COSE212: Programming Languages

Lecture 7 — Design and Implementation of PLs (3) Scoping and Binding

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References and Declarations

In programming languages, variables appear in two different ways:

- A variable *reference* is a use of the variable.
- A variable *declaration* introduces the variable as a name for some value.
- In well-formed programs, a variable reference is *bound by* some declaration (where the variable is *bound to* its value).
- Examples:

```
proc (x) (x + 3)

let x = y + 7 in x + 3
```

Binding

- Binding: the association between a variable and its value; i.e., an environment is a collection of variable bindings.
- In LETREC, bindings are created in
  - let expressions:

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad [x \mapsto v_1]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{let}\ x = E_1\ \texttt{in}\ E_2 \Rightarrow v}$$

  - letrec expressions:

$$\frac{[f \mapsto (f, x, E_1, \rho)]\rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{letrec}\ f(x) = E_1\ \texttt{in}\ E_2 \Rightarrow v}$$

  - procedure calls:

$$\frac{\rho \vdash E_1 \vdash (x, E, \rho') \qquad \rho \vdash E_2 \Rightarrow v \qquad [x \mapsto v]\rho' \vdash E \Rightarrow v'}{\rho \vdash E_1\ E_2 \Rightarrow v'}$$

Scoping Rules

- How to determine the corresponding declaration of a variable reference? By *scoping rules*.
- Most programming languages use *lexical scoping* rules, where the declaration of a reference is found by searching outward from the reference until we find a declaration of the variable:

```
let x = 3                     // call this x1
  in let y = 4
     in (let x = y + 5        // call this x2
          in x * y)           // Here x refers to x2
         + x                  // Here x refers to x1
```

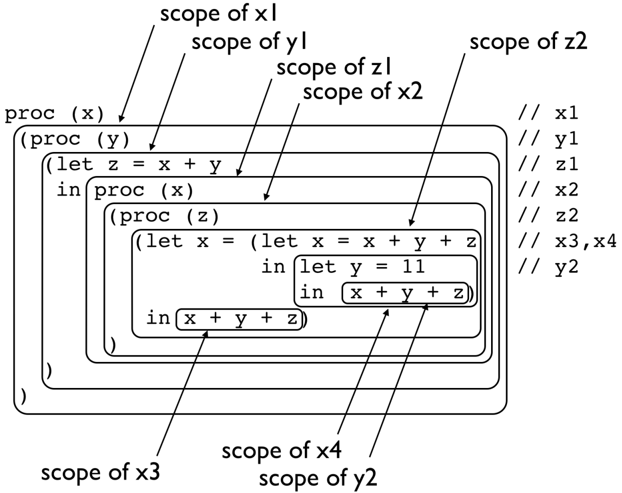
Scopes of Variables

Declarations have limited *scopes*, each of which lies entirely within another:

```
proc (x)                                        // x1
 (proc (y)                                      // y1
   (let z = x + y                               // z1
    in proc (x)                                 // x2
        (proc (z)                               // z2
          (let x = (let x = x + y + z   // x3,x4
                     in let y = 11              // y2
                        in  x + y + z)
           in x + y + z)
        )
   )
 )
```

Scopes of Variables

Declarations have limited *scopes*, each of which lies entirely within another:



Static vs. Dynamic Properties of Programs

- Static properties can be determined at compile-time.
  - ex) declaration, scope, etc
- Dynamic properties are only determined at run-time.
  - ex) values, types, the absence of bugs, etc.

Lexical Address

- *Lexical depth* of a variable reference is the number of declarations crossed to find the associated declaration.

```
let x = 1
  in let y = 2
     in x + y
```

- The lexical depth of a variable reference uniquely identifes the declaration to which it refers.
- Therefore, variable names are entirely removed from the program, and variable references are replaced by their *lexical address*:

```
let 1
  in let 2
     in #1 + #0
```

"*Nameless*" or "*De Bruijn*" representation.

