

COSE212: Programming Languages

Lecture 3 — Functional Programming in OCaml

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Why learn ML?

Learning ML is a good way of experiencing modern language features:

- functional programming: scala, java8, haskell, python, JavaScript, etc
- static type system: scala, java, haskell, etc
- automatic type inference: scala, haskell, etc
- pattern matching: scala, etc
- algebraic data types, module system, etc

Basics of the Language

- Expressions
- Names
- Functions
- Pattern matching
- Type inference
- Tuples and lists
- Data types
- Exceptions

Write and run all examples in the slides by yourself!

An OCaml Program is an Expression

Statement and expressions:

- A statement *does something*.
- An expression *evaluates to a value*.

Programming languages can be classified into

- statement-oriented: C, C++, Java, Python, JavaScript, etc
 - ▶ often called “imperative languages”
- expression-oriented: ML, Haskell, Scala, Lisp, etc
 - ▶ often called “functional languages”

Arithmetic Expressions

- Arithmetic expressions evaluate to numbers: e.g., $1+2*3$, $1+5$, 7
- Try to evaluate expressions in the REPL:

```
# 1+2*3;;
```

```
- : int = 7
```

- Arithmetic operators on integers:

$a + b$	addition
$a - b$	subtraction
$a * b$	multiplication
a / b	divide a by b , returning the whole part
$a \bmod b$	divide a by b , returning the remaining part

Boolean Expressions

- Boolean expressions evaluate to boolean values (i.e., true, false).
- Try to evaluate boolean expressions:

```
# true;;
```

```
- : bool = true
```

```
# true;;
```

```
- : bool = true
```

```
# 1 > 2;;
```

```
- : bool = false
```

- Comparison operators produces boolean values:

$a = b$ true if a and b are equal

$a \neq b$ true if a and b are not equal

$a < b$ true if a is less than b

$a \leq b$ true if a is less than or equal to b

$a > b$ true if a is greater than b

$a \geq b$ true if a is greater than or equal to b

Boolean Operators

- Boolean expressions are combined by boolean operators:

```
# true && false;;
```

```
- : bool = false
```

```
# true || false;;
```

```
- : bool = true
```

```
# (2 > 1) && (3 > 2);;
```

```
- : bool = true
```

ML is a Statically Typed Language

If you try to evaluate an expression that does not make sense, OCaml rejects and does not evaluate the program: e.g.,

```
# 1 + true;;
```

```
Error: This expression has type bool but an expression was  
expected of type int
```


cf) Static Types and Dynamic Types

Programming languages are classified into:

- *Statically typed languages*: type checking is done at compile-time.
 - ▶ type errors are detected before program executions
 - ▶ C, C++, Java, ML, Scala, etc
- *Dynamically typed languages*: type checking is done at run-time.
 - ▶ type errors are detected during program executions
 - ▶ Python, JavaScript, Ruby, Lisp, etc

Statically typed languages are further classified into:

- *Type-safe languages* guarantee that compiled programs do not have type errors at run-time.
 - ▶ All type errors are detected at compile time.
 - ▶ Compiled programs do not stuck.
 - ▶ ML, Haskell, Scala
- *Unsafe languages* do not provide such a guarantee.
 - ▶ Some type errors remain at run-time.
 - ▶ C, C++

Which one is better?

Statically typed languages:

- (+) Type errors are caught early in the development cycle.
- (+) Program execution is efficient by omitting runtime checks.
- (−) Less flexible than dynamic languages.

Dynamically typed languages:

- (−) Type errors appear at run-time, often unexpectedly.
- (+) Provide more flexible language features.
- (+) Easy and fast prototyping.

