

COSE212: Programming Languages

Lecture 8 — Design and Implementation of PLs (4) States

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Review: Our Language So Far

Our language has expressions and procedures.

Syntax

$$\begin{array}{lcl} P & \rightarrow & E \\ E & \rightarrow & n \\ & | & x \\ & | & E + E \\ & | & E - E \\ & | & \text{iszero } E \\ & | & \text{if } E \text{ then } E \text{ else } E \\ & | & \text{let } x = E \text{ in } E \\ & | & \text{read} \\ & | & \text{letrec } f(x) = E \text{ in } E \\ & | & \text{proc } x \ E \\ & | & E \ E \end{array}$$

Review: Our Language So Far

Semantics

$$\frac{}{\rho \vdash n \Rightarrow n} \quad \frac{}{\rho \vdash x \Rightarrow \rho(x)} \quad \frac{\rho \vdash E_1 \Rightarrow n_1 \quad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2}$$

$$\frac{\rho \vdash E \Rightarrow 0}{\rho \vdash \text{iszero } E \Rightarrow \text{true}} \quad \frac{\rho \vdash E \Rightarrow n}{\rho \vdash \text{iszero } E \Rightarrow \text{false}} \quad n \neq 0 \quad \frac{}{\rho \vdash \text{read} \Rightarrow n}$$

$$\frac{\rho \vdash E_1 \Rightarrow \text{true} \quad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v} \quad \frac{\rho \vdash E_1 \Rightarrow \text{false} \quad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \quad [x \mapsto v_1]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v} \quad \frac{[f \mapsto (f, x, E_1, \rho)]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \text{letrec } f(x) = E_1 \text{ in } E_2 \Rightarrow v}$$

$$\frac{}{\rho \vdash \text{proc } x \ E \Rightarrow (x, E, \rho)}$$

$$\frac{\rho \vdash E_1 \Rightarrow (x, E, \rho') \quad \rho \vdash E_2 \Rightarrow v \quad [x \mapsto v]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1 \ E_2 \Rightarrow v'}$$

$$\frac{\rho \vdash E_1 \Rightarrow (f, x, E, \rho') \quad \rho \vdash E_2 \Rightarrow v \quad [x \mapsto v, f \mapsto (f, x, E, \rho')]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1 \ E_2 \Rightarrow v'}$$

This Lecture: Adding States to the Language

- So far, our language only had the values produced by computation.
- But computation also has *effects*: it may change the state of memory.
- We will extend the language to support computational effects:
 - ▶ Syntax for creating and using memory locations
 - ▶ Semantics for manipulating memory states

Motivating Example

- How can we compute the number of times `f` has been called?

```
let f = proc (x) (x)
in (f (f 1))
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- Does the following program work?

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let counter = 0
in let f = proc (x) (let counter = counter + 1
                    in x)
   in let a = (f (f 1))
      in counter
```

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      in counter
```

- The binding of `counter` is local. We need global *effects*.
- Effects are implemented by introducing *memory (store)* and *locations (reference)*.

Two Approaches

Programming languages support references explicitly or implicitly.

- Languages with explicit references provide a clear account of allocation, dereference, and mutation of memory cells.
 - ▶ e.g., OCaml, F#
- In languages with implicit references, references are built-in. References are not explicitly manipulated.
 - ▶ e.g., C and Java.

EXPLICIT-REFS: A Language with Explicit References

$$\begin{array}{lcl} P & \rightarrow & E \\ E & \rightarrow & n \mid x \\ & & | \quad E + E \mid E - E \\ & & | \quad \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ & & | \quad \text{let } x = E \text{ in } E \\ & & | \quad \text{proc } x \ E \mid E \ E \\ & & | \quad \text{ref } E \\ & & | \quad ! E \\ & & | \quad E := E \\ & & | \quad E; E \end{array}$$

- $\text{ref } E$ allocates a new location, store the value of E in it, and returns it.
- $! E$ returns the contents of the location that E refers to.
- $E_1 := E_2$ changes the contents of the location (E_1) by the value of E_2 .
- $E_1; E_2$ executes E_1 and then E_2 while accumulating effects.

Example 1

- ```
let counter = ref 0
 in let f = proc (x) (counter := !counter + 1; !counter)
 in let a = (f 0)
 in let b = (f 0)
 in (a - b)
```

## Example 1

- `let counter = ref 0`  
  `in let f = proc (x) (counter := !counter + 1; !counter)`  
    `in let a = (f 0)`  
      `in let b = (f 0)`  
        `in (a - b)`
- `let f = let counter = ref 0`  
  `in proc (x) (counter := !counter + 1; !counter)`  
`in let a = (f 0)`  
  `in let b = (f 0)`  
    `in (a - b)`

## Example 1

- ```
let counter = ref 0
  in let f = proc (x) (counter := !counter + 1; !counter)
    in let a = (f 0)
      in let b = (f 0)
        in (a - b)
```
- ```
let f = let counter = ref 0
 in proc (x) (counter := !counter + 1; !counter)
in let a = (f 0)
 in let b = (f 0)
 in (a - b)
```
- ```
let f = proc (x) (let counter = ref 0
  in (counter := !counter + 1; !counter))
in let a = (f 0)
  in let b = (f 0)
    in (a - b)
```

Example 2

We can make chains of references:

