AAA616: Program Analysis

Lecture 4 — Abstract Interpretation Framework

Hakjoo Oh
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Abstract Interpretation Framework

A powerful framework for designing correct static analysis

- "framework": correct static analysis comes out, reusable
- "powerful": all static analyses are understood in this framework
- "simple": prescription is simple
- "eye-opening": any static analysis is an abstract interpretation


ABSTRACT INTERPRETATION : A UNIFIED LATTICE MODEL FOR STATIC ANALYSIS OF PROGRAMS BY CONSTRUCTION OR APPROXIMATION OF FIXPOINTS

Patrick Cousot and Radhia Cousot

Laboratoire d'Informatique, U.S.M.G., BP. 53 38041 Grenoble cedex, France

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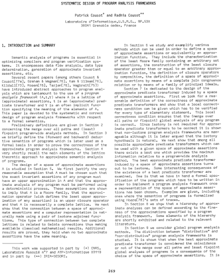
SYSTEMATIC DESIGN OF PROGRAM ANALYSIS FRAMEWORKS

Patrick Cousot and Radhia Cousot

Laboratoire d'Informatique,U.S.M.G., BP.53X 38041 Grenoble cedex, France

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Step 1: Define Concrete Semantics

The concrete semantics describes the real executions of the program.
Described by semantic domain and function.

- A *semantic domain* D, which is a CPO:
  - D is a partially ordered set with a least element $\bot$.
  - Any increasing chain $d_0 \sqsubseteq d_1 \sqsubseteq \ldots$ in D has a least upper bound $\bigsqcup_{n \geq 0} d_n$ in D.
- A *semantic function* $F : D \to D$, which is continuous: for all chains $d_0 \sqsubseteq d_1 \sqsubseteq \ldots$,

$$F(\bigsqcup_{n \geq 0} d_i) = \bigsqcup_{n \geq 0} F(d_n).$$

Then, the concrete semantics (or collecting semantics) is defined as the least fixed point of *semantic function* $F : D \to D$:

$$\mathit{fix} F = \bigsqcup_{i \in N} F^i(\bot).$$

Example: Concrete Semantics

- Program representation:
  - P is represented by control flow graph $(\mathbb{C}, \rightarrow, c_0)$
  - Each program point c is associated with a command $\mathbf{cmd}(c)$

$$
\begin{array}{rcl}
cmd & \rightarrow & skip \mid x := e \\
e & \rightarrow & n \mid x \mid e + e \mid e - e.
\end{array}
$$

- Semantics of commands:
- Concrete memory states: $\mathbb{M} = \textbf{Var} \rightarrow \mathbb{Z}$
- Concrete semantics:

$$
\llbracket c \rrbracket \quad : \quad \mathbb{M} \rightarrow \mathbb{M}
$$

$$
\begin{array}{rcl}
\llbracket skip \rrbracket(m) & = & m \\
\llbracket x := e \rrbracket(m) & = & m[x \mapsto \llbracket e \rrbracket(s)]
\end{array}
$$

$$
\llbracket e \rrbracket \quad : \quad \mathbb{M} \rightarrow \mathbb{Z}
$$

$$
\begin{array}{rcl}
\llbracket n \rrbracket(m) & = & n \\
\llbracket x \rrbracket(m) & = & m(x) \\
\llbracket e_1 + e_2 \rrbracket(m) & = & \llbracket e_1 \rrbracket(m) + \llbracket e_2 \rrbracket(m) \\
\llbracket e_1 - e_2 \rrbracket(m) & = & \llbracket e_1 \rrbracket(m) + \llbracket e_2 \rrbracket(m)
\end{array}
$$

Example: Concrete Semantics

- Program states: $\textbf{State} = \mathbb{C} \times \mathbb{M}$
- A trace $\sigma \in \textbf{State}^+$ is a (partial) execution sequence of the program:

$$\sigma_0 \in I \ \wedge \ \forall k. \sigma_k \rightsquigarrow \sigma_{k+1}$$

where $I \subseteq \textbf{State}$ is the initial program states

$$I = \{(c_0, m_0) \mid m_0 \in \mathbb{M}\}$$

and $(\rightsquigarrow) \subseteq \textbf{State} \times \textbf{State}$ is the relation for the one-step execution:

$$(c_i, s_i) \rightsquigarrow (c_j, s_j) \iff c_i \to c_j \ \wedge \ s_j = [\![\textbf{cmd}(c_j)]\!](s_i)$$