Conversion between Different Types

- In OCaml, different types of values are distinguished:

```
# 3 + 2.0;;
```

```
Error: This expression has type float but an expression
was expected of type int
```

- Types must be explicitly converted:

```
# 3 + int_of_float 2.0;;
```

```
- : int = 5
```

- Operators for floating point numbers:

```
# 1.2 +. 2.3;;
```

```
- : float = 3.5
```

```
# 1.5 *. 2.0;;
```

```
- : float = 3.
```

```
# float_of_int 1 +. 2.2;;
```

```
- : float = 3.2
```

Other Primitive Values

- OCaml provides six primitive values: integers, booleans, floating point numbers, characters, strings, and unit.

```
# 'c';;
```

```
- : char = 'c'
```

```
# "cose212";;
```

```
- : string = "cose212"
```

```
# ();;
```

```
- : unit = ()
```

Conditional Expressions

if be then e_1 else e_2

- If be is true, the value of the conditional expression is the value of e_1 .
- If be is false, the value of the expression is the value of e_2 .
- # if 2 > 1 then 0 else 1;;
- : int = 0
if 2 < 1 then 0 else 1;;
- : int = 1
- be must be a boolean expression.
- types of e_1 and e_2 must be equivalent.
- # if 1 then 1 else 2;;
Error: ...
if true then 1 else true;;
Error: ...
if true then true else false;;
- : bool = true

Names and Functions

- Create a global variable with the `let` keyword:

```
# let x = 3 + 4;;  
val x : int = 7
```

We say a variable x is *bound* to value **7**.

```
# let y = x + x;;  
val y : int = 14
```

- Create a local variable with `let ... in ...` construct:

$$\text{let } x = e_1 \text{ in } e_2$$

- ▶ x is bound to the value of e_1
- ▶ the scope of x is e_2
- ▶ the value of e_2 becomes the value of the entire expression

Examples

- ```
let a = 1 in a;;
- : int = 1
let a = 1 in a * 2;;
- : int = 2
```
- ```
# let a = 1 in  
  let b = a + a in  
  let c = b + b in  
    c + c;;  
- : int = 8
```
- ```
let d =
 let a = 1 in
 let b = a + a in
 let c = b + b in
 c + c;;
val d : int = 8
```

# Functions

- Define a function with `let`:

```
let square x = x * x;;
val square : int -> int = <fun>
```

- Apply the function:

```
square 2;;
- : int = 4
square (2 + 5);;
- : int = 49
square (square 2);;
- : int = 16
```

- The body can be any expression:

```
let neg x = if x < 0 then true else false;;
val neg : int -> bool = <fun>
neg 1;;
- : bool = false
neg (-1);;
- : bool = true
```



# Functions

- Functions with multiple arguments:

```
let sum_of_squares x y = (square x) + (square y);;
val sum_of_squares : int -> int -> int = <fun>
sum_of_squares 3 4;;
- : int = 25
```

- Recursive functions are defined with `let rec` construct:

```
let rec factorial a =
 if a = 1 then 1 else a * factorial (a - 1);;
val factorial : int -> int = <fun>
factorial 5;;
- : int = 120
```

# Nameless Functions

- Many modern programming languages provide nameless functions, e.g., ML, Scala, Java8, JavaScript, Python, etc.
- In OCaml, a function can be defined without names:

```
fun x -> x * x;;
- : int -> int = <fun>
```

Called *nameless* or *anonymous* functions.

- Apply nameless function as usual:

```
(fun x -> x * x) 2;;
- : int = 4
```

- A variable can be bound to functions:

```
let square = fun x -> x * x;;
val square : int -> int = <fun>
```

- The followings are equivalent:

```
let square = fun x -> x * x
let square x = x * x
```

## Example: Square Roots with Newton's Method

We implement the function

```
sqrt : float -> float
```

using Newton's method. To compute  $\text{sqrt}(x)$ ,

- Start with an initial guess  $y$  for the value of the square root of  $x$  (pick  $y = 1$ ).
- Repeatedly improve the estimate by taking the mean of  $y$  and  $x/y$ .

