COSE212: Programming Languages

Lecture 7 — Design and Implementation of PLs (3) Lexical Scoping of Variables

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Goal

Understand lexical scoping in a more systematic way.

- Variable declaration and use
- Scoping rule
- Lexical address
- Nameless representation

References and Declarations

In programming languages, variables appear in two different ways:

- A variable *reference* is a use of the variable.
- A variable *declaration* introduces the variable as a name for some value.
- Examples:

```
(f x y)

proc (x) (x + 3)

let x = y + 7 in x + 3
```

- We say a variable reference is *bound by* the declaration with which it is associated, and that the variable is *bound to* its value.

Scoping Rules

- Every programming language has some rules to determine the corresponding declaration of a variable reference. Called *scoping rules*.
- Most programming languages use *lexical scoping* rules, where the declaration of a reference is found by searching outward from the reference until we find a declaration of the variable:

```
let x = 3                      // call this x1
  in let y = 4
     in (let x = y + 5         // call this x2
          in x * y)            // Here x refers to x2
         + x                   // Here x refers to x1
```

- We can determine the declaration of each variable reference without executing the program.

Static vs. Dynamic Properties of Programs

- Properties of programs are classified into static and dynamic properties.
- Properties that can be computed without executing the program are called *static properties*.
  - ex) declaration, scope, etc
- Properties that cannot be computed without executing the program are called *dynamic properties*. Dynamic properties are only determined at run-time.
  - ex) values, types, the absence of bugs, etc.

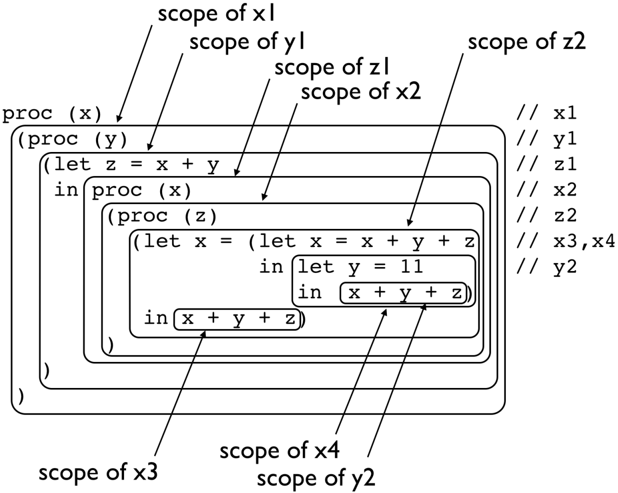
Example: Lexical Scopes of Variables

Declarations have limited *scopes*, each of which lies entirely within another:

```
proc (x)                                      // x1
 (proc (y)                                    // y1
   (let z = x + y                             // z1
    in proc (x)                               // x2
        (proc (z)                             // z2
          (let x = (let x = x + y + z         // x3,x4
                     in let y = 11            // y2
                        in  x + y + z)
           in x + y + z)
        )
   )
 )
```

Example: Lexical Scopes of Variables

Declarations have limited *scopes*, each of which lies entirely within another:



Lexical Address

- Execution of the scoping algorithm can be viewed as a search outward from a variable reference.
- The number of declarations crossed to find the associated declaration is called the *lexical depth* of a variable reference.

```
let x = 1
  in let y = 2
     in x + y
```

- The lexical depth of a variable reference uniquely identifes the declaration to which it refers.
- Therefore, variable names are entirely removed from the program, and variable references are replaced by their *lexical address*:

```
let 1
  in let 2
     in #1 + #0
```

"*Nameless*" or "*De Bruijn*" representation.

