Homework 2
AAA 616: Program Analysis, Fall 2016

Hakjoo Oh

Due: 10/11 (in class)

Problem 1 Assume that $(D_1, \sqsubseteq_1)$ and $(D_2, \sqsubseteq_2)$ are CPOs, and assume that the funtion $f : D_1 \to D_2$ preserves the least upper bounds, i.e.,

$$\bigsqcup f(Y) = f(\bigsqcup Y),$$

for all non-empty chains Y of D_1. Prove that f is monotone.

Problem 2 Consider the CPO of 'vertical natural numbers', denoted $(\Omega, \sqsubseteq)$, where

$$\Omega = \mathbb{N} \cup \{\omega\}, \qquad d \sqsubseteq d' \text{ iff } \begin{cases} & d, d' \in \mathbb{N} \ \wedge \ d \leq d' \\ \text{or} & d \in \mathbb{N} \ \wedge \ d' = \omega \\ \text{or} & d = d' = \omega \end{cases}$$

and $\bigsqcup Y$ for a chain Y is given by

$$\bigsqcup Y = \begin{cases} X_0 & \cdots Y \text{ finite} \\ \omega & \cdots \textit{otherwise} \end{cases}$$

1. Prove that the function $f : \Omega \to \Omega$ defined by

   $$f(x) = \begin{cases} 0 & x \in \mathbb{N} \\ \omega & \text{otherwise} \end{cases}$$

   is monotone but not continuous.

2. Prove that the function $f : \Omega \to \Omega$ defined by

   $$f(x) = \begin{cases} x & x \in \mathbb{N} \\ \omega & \text{otherwise} \end{cases}$$

   is continuous.

Problem 3 Design a constant propagation analysis for the while language, which statically predicts whether arithmetic and boolean expressions always produce constant values. In this analysis, a set of integers is abstracted to an element from the complete lattice $(\widehat{\mathsf{Z}}, \sqsubseteq)$ such that

$$\widehat{\mathsf{Z}} = \{\top, \bot\} \cup \mathsf{Z}$$

and $c_1 \sqsubseteq c_2$ iff $c_1 = \bot$ or $c_2 = \top$. The abstract states are defined as follows:

$$\widehat{\mathsf{State}} = \mathsf{Var} \to \widehat{\mathsf{Z}}.$$

Let $(\widehat{\mathsf{T}}, \sqsubseteq)$ be the abstract domain (complete lattice) for truth values (see lecture slides). Define the abstract semantics for the constant propagation analysis:

$$\begin{array}{rcl} \widehat{\mathcal{A}}[\![a]\!] & : & \widehat{\mathsf{State}} \to \widehat{\mathsf{Z}} \\ \widehat{\mathcal{B}}[\![b]\!] & : & \widehat{\mathsf{State}} \to \widehat{\mathsf{T}} \\ \widehat{\mathcal{C}}[\![c]\!] & : & \widehat{\mathsf{State}} \to \widehat{\mathsf{State}} \end{array}$$