Example: Concrete Semantics

The collecting semantics of program P is defined as the set of all finite traces of the program:

$$[\![P]\!] = \{\sigma \in \mathbf{State}^{+} \mid \sigma_0 \in I \;\wedge\; \forall k.\sigma_k \rightsquigarrow \sigma_{k+1}\}$$

The semantic domain:

$$D = \wp(\mathbf{State}^{+})$$

The semantic function:

$$\begin{array}{rcl} F & : & \wp(\mathbf{State}^{+}) \to \wp(\mathbf{State}^{+}) \\ F(\Sigma) & = & I \cup \{\sigma \cdot (c, m) \mid \sigma \in \Sigma \;\wedge\; \sigma_{\dashv} \rightsquigarrow (c, m)\} \end{array}$$

Lemma

$[\![P]\!] = \mathit{fix} F$.

Step 2: Define Abstract Semantics

Define the abstract semantics of the input program.

- Define an *abstract semantic domain* CPO $\hat{D}$.
  - Intuition: $\hat{D}$ is an abstraction of D
- Define an *abstract semantic function* $\hat{F} : \hat{D} \to \hat{D}$.
  - Intuition: $\hat{F}$ is an abstraction of F.
  - $\hat{F}$ must be monotone:

$$\forall \hat{x}, \hat{y} \in \hat{D}.\ \hat{x} \sqsubseteq \hat{y} \implies \hat{F}(\hat{x}) \sqsubseteq \hat{F}(\hat{y})$$

(or extensive: $\forall x \in \hat{D}.\ x \sqsubseteq \hat{F}(x)$)

Then, static analysis is to compute an upper bound of:

$$\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\bot)$$

How can we ensure that the result soundly approximate the concrete semantics?

Requirement 1: Galois Connection

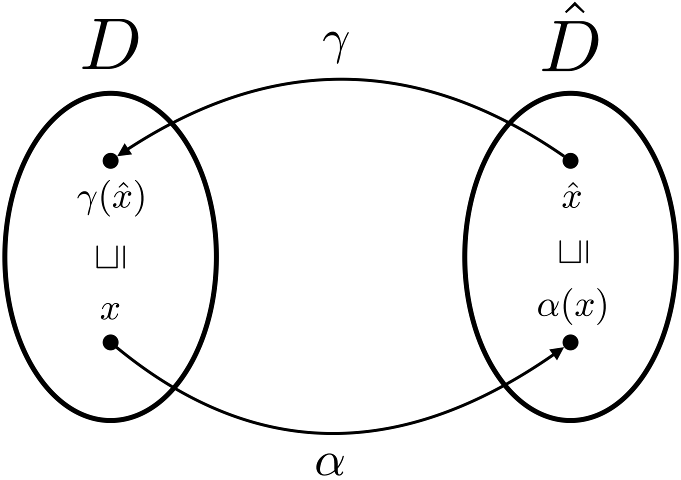
D and $\hat{D}$ must be related with Galois-connection:

$$D \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \hat{D}$$

That is, we have

- *abstraction function*: $\alpha \in D \to \hat{D}$
  - represents elements in D as elements of $\hat{D}$
- *concretization function*: $\gamma \in \hat{D} \to D$
  - gives the meaning of elements of $\hat{D}$ in terms of D
- $\forall x \in D, \hat{x} \in \hat{D}.\ \alpha(x) \sqsubseteq \hat{x} \iff x \sqsubseteq \gamma(\hat{x})$
  - α and γ respect the orderings of D and $\hat{D}$

Galois-Connection



Example: Sign Abstraction

Sign abstraction:

$$\wp(\mathbb{Z}) \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \{\bot, +, 0, -\top\}$$

where

$$\begin{array}{rcl}
\alpha(Z) & = & \begin{cases} \bot & Z = \emptyset \\ + & \forall z \in Z.\ z > 0 \\ 0 & Z = \{0\} \\ - & \forall z \in Z.\ z < 0 \\ \top & \text{otherwise} \end{cases} \\
\gamma(\bot) & = & \emptyset \\
\gamma(\top) & = & \mathbb{Z} \\
\gamma(+) & = & \{z \in \mathbb{Z} \mid z > 0\} \\
\gamma(0) & = & \{0\} \\
\gamma(-) & = & \{z \in \mathbb{Z} \mid z < 0\}
\end{array}$$