Examples: Nameless Representation

- `(let a = 5 in proc (x) (x-a)) 7`
- ```
  (let x = 37
   in proc (y)
       let z = (y - x)
       in (x - y)) 10
  ```
  ```

Lexical Address

- The lexical address of a variable indicates the position of the variable in the environment.
- ```
  let x = 1
   in let y = 2
      in x + y
  ```
- `(let a = 5 in proc (x) (x-a)) 7`
  ```

Nameless Proc

Syntax

$$
\begin{array}{rcl}
P & \rightarrow & E \\
E & \rightarrow & n \\
& | & \#n \\
& | & E + E \\
& | & E - E \\
& | & \texttt{iszero}\ E \\
& | & \texttt{if}\ E\ \texttt{then}\ E\ \texttt{else}\ E \\
& | & \texttt{let}\ E\ \texttt{in}\ E \\
& | & \texttt{proc}\ E \\
& | & E\ E
\end{array}
$$

Nameless Proc

Semantics

$$\begin{array}{rcl} Val & = & \mathbb{Z} + Bool + Procedure \\ Procedure & = & E \times Env \\ Env & = & Val^* \end{array}$$

$$\frac{}{\rho \vdash n \Rightarrow n} \qquad \frac{}{\rho \vdash \#n \Rightarrow \rho_n} \qquad \frac{\rho \vdash E_1 \Rightarrow n_1 \qquad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2}$$

$$\frac{\rho \vdash E \Rightarrow 0}{\rho \vdash \texttt{iszero}\ E \Rightarrow true} \qquad \frac{\rho \vdash E \Rightarrow n}{\rho \vdash \texttt{iszero}\ E \Rightarrow false}\ n \neq 0$$

$$\frac{\rho \vdash E_1 \Rightarrow true \qquad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v} \qquad \frac{\rho \vdash E_1 \Rightarrow false \qquad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad v_1 :: \rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{let}\ E_1\ \texttt{in}\ E_2 \Rightarrow v}$$

$$\frac{}{\rho \vdash \texttt{proc}\ E \Rightarrow (E, \rho)}$$

$$\frac{\rho \vdash E_1 \Rightarrow (E, \rho') \qquad \rho \vdash E_2 \Rightarrow v \qquad v :: \rho' \vdash E \Rightarrow v'}{\rho \vdash E_1\ E_2 \Rightarrow v'}$$

Example

$$\overline{[] \vdash \texttt{(let 37 in proc (let (\#0 -\#1) in (\#2 - \#1))) 10} \Rightarrow \mathbf{27}}$$

Translation

The nameless version of a program P is defined to be **trans**$(E)([])$:

$$
\begin{array}{rcl}
\textbf{trans}(n)(\rho) & = & n \\
\textbf{trans}(x)(\rho) & = & \#n \qquad (n \text{ is the first position of } x \text{ in } \rho) \\
\textbf{trans}(E_1 + E_2)(\rho) & = & \textbf{trans}(E_1)(\rho) + \textbf{trans}(E_2)(\rho) \\
\textbf{trans}(\texttt{iszero} E)(\rho) & = & \texttt{iszero}\ (\textbf{trans}(E)(\rho)) \\
\textbf{trans}(\texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3)(\rho) & = & \texttt{if}\ \textbf{trans}(E_1)(\rho) \\
 & & \texttt{then}\ \textbf{trans}(E_2)(\rho)\ \texttt{else}\ \textbf{trans}(E_3)(\rho) \\
\textbf{trans}(\texttt{let}\ x = E_1\ \texttt{in}\ E_2)(\rho) & = & \texttt{let}\ \textbf{trans}(E_1)(\rho)\ \texttt{in}\ \textbf{trans}(E_2)(x :: \rho) \\
\textbf{trans}(\texttt{proc}(x)\ E)(\rho) & = & \texttt{proc}\ \textbf{trans}(E)(x :: \rho) \\
\textbf{trans}(E_1\ E_2)(\rho) & = & \textbf{trans}(E_1)(\rho)\ \textbf{trans}(E_2)(\rho)
\end{array}
$$

Example

$$\textbf{trans}\left(\begin{array}{l}(\texttt{let x} = 37\\ \;\texttt{in proc (y)}\\ \quad\;\; \texttt{let z} = (\texttt{y} - \texttt{x})\\ \quad\;\; \texttt{in} \ (\texttt{x} - \texttt{y}))\ \texttt{10}\end{array}\right)([]) =$$

Summary

- In lexical scoping, scoping rules are static properties: nameless representation with lexical addresses.
- Lexical address predicts the place of the variable in the environment.
- Compilers routinely use the nameless representation: Given an input program P,
  1. translate it to $\textbf{trans}(P)([])$,
  2. execute the nameless program.