Examples: Nameless Representation

- `(let a = 5 in proc (x) (x-a)) 7`
- ```
  (let x = 37
   in proc (y)
        let z = (y - x)
        in (x - y)) 10
  ```
  ```

Lexical Address

- The lexical address of a variable indicates the position of the variable in the environment.
- ```
  let x = 1
   in let y = 2
      in x + y
  ```
- `(let a = 5 in proc (x) (x-a)) 7`
  ```

Nameless Proc

$\boxed{\text{Syntax}}$

$$
\begin{array}{rcl}
P & \rightarrow & E \\
E & \rightarrow & n \\
& | & \#n \\
& | & E + E \\
& | & E - E \\
& | & \texttt{zero?}\ E \\
& | & \texttt{if}\ E\ \texttt{then}\ E\ \texttt{else}\ E \\
& | & \texttt{let}\ E\ \texttt{in}\ E \\
& | & \texttt{proc}\ E \\
& | & E\ E
\end{array}
$$

Nameless Proc

Semantics

$$\begin{array}{rcl} Val & = & \mathbb{Z} + Bool + Procedure \\ Procedure & = & E \times Env \\ Env & = & Val^* \end{array}$$

$$\frac{}{\rho \vdash n \Rightarrow n} \qquad \frac{}{\rho \vdash \#n \Rightarrow \rho_n} \qquad \frac{\rho \vdash E_1 \Rightarrow n_1 \qquad \rho \vdash E_2 \Rightarrow n_2}{\rho \vdash E_1 + E_2 \Rightarrow n_1 + n_2}$$

$$\frac{\rho \vdash E \Rightarrow 0}{\rho \vdash \texttt{zero?}\ E \Rightarrow true} \qquad \frac{\rho \vdash E \Rightarrow n}{\rho \vdash \texttt{zero?}\ E \Rightarrow false}\ n \neq 0$$

$$\frac{\rho \vdash E_1 \Rightarrow true \qquad \rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v} \qquad \frac{\rho \vdash E_1 \Rightarrow false \qquad \rho \vdash E_3 \Rightarrow v}{\rho \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v}$$

$$\frac{\rho \vdash E_1 \Rightarrow v_1 \qquad v_1 :: \rho \vdash E_2 \Rightarrow v}{\rho \vdash \texttt{let}\ E_1\ \texttt{in}\ E_2 \Rightarrow v}$$

$$\frac{}{\rho \vdash \texttt{proc}\ E \Rightarrow (E, \rho)}$$

$$\frac{\rho \vdash E_1 \vdash (E, \rho') \qquad \rho \vdash E_2 \Rightarrow v \qquad v :: \rho' \vdash E \Rightarrow v'}{\rho \vdash E_1\ E_2 \Rightarrow v'}$$

Example

$$\frac{}{[\,] \vdash \texttt{(let 37 in proc (let (\#0 -\#1) in (\#2 - \#1))) 10} \Rightarrow \mathbf{27}}$$

Translation

The nameless version of a program P is defined to be $\mathbf{T}(E)([])$:

$$
\begin{array}{rcl}
\mathbf{T}(n)(\rho) & = & n \\
\mathbf{T}(x)(\rho) & = & \#n \qquad (n \text{ is the first position of } x \text{ in } \rho) \\
\mathbf{T}(E_1 + E_2)(\rho) & = & \mathbf{T}(E_1)(\rho) + \mathbf{T}(E_2)(\rho) \\
\mathbf{T}(\texttt{zero?}E)(\rho) & = & \texttt{zero?}\ (\mathbf{T}(E)(\rho)) \\
\mathbf{T}(\texttt{if } E_1 \texttt{ then } E_2 \texttt{ else } E_3)(\rho) & = & \texttt{if } \mathbf{T}(E_1)(\rho) \texttt{ then } \mathbf{T}(E_2)(\rho) \texttt{ else } \mathbf{T}(E_3)(\rho) \\
\mathbf{T}(\texttt{let } x = E_1 \texttt{ in } E_2)(\rho) & = & \texttt{let } \mathbf{T}(E_1)(\rho) \texttt{ in } \mathbf{T}(E_2)(x :: \rho) \\
\mathbf{T}(\texttt{proc}(x)\ E)(\rho) & = & \texttt{proc } \mathbf{T}(E)(x :: \rho) \\
\mathbf{T}(E_1\ E_2)(\rho) & = & \mathbf{T}(E_1)(\rho)\ \mathbf{T}(E_2)(\rho)
\end{array}
$$

Example

$$\mathbf{T}\left(\begin{array}{l}(\texttt{let x} = 37\\ \ \texttt{in proc}\ (\texttt{y})\\ \quad\ \texttt{let z} = (\texttt{y} - \texttt{x})\\ \quad\ \texttt{in}\ (\texttt{x} - \texttt{y}))\ \texttt{10}\end{array}\right)([]) =$$

Summary

- In lexical scoping, scoping rules are static properties: nameless representation with lexical addresses.
- Lexical address predicts the place of the variable in the environment.
- Compilers routinely use the nameless representation: Given an input program P,
  1. translate it to $\mathbf{T}(P)([])$,
  2. execute the nameless program.