```
let x = ref (ref 0)
in (!x := 11; !(!x))
```

Semantics

Memory is modeled as a finite map from locations to values:

$$\begin{aligned}Val &= \mathbb{Z} + Bool + Procedure + Loc \\ Procedure &= Id \times E \times Env \\ \rho \in Env &= Id \rightarrow Val \\ \sigma \in Mem &= Loc \rightarrow Val\end{aligned}$$

Semantics rules additionally describe memory effects:

$$\rho, \sigma \vdash E \Rightarrow v, \sigma'$$

Semantics

Existing rules are enriched with memory effects:

$$\frac{}{\rho, \sigma \vdash n \Rightarrow n, \sigma} \quad \frac{}{\rho, \sigma \vdash x \Rightarrow \rho(x), \sigma}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2}{\rho, \sigma_0 \vdash E_1 + E_2 \Rightarrow n_1 + n_2, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{true}, \sigma_1} \quad \frac{\rho, \sigma_0 \vdash E \Rightarrow n, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{false}, \sigma_1} \quad n \neq 0$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \text{true}, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \text{false}, \sigma_1 \quad \rho, \sigma_1 \vdash E_3 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{if } E_1 \text{ then } E_2 \text{ else } E_3 \Rightarrow v, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad [x \mapsto v_1]\rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v, \sigma_2}$$

$$\frac{}{\rho, \sigma \vdash \text{proc } x \ E \Rightarrow (x, E, \rho), \sigma}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \quad [x \mapsto v]\rho', \sigma_2 \vdash E \Rightarrow v', \sigma_3}{\rho, \sigma_0 \vdash E_1 \ E_2 \Rightarrow v', \sigma_3}$$

Semantics

Rules for new constructs:

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow v, \sigma_1}{\rho, \sigma_0 \vdash \text{ref } E \Rightarrow l, [l \mapsto v]\sigma_1} \quad l \notin \text{Dom}(\sigma_1)$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow l, \sigma_1}{\rho, \sigma_0 \vdash ! E \Rightarrow \sigma_1(l), \sigma_1}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow l, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash E_1 := E_2 \Rightarrow v, [l \mapsto v]\sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v_2, \sigma_2}{\rho, \sigma_0 \vdash E_1; E_2 \Rightarrow v_2, \sigma_2}$$

Example

$$\frac{}{\rho, \sigma_0 \vdash \text{let } x = \text{ref } (\text{ref } 0) \text{ in } (!x := 11; !(!x)) \Rightarrow}$$

Exercise

Extend the language with recursive procedures:

$$\begin{array}{lcl} P & \rightarrow & E \\ E & \rightarrow & n \mid x \\ & & E + E \mid E - E \\ & & \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ & & \text{let } x = E \text{ in } E \\ & & \text{letrec } f(x) = E \text{ in } E \\ & & \text{proc } x E \mid E E \\ & & \text{ref } E \\ & & ! E \\ & & E := E \\ & & E; E \end{array}$$

Exercise (Continued)

- Domain:

$$\begin{aligned}Val &= \mathbb{Z} + Bool + Procedure + Loc \\ Procedure &= Id \times E \times Env \\ \rho \in Env &= Id \rightarrow Val \\ \sigma \in Mem &= Loc \rightarrow Val\end{aligned}$$

- Semantics rules:

$$\frac{}{\rho, \sigma_0 \vdash \text{letrec } f(x) = E_1 \text{ in } E_2 \Rightarrow}$$

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IMPLICIT-REFS: A Language with Implicit References

$$\begin{array}{lcl} P & \rightarrow & E \\ E & \rightarrow & n \mid x \\ & & | \quad E + E \mid E - E \\ & & | \quad \text{iszero } E \mid \text{if } E \text{ then } E \text{ else } E \\ & & | \quad \text{let } x = E \text{ in } E \\ & & | \quad \text{proc } x \ E \mid E \ E \\ & & | \quad x := E \\ & & | \quad E; E \end{array}$$

- In this design, every variable denotes a reference and is mutable.
- $x := E$ changes the contents of x by the value of E .

Examples

Computing the number of times `f` has been called:

- ```
let counter = 0
in let f = proc (x) (counter := counter + 1; counter)
 in let a = (f 0)
 in let b = (f 0)
 in (a-b)
```

## Examples

Computing the number of times `f` has been called:

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let counter = 0
  in let f = proc (x) (counter := counter + 1; counter)
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          in let b = (f 0)
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- ```
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Computing the number of times `f` has been called:

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- ```
let f = let counter = 0
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```
- ```
let f = proc (x) (let counter = 0
                  in (counter := counter + 1; counter))
  in let a = (f 0)
      in let b = (f 0)
          in (a-b)
```

Exercise

What is the result of the program?