Example: Interval Abstraction

$$\wp(\mathbb{Z}) \xleftrightarrows[\alpha]{\gamma} \{\perp\} \cup \{[a,b] \mid a \in \mathbb{Z} \cup \{-\infty\}, b \in \mathbb{Z} \cup \{+\infty\}\}$$

$$\begin{array}{rcl}
\gamma(\perp) & = & \emptyset \\
\gamma([a,b]) & = & \{z \in \mathbb{Z} \mid a \le z \le b\} \\
\gamma([a,+\infty]) & = & \{z \in \mathbb{Z} \mid z \ge a\} \\
\gamma([-\infty,b]) & = & \{z \in \mathbb{Z} \mid z \le b\} \\
\gamma([-\infty,+\infty]) & = & \mathbb{Z}
\end{array}$$

Requirement 2: $\hat{F}$ and F

- $\hat{F}$ and F must satisfy

$$\alpha \circ F \sqsubseteq \hat{F} \circ \alpha \quad (\text{i.e., } F \circ \gamma \sqsubseteq \gamma \circ \hat{F})$$

- or, alternatively,

$$\alpha(x) \sqsubseteq \hat{x} \implies \alpha(F(x)) \sqsubseteq \hat{F}(\hat{x})$$

Soundness Guarantee

Theorem (Fixpoint Transfer)

Let D *and* $\hat{D}$ *be related by Galois-connection* $D \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \hat{D}$. *Let* $F : D \to D$ *be a continuous function and* $\hat{F} : \hat{D} \to \hat{D}$ *be a monotone function such that* $\alpha \circ F \sqsubseteq \hat{F} \circ \alpha$. *Then,*

$$\alpha(\mathit{fix} F) \sqsubseteq \bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot}).$$

Theorem (Fixpoint Transfer2)

Let D *and* $\hat{D}$ *be related by Galois-connection* $D \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} \hat{D}$. *Let* $F : D \to D$ *be a continuous function and* $\hat{F} : \hat{D} \to \hat{D}$ *be a monotone function such that* $\alpha(x) \sqsubseteq \hat{x} \implies \alpha(F(x)) \sqsubseteq \hat{F}(\hat{x})$. *Then,*

$$\alpha(\mathit{fix} F) \sqsubseteq \bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot}).$$

A Property of Galois-Connection

The functional composition of two Galois-connections is also Galois-connection:

Lemma

If $D_1 \underset{\alpha_1}{\overset{\gamma_1}{\leftrightarrows}} D_2$ *and* $D_2 \underset{\alpha_2}{\overset{\gamma_2}{\leftrightarrows}} D_3$, *then*

$$D_1 \underset{\alpha_2 \circ \alpha_1}{\overset{\gamma_1 \circ \gamma_2}{\leftrightarrows}} D_3.$$

Proof.

Exercise □

Example: Partitioning Abstraction

Galois-connection: $\wp(\textbf{State}^+) \underset{\alpha_1}{\overset{\gamma_1}{\leftrightarrows}} \mathbb{C} \to \wp(\mathbb{M})$

$$\alpha_1(\Sigma) \;=\; \lambda c.\{m \in \mathbb{M} \mid \exists \sigma \in \Sigma \;\wedge\; \exists i.\sigma_i = (c, m)\}$$

Semantic function:

$$\hat{F}_1 : (\mathbb{C} \to \wp(\mathbb{M})) \to (\mathbb{C} \to \wp(\mathbb{M}))$$

$$\hat{F}_1(X) = \alpha_1(I) \sqcup \lambda c \in \mathbb{C}.\; f_c(\bigcup_{c' \to c} X(c'))$$

where $f_c : \wp(\mathbb{M}) \to \wp(\mathbb{M})$ is a *transfer function* at program point c:

$$f_c(M) \;=\; \{m' \mid m \in M \;\wedge\; m' = [\![\textbf{cmd}(c)]\!](m)\}$$

Lemma (Soundness of Partitioning Abstraction)

$\alpha_1(\mathit{fix}\,F) \sqsubseteq \bigsqcup_{i \in \mathbb{N}} \hat{F}_1^i(\bot)$.