ex)

| Estimation | Quotient            | Mean   |
|------------|---------------------|--------|
| 1          | $2/1 = 2$           | 1.5    |
| 1.5        | $2/1.5 = 1.3333$    | 1.4167 |
| 1.4167     | $2/1.4167 = 1.4118$ | 1.4142 |
| 1.4142     | ...                 | ...    |

## Example: Square Roots with Newton's Method

```
let is_good_enough guess x =
 abs_float (guess *. guess -. x) < 0.001

let improve guess x = (guess +. x /. guess) /. 2.0

let rec sqrt_iter guess x =
 if is_good_enough guess x then guess
 else sqrt_iter (improve guess x) x

let sqrt x = sqrt_iter 1.0 x
```

## Example: Square Roots with Newton's Method

```
#use "sqrt.ml";;
val is_good_enough : float -> float -> bool = <fun>
val improve : float -> float -> float = <fun>
val sqrt_iter : float -> float -> float = <fun>
val sqrt : float -> float = <fun>
sqrt 9.0;;
- : float = 3.00009155413138
sqrt (sqrt 2.0 +. sqrt 3.0);;
- : float = 1.77392790232078923
sqrt 1000.0 *. sqrt 1000.0;;
- : float = 1000.00036992436605
```

## Functions are First-Class in OCaml

In programming languages, a value is *first-class*, if the value can be

- stored in a variable,
- passed as an argument of a function, and
- returned from other functions.

A language is often called *functional*, if functions are first class values, e.g., ML, Scala, Java8, JavaScript, Python, Lisp, etc.

# Functions are First-Class in OCaml

- Functions can be stored in variables:

```
let square = fun x -> x * x;;
square 2;;
- : int = 4
```

- Functions can be passed to other functions:

```
let sum_if_true test first second =
 (if test first then first else 0)
 + (if test second then second else 0);;
val sum_if_true : (int -> bool) -> int -> int -> int = <fun>

let even x = x mod 2 = 0;;
val even : int -> bool = <fun>
sum_if_true even 3 4;;
- : int = 4
sum_if_true even 2 4;;
- : int = 6
```

## Functions are First-Class in OCaml

- Functions can be also returned from a procedure:

```
let plus_a a = fun b -> a + b;;
val plus_a : int -> int -> int = <fun>

let f = plus_a 3;;
val f : int -> int = <fun>
f 1;;
- : int = 4
f 2;;
- : int = 5
```

Functions that manipulate functions are called *higher-order functions*.

- i.e., functions that take as argument functions or return functions
- greatly increase the expressiveness of the language



# Pattern Matching

- An elegant way of doing case analysis.
- E.g., using pattern-matching, the factorial function

```
let rec factorial a =
 if a = 1 then 1 else a * factorial (a - 1)
```

can be written as follows:

```
let factorial a =
 match a with
 | 1 -> 1
 | _ -> a * factorial (a - 1)
```

# Pattern Matching

The nested if-then-else expression

```
let isabc c = if c = 'a' then true
 else if c = 'b' then true
 else if c = 'c' then true
 else false
```

can be written using pattern matching:

```
let isabc c =
 match c with
 'a' -> true
| 'b' -> true
| 'c' -> true
| _ -> false
```

or simply,

```
let isabc c =
 match c with
 'a' | 'b' | 'c' -> true
| _ -> false
```

# Type Inference

In C or Java, types must be annotated:

```
public static int f(int n)
{
 int a = 2;
 return a * n;
}
```

In OCaml, type annotations are not mandatory:

```
let f n =
 let a = 2 in
 a * n;;
val f : int -> int = <fun>
```

# Type Inference

OCaml can infer types, no matter how complex the program is:

```
let sum_if_true test first second =
 (if test first then first else 0)
 + (if test second then second else 0);;
val sum_if_true : (int -> bool) -> int -> int -> int = <fun>
```

OCaml compiler infers the type through the following reasoning steps:

- 1 the types of `first` and `second` must be `int`, because both branches of a conditional expression must have the same type,
- 2 the type of `test` is a function type  $\alpha \rightarrow \beta$ , because `test` is used as a function,
- 3  $\alpha$  must be of `int`, because `test` is applied to `first`, a value of `int`,
- 4  $\beta$  must be of `bool`, because conditions must be boolean expressions,
- 5 the return value of the function has type `int`, because the two conditional expressions are of `int` and their addition gives `int`.