```
let f = proc (x)
  proc (y)
    (x := x + 1; x - y)
in ((f 44) 33)
```


Semantics

References are no longer values and every variable denotes a reference:

$$\begin{aligned}Val &= \mathbb{Z} + Bool + Procedure \\ Procedure &= Id \times E \times Env \\ \rho \in Env &= Id \rightarrow Loc \\ \sigma \in Mem &= Loc \rightarrow Val\end{aligned}$$

Semantics

$$\begin{array}{c}
 \overline{\rho, \sigma \vdash n \Rightarrow n, \sigma} \quad \overline{\rho, \sigma \vdash x \Rightarrow \sigma(\rho(x)), \sigma} \\
 \\
 \frac{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2}{\rho, \sigma_0 \vdash E_1 + E_2 \Rightarrow n_1 + n_2, \sigma_2} \quad \frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \text{iszero } E \Rightarrow \text{true}, \sigma_1} \\
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 \\
 \frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \quad [x \mapsto l]\rho, [l \mapsto v_1]\sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \text{let } x = E_1 \text{ in } E_2 \Rightarrow v, \sigma_2} \quad l \notin \text{Dom}(\sigma_1) \\
 \\
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 \end{array}$$

Example

```
let f = let count = 0
        in proc (x) (count := count + 1; count)
in let a = (f 0)
    in let b = (f 0)
        in a - b
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Parameter-Passing Variations

- Our current strategy of calling a procedure is *call-by-value*. The formal parameter refers to a new location containing the value of the actual parameter:

$$\frac{\begin{array}{c} \rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \\ [x \mapsto l]\rho', [l \mapsto v]\sigma_2 \vdash E \Rightarrow v', \sigma_3 \end{array}}{\rho, \sigma_0 \vdash E_1 E_2 \Rightarrow v', \sigma_3} \quad l \notin \text{Dom}(\sigma_2)$$

- The most commonly used form of parameter-passing.

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- The most commonly used form of parameter-passing.
- For example, the assignment to x has no effect on the contents of a:

```
let p = proc (x) (x := 4)
in let a = 3
    in ((p a); a)
```

Parameter-Passing Variations

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- The most commonly used form of parameter-passing.
- For example, the assignment to `x` has no effect on the contents of `a`:

```
let p = proc (x) (x := 4)
in let a = 3
    in ((p a); a)
```
- Under *call-by-reference*, the assignment changes the value of `a` after the call.

Call-By-Reference Parameter-Passing

The location of the caller's variable is passed, rather than the contents of the variable.

- Extend the syntax:

$$\begin{array}{c} E \rightarrow \vdots \\ | \quad E \ E \\ | \quad E \ \langle y \rangle \end{array}$$

- Extend the semantics:

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \quad [x \mapsto \rho(y)]\rho', \sigma_1 \vdash E \Rightarrow v', \sigma_2}{\rho, \sigma_0 \vdash E_1 \ \langle y \rangle \Rightarrow v', \sigma_2}$$

What is the benefit of call-by-reference compared to call-by-value?

Examples

- ```
let p = proc (x) (x := 4)
 in let a = 3
 in ((p <a>); a)
```

## Examples

- `let p = proc (x) (x := 4)`  
  `in let a = 3`  
    `in ((p <a>); a)`
- `let f = proc (x) (x := 44)`  
  `in let g = proc (y) (f <y>)`  
    `in let z = 55`  
      `in ((g <z>); z)`

## Examples

- ```
let p = proc (x) (x := 4)
  in let a = 3
    in ((p <a>); a)
```
- ```
let f = proc (x) (x := 44)
 in let g = proc (y) (f <y>)
 in let z = 55
 in ((g <z>); z)
```
- ```
let swap = proc (x) proc (y)
  let temp = x
  in (x := y; y := temp)
in let a = 33
  in let b = 44
    in (((swap <a>) <b>); (a-b))
```

Variable Aliasing

More than one call-by-reference parameter may refer to the same location:

```
let b = 3
in let p = proc (x) proc (y)
      (x := 4; y)
  in ((p <b>) <b>)
```

- A *variable aliasing* is created: x and y refer to the same location
- With aliasing, reasoning about program behavior is very difficult, because an assignment to one variable may change the value of another.

Lazy Evaluation

- So far all the parameter-passing strategies are *eager* in that they always evaluate the actual parameter before calling a procedure.
- In eager evaluation, procedure arguments are completely evaluated before passing them to the procedure.
- On the other hand, *lazy evaluation* delays the evaluation of arguments until it is actually needed. If the procedure body never uses the parameter, it will never be evaluated.

- Lazy evaluation potentially avoids non-termination:

```
letrec infinite(x) = (infinite x)
in let f = proc (x) (1)
    in (f (infinite 0))
```

- Lazy evaluation is popular in functional languages, because lazy evaluation makes it difficult to determine the order of evaluation, which is essential to understanding a program with effects.

Summary

Our language is now (somewhat) realistic:

- expressions, procedures, recursion,
- states with explicit/implicit references
- parameter-passing variations