Example: Memory State Abstraction

Galois-connection:

$$\mathbb{C} \to \wp(\mathbb{M}) \underset{\alpha_2}{\overset{\gamma_2}{\leftrightarrows}} \mathbb{C} \to \hat{\mathbb{M}}$$

$$\begin{array}{rcl} \alpha_2(f) & = & \lambda c.\ \alpha_m(f(c)) \\ \gamma_1(\hat{f}) & = & \lambda c.\ \gamma_m(\hat{f}(c)) \end{array}$$

where we assume

$$\wp(\mathbb{M}) \underset{\alpha_m}{\overset{\gamma_m}{\leftrightarrows}} \hat{\mathbb{M}}$$

Semantic function $\hat{F} : (\mathbb{C} \to \hat{\mathbb{M}}) \to (\mathbb{C} \to \hat{\mathbb{M}})$:

$$\hat{F}(X) = (\alpha_2 \circ \alpha_1)(I) \sqcup \lambda c \in \mathbb{C}.\ \hat{f}_c(\bigsqcup_{c' \to c} X(c'))$$

where *abstract transfer function* $\hat{f}_c : \hat{\mathbb{M}} \to \hat{\mathbb{M}}$ is given such that

$$\alpha_m \circ f_c \sqsubseteq \hat{f}_c \circ \alpha_m \qquad (1)$$

Theorem (Soundness)

$\alpha(\mathit{fix} F) \sqsubseteq \bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\bot)$ *where* $\alpha = \alpha_2 \circ \alpha_1$.

Example: Sign Analysis

Memory state abstraction:

$$\wp(\mathbb{M}) \xleftarrow[\alpha_m]{\gamma_m} \hat{\mathbb{M}}$$

$$\alpha_m(M) \quad = \quad \lambda x \in \textbf{Var}.\ \alpha_s(\{m(x) \mid m \in M\})$$

where α_s is the sign abstraction:

$$\wp(\mathbb{Z}) \xleftarrow[\alpha_s]{\gamma_s} \hat{\mathbb{Z}}$$

The transfer function $\hat{f}_c : \hat{\mathbb{M}} \to \hat{\mathbb{M}}$:

$$\begin{array}{rcll} \hat{f}_c(\hat{m}) & = & \hat{m} & c = skip \\ \hat{f}_c(\hat{m}) & = & \hat{m}[x \mapsto \hat{\mathcal{V}}(e)(\hat{m})] & c = x := e \end{array}$$

$$\begin{array}{rcl} \hat{\mathcal{V}}(n)(\hat{m}) & = & \alpha_s(\{n\}) \\ \hat{\mathcal{V}}(x)(\hat{m}) & = & \hat{m}(x) \\ \hat{\mathcal{V}}(e_1 + e_2) & = & \hat{\mathcal{V}}(e_1)(\hat{m}) \mathbin{\hat{+}} \hat{\mathcal{V}}(e_2)(\hat{m}) \\ \hat{\mathcal{V}}(e_1 - e_2) & = & \hat{\mathcal{V}}(e_1)(\hat{m}) \mathbin{\hat{-}} \hat{\mathcal{V}}(e_2)(\hat{m}) \end{array}$$

Lemma

$\alpha_m \circ f_c \sqsubseteq \hat{f}_c \circ \alpha_m$

Example: Interval Analysis

Memory state abstraction:

$$\alpha_m(M) \quad = \quad \lambda x \in \textbf{Var}.\ \alpha_n(\{m(x) \mid m \in M\})$$

where α_n is the interval abstraction:

$$\wp(\mathbb{Z}) \underset{\alpha_n}{\overset{\gamma_n}{\leftrightarrows}} \hat{\mathbb{Z}}$$

$$\hat{\mathbb{Z}} = \{\bot\} \cup \{[l, u] \mid l, u \in \mathbb{Z} \cup \{-\infty, +\infty\} \wedge l \leq u\}$$

The transfer function $\hat{f}_c : \hat{\mathbb{M}} \to \hat{\mathbb{M}}$:

$$\begin{array}{rcll} \hat{f}_c(\hat{m}) & = & \hat{m} & c = skip \\ \hat{f}_c(\hat{m}) & = & \hat{m}[x \mapsto \hat{\mathcal{V}}(e)(\hat{m})] & c = x := e \end{array}$$

$$\begin{array}{rcl} \hat{\mathcal{V}}(n)(\hat{m}) & = & \alpha_s(\{n\}) \\ \hat{\mathcal{V}}(x)(\hat{m}) & = & \hat{m}(x) \\ \hat{\mathcal{V}}(e_1 + e_2) & = & \hat{\mathcal{V}}(e_1)(\hat{m}) \mathbin{\hat{+}} \hat{\mathcal{V}}(e_2)(\hat{m}) \\ \hat{\mathcal{V}}(e_1 - e_2) & = & \hat{\mathcal{V}}(e_1)(\hat{m}) \mathbin{\hat{-}} \hat{\mathcal{V}}(e_2)(\hat{m}) \end{array}$$