# Type Annotation

Explicit type annotations are possible:

```
let sum_if_true (test : int -> bool) (x : int) (y : int) : int =
 (if test x then x else 0) + (if test y then y else 0);;
val sum_if_true : (int -> bool) -> int -> int -> int = <fun>
```

If the annotation is wrong, OCaml finds the error and report it:

```
let sum_if_true (test : int -> int) (x : int) (y : int) : int =
 (if test x then x else 0) + (if test y then y else 0);;
```

Error: The expression (test x) has type int but an expression  
was expected of type bool

# Polymorphic Types

- What is the type of the program?

```
let id x = x
```

See how OCaml infers its type:

```
let id x = x;;
val id : 'a -> 'a = <fun>
```

The function works for values of any type:

```
id 1;;
- : int = 1
id "abc";;
- : string = "abc"
id true;;
- : bool = true
```

- Such a function is called *polymorphic* and 'a is a *type variable*.

## Polymorphic Types

Quiz) What is the type of the function?

```
let first_if_true test x y =
 if test x then x else y
```

# Tuples

- An ordered collection of values, each of which can be a different types, e.g.,

```
let x = (1, "one");;
```

```
val x : int * string = (1, "one")
```

```
let y = (2, "two", true);;
```

```
val y : int * string * bool = (2, "two", true)
```

- Extract each component using pattern-matching:

```
let fst p = match p with (x,_) -> x;;
```

```
val fst : 'a * 'b -> 'a = <fun>
```

```
let snd p = match p with (_,x) -> x;;
```

```
val snd : 'a * 'b -> 'b = <fun>
```

or equivalently,

```
let fst (x,_) = x;;
```

```
val fst : 'a * 'b -> 'a = <fun>
```

```
let snd (_,x) = x;;
```

```
val snd : 'a * 'b -> 'b = <fun>
```



# Tuples

- Patterns can be used in let:

```
let p = (1, true);;
val p : int * bool = (1, true)
let (x,y) = p;;
val x : int = 1
val y : bool = true
```

# Lists

- A finite sequence of elements, each of which has the same type, e.g.,

[1; 2; 3]

is a list of integers:

```
[1; 2; 3];;
```

```
- : int list = [1; 2; 3]
```

Note that

- ▶ all elements must have the same type, e.g., [1; true; 2] is not a list,
  - ▶ the elements are ordered, e.g., [1; 2; 3]  $\neq$  [2; 3; 1], and
  - ▶ the first element is called *head*, the rest *tail*.
- []: the empty list, i.e., nil. What are head and tail of []?
  - [5]: a list with a single element. What are head and tail of [5]?

## List Examples

- # [1;2;3;4;5];;  
- : int list = [1; 2; 3; 4; 5]
- # ["OCaml"; "Java"; "C"];;  
- : string list = ["OCaml"; "Java"; "C"]
- # [(1,"one"); (2,"two"); (3,"three")];;  
- : (int \* string) list = [(1, "one"); (2, "two"); (3, "three")]
- # [[1;2;3];[2;3;4];[4;5;6]];;  
- : int list list = [[1; 2; 3]; [2; 3; 4]; [4; 5; 6]]
- # [1;"OCaml";3] ;;  
Error: This expression has type string but an expression was expected of type int

## List Operators

- `::` (cons): add a single element to the front of a list, e.g.,

```
1::[2;3];;
```

```
- : int list = [1; 2; 3]
```

```
1::2::3::[];;
```

```
- : int list = [1; 2; 3]
```

(`[1; 2; 3]` is a shorthand for `1::2::3::[]`)

- `@` (append): combine two lists, e.g.,

```
[1; 2] @ [3; 4; 5];;
```

```
- : int list = [1; 2; 3; 4; 5]
```

## Patterns for Lists

Pattern matching is useful for manipulating lists.

