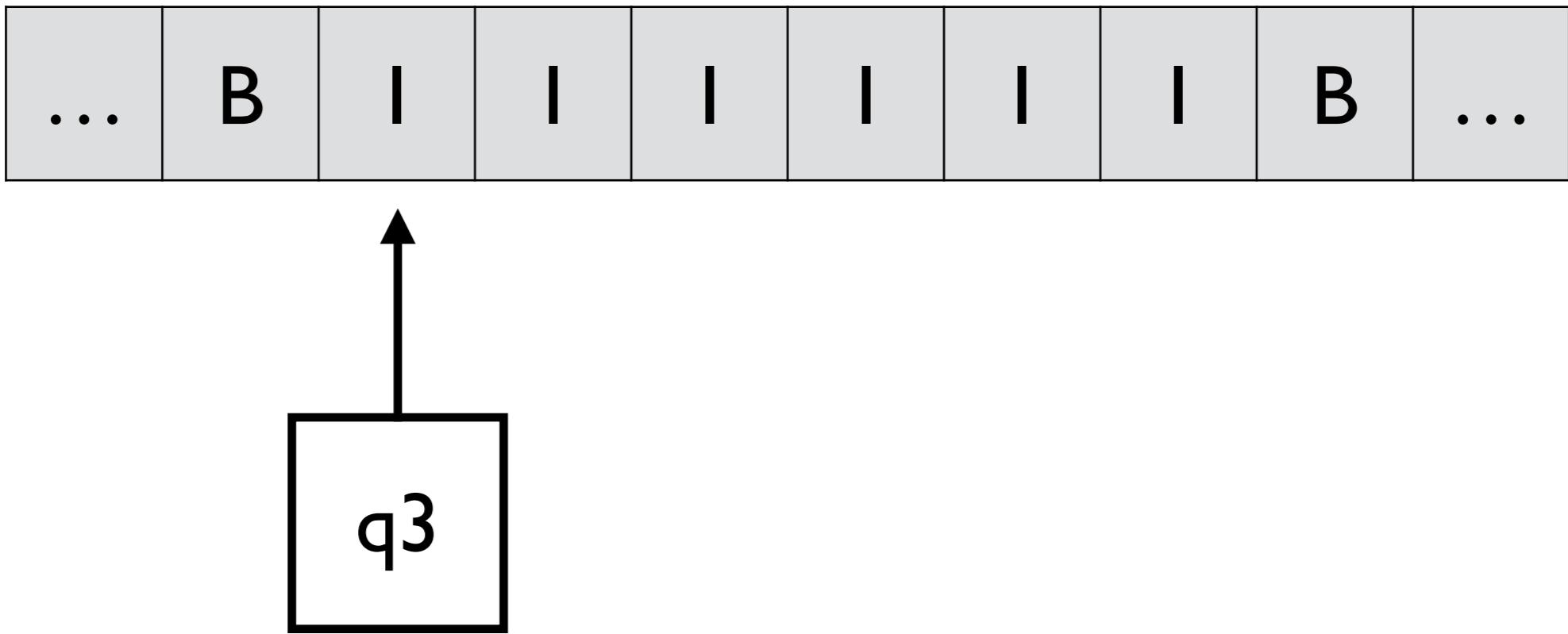
$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$



“final state”

$$\begin{aligned}\delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R)\end{aligned}$$