Lemma

$\alpha_m \circ f_c \sqsubseteq \hat{f}_c \circ \alpha_m$

Computing an upper bound of $\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot})$

- If the abstract domain $\hat{D}$ has finite height (i.e., all chains are finite), we can directly calculate

$$\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot}).$$

- If the domain $\hat{D}$ has infinite height, the computation may not terminate. In this case, we find a finite chain $\hat{X}_0 \sqsubseteq \hat{X}_1 \sqsubseteq \hat{X}_2 \sqsubseteq \ldots$ such that

$$\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot}) \sqsubseteq \lim_{i \in \mathbb{N}} \hat{X}_i$$

Finite Chain $\hat{X}_i$

Define finite chain $\hat{X}_i$ by an widening operator $\triangledown : \hat{D} \times \hat{D} \to \hat{D}$:

$$\begin{array}{rcll} \hat{X}_0 & = & \bot & \\ \hat{X}_i & = & \hat{X}_{i-1} & \text{if } \hat{F}(\hat{X}_{i-1}) \sqsubseteq \hat{X}_{i-1} \\ & = & \hat{X}_{i-1} \triangledown \hat{F}(\hat{X}_{i-1}) & \text{otherwise} \end{array} \tag{2}$$

Conditions on $\triangledown$:

- $\forall a, b \in \hat{D}.\ (a \sqsubseteq a \triangledown b)\ \wedge\ (b \sqsubseteq a \triangledown b)$
- For all increasing chains $(x_i)_i$, the increasing chain $(y_i)_i$ defined as

$$y_i = \begin{cases} x_0 & \text{if } i = 0 \\ y_{i-1} \triangledown x_i & \text{if } i > 0 \end{cases}$$

  eventually stabilizes (i.e., the chain is finite).

Then, the limit of the chain is safe analysis result.

Theorem (Widening's Safety)

Let $\hat{D}$ be a CPO, $\hat{F} : \hat{D} \to \hat{D}$ a monotone function, $\triangledown : \hat{D} \times \hat{D} \to \hat{D}$ a widening operator. Then, chain $(\hat{X}_i)_i$ defined as (2) eventually stabilizes and

$$\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot}) \sqsubseteq \lim_{i \in \mathbb{N}} \hat{X}_i.$$

Narrowing

- We can refine the widening result $\mathbf{lim}_{i\in\mathbb{N}}\, \hat{X}_i$ by a narrowing operator $\triangle : \hat{D} \times \hat{D} \to \hat{D}$.
- Compute chain $(\hat{Y}_i)_i$

$$\hat{Y}_i = \begin{cases} \mathbf{lim}_{i\in\mathbb{N}}\, \hat{X}_i & \text{if } i = 0 \\ \hat{Y}_{i-1} \triangle \hat{F}(\hat{Y}_{i-1}) & \text{if } i > 0 \end{cases} \tag{3}$$

- Conditions on $\triangle$
  - $\forall a, b \in \hat{D}.\ a \sqsubseteq b \implies a \sqsubseteq a \triangle b \sqsubseteq b$
  - For all decreasing chain $(x_i)_i$, the decreasing chain $(y_i)_i$ defined as

$$y_i = \begin{cases} x_i & \text{if i} = 0 \\ y_{i-1} \triangle x_i & \text{if } i > 0 \end{cases}$$

    eventually stabilizes.