- A function to check if a list is empty:

```
let isnil l =
 match l with
 [] -> true
 | _ -> false;;
val isnil : 'a list -> bool = <fun>
isnil [1];;
- : bool = false
isnil [];;
- : bool = true
```

## Patterns for Lists

- A function that computes the length of lists:

```
let rec length l =
 match l with
 [] -> 0
 |h::t -> 1 + length t;;
val length : 'a list -> int = <fun>
length [1;2;3];;
- : int = 3
```

We can replace pattern `h` by `_`:

```
let rec length l =
 match l with
 [] -> 0
 |_::t -> 1 + length t;;
```

# Data Types

- If data elements are finite, just enumerate them, e.g., “days”:

```
type days = Mon | Tue | Wed | Thu | Fri | Sat | Sun;;
type days = Mon | Tue | Wed | Thu | Fri | Sat | Sun
```

Construct values of the type:

```
Mon;;
- : days = Mon
Tue;;
- : days = Tue
```

A function that manipulates the defined data:

```
let nextday d =
 match d with
 | Mon -> Tue | Tue -> Wed | Wed -> Thu | Thu -> Fri
 | Fri -> Sa | Sat -> Sun | Sun -> Mon ;;
val nextday : days -> days = <fun>
nextday Mon;;
- : days = Tue
```

# Data Types

- Constructors can be associated with values, e.g.,

```
type shape = Rect of int * int | Circle of int;;
type shape = Rect of int * int | Circle of int
```

Construct values of the type:

```
Rect (2,3);;
- : shape = Rect (2, 3)
Circle 5;;
- : shape = Circle 5
```

A function that manipulates the data:

```
let area s =
 match s with
 | Rect (w,h) -> w * h
 | Circle r -> r * r * 3;;
val area : shape -> int = <fun>
area (Rect (2,3));;
- : int = 6
area (Circle 5);;
- : int = 75
```



# Data Types

- Inductive data types, e.g.,

```
type mylist = Nil | List of int * mylist;;
type mylist = Nil | List of int * mylist
```

Construct values of the type:

```
Nil;;
- : mylist = Nil
List (1, Nil);;
- : mylist = List (1, Nil)
List (1, List (2, Nil));;
- : mylist = List (1, List (2, Nil))
```

A function that manipulates the data:

```
let rec mylength l =
 match l with
 | Nil -> 0
 | List (_, l') -> 1 + mylength l';;
val mylength : mylist -> int = <fun>
mylength (List (1, List (2, Nil)));;
- : int = 2
```

# Exceptions

- An exception means a run-time error: e.g.,

```
let div a b = a / b;;
val div : int -> int -> int = <fun>
div 10 5;;
- : int = 2
div 10 0;;
Exception: Division_by_zero.
```

- The exception can be handled with `try ... with` constructs.

```
let div a b =
 try
 a / b
 with Division_by_zero -> 0;;
val div : int -> int -> int = <fun>
div 10 5;;
- : int = 2
div 10 0;;
- : int = 0
```

# Exceptions

- User-defined exceptions: e.g.,

```
exception Problem;;
exception Problem
let div a b =
 if b = 0 then raise Problem
 else a / b;;
val div : int -> int -> int = <fun>
div 10 5;;
- : int = 2
div 10 0;;
Exception: Problem.
try
 div 10 0
with Problem -> 0;;
- : int = 0
```

# Summary

We've gone through the basics of OCaml programming:

- Expressions
- Names
- Functions
- Pattern matching
- Type inference
- Tuples and lists
- Data types
- Exceptions