Theorem (Narrowing's Safety)

Let $\hat{D}$ be a CPO, $\hat{F} : \hat{D} \to \hat{D}$ a monotone function, $\triangle : \hat{D} \times \hat{D} \to \hat{D}$ a narrowing operator. Then, chain $(\hat{Y}_i)_i$ defined as (3) eventually stabilizes and

$$\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot}) \sqsubseteq \lim_{i \in \mathbb{N}} \hat{Y}_i.$$

Widening/Narrowing Example

```
i = 0;
while (i<10)
  i++;
```

- Abstract equation:

$$\begin{array}{rcl} X_1 & = & [0,0] \\ X_2 & = & (X_1 \sqcup X_3] \sqcap [-\infty, 9] \\ X_3 & = & X_2 \hat{+} [1,1] \\ X_4 & = & (X_1 \sqcup X_3) \sqcap [10, +\infty] \end{array}$$

- Abstract domain $\hat{D} =$ Interval $\times$ Interval $\times$ Interval $\times$ Interval
- Semantic function $\hat{F} : \hat{D} \to \hat{D}$ such that

$$(X_1, X_2, X_3, X_4) = \hat{F}(X_1, X_2, X_3, X_4)$$

Widening/Narrowing Example

$$\begin{array}{rcl} X_1 & = & [0,0] \\ X_2 & = & (X_1 \sqcup X_3] \sqcap [-\infty, 9] \\ X_3 & = & X_2 \hat{+} [1,1] \\ X_4 & = & (X_1 \sqcup X_3) \sqcap [10, +\infty] \end{array}$$

$\bigsqcup_{i \in \mathbb{N}} \hat{F}^i(\hat{\bot})$:

	0	1	2	3	4	5	6	...	
X_1	$\hat{\bot}$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$		$[0,0]$
X_2	$\hat{\bot}$	$\hat{\bot}$	$[0,0]$	$[0,0]$	$[0,1]$	$[0,1]$	$[0,2]$		$[0,9]$
X_3	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$[1,1]$	$[1,1]$	$[1,2]$	$[1,2]$		$[1,10]$
X_4	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$		$[10,10]$

Widening/Narrowing Example

A simple widening operator for the Interval domain:

$$\begin{array}{rcll} [a,b] & \bigtriangledown & \bot & = [a,b] \\ \bot & \bigtriangledown & [c,d] & = [c,d] \\ [a,b] & \bigtriangledown & [c,d] & = [(c < a ? -\infty : a), (b < d ? +\infty : b)] \end{array}$$

A simple narrowing operator:

$$\begin{array}{rcll} [a,b] & \bigtriangleup & \bot & = \bot \\ \bot & \bigtriangleup & [c,d] & = \bot \\ [a,b] & \bigtriangleup & [c,d] & = [(a = -\infty ? c : a), (b = +\infty ? d : b)] \end{array}$$

Widening/Narrowing Example

Widening iteration:

	0	1	2	3	4	5	6	7
X_1	$\hat{\bot}$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$
X_2	$\hat{\bot}$	$\hat{\bot}$	$[0,0]$	$[0,0]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$	$[0,+\infty]$
X_3	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$[1,1]$	$[1,1]$	$[1,+\infty]$	$[1,+\infty]$	$[1,+\infty]$
X_4	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$\hat{\bot}$	$[10,+\infty]$	$[10,+\infty]$

Narrowing iteration:

	0	1	2	3	4
X_1	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$	$[0,0]$
X_2	$[0,+\infty]$	$[0,9]$	$[0,9]$	$[0,9]$	$[0,9]$
X_3	$[1,+\infty]$	$[1,+\infty]$	$[1,10]$	$[1,10]$	$[1,10]$
X_4	$[10,+\infty]$	$[10,+\infty]$	$[10,+\infty]$	$[10,10]$	$[10,10]$

Worklist Algorithm

$$
\begin{array}{l}
W \in \mathit{Worklist} = \wp(\mathbb{C}) \\
T \in \mathbb{C} \to \hat{\mathbb{S}} \\
\hat{f}_c \in \hat{\mathbb{S}} \to \hat{\mathbb{S}} \\
\\
W := \mathbb{C} \\
T := \lambda c.\bot \\
\textbf{repeat} \\
\quad c := \text{choose}(W) \\
\quad W := W - \{c\} \\
\quad \hat{s}_{in} := \bigsqcup_{c' \to c} \hat{f}_{c'}(T(c')) \\
\quad \textbf{if } \hat{s}_{in} \not\sqsubseteq \hat{X}(c) \\
\qquad \textbf{if } c \text{ is a head of a flow cycle} \\
\qquad\quad \hat{s}_{in} := T(c) \bigtriangledown \hat{s}_{in} \\
\qquad \hat{X}(c) := \hat{s}_{in} \\
\qquad W := W \cup \{c' \mid c \to c'\} \\
\textbf{until } W = \emptyset
\end{array}
$$

Example