

Computer-Aided Reasoning for Software

# **Practical Applications of SAT**

[courses.cs.washington.edu/courses/cse507/18sp/](https://courses.cs.washington.edu/courses/cse507/18sp/)

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# Today

## Past 2 lectures

- The theory and mechanics of SAT solving

## Today

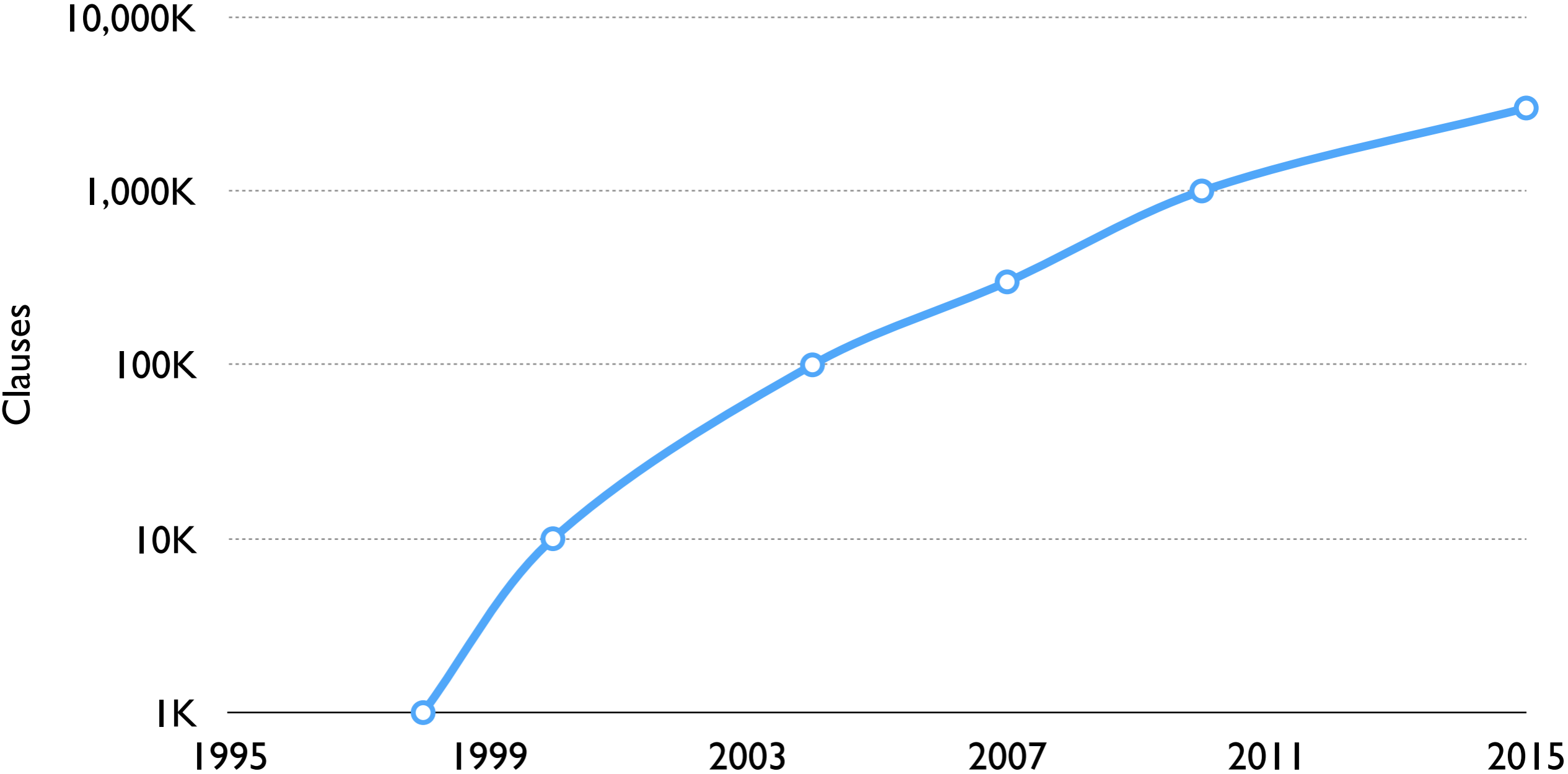
- Practical applications of SAT
- Variants of the SAT problem
- Motivating the next lecture on SMT

## But first ...

- A brief Q&A session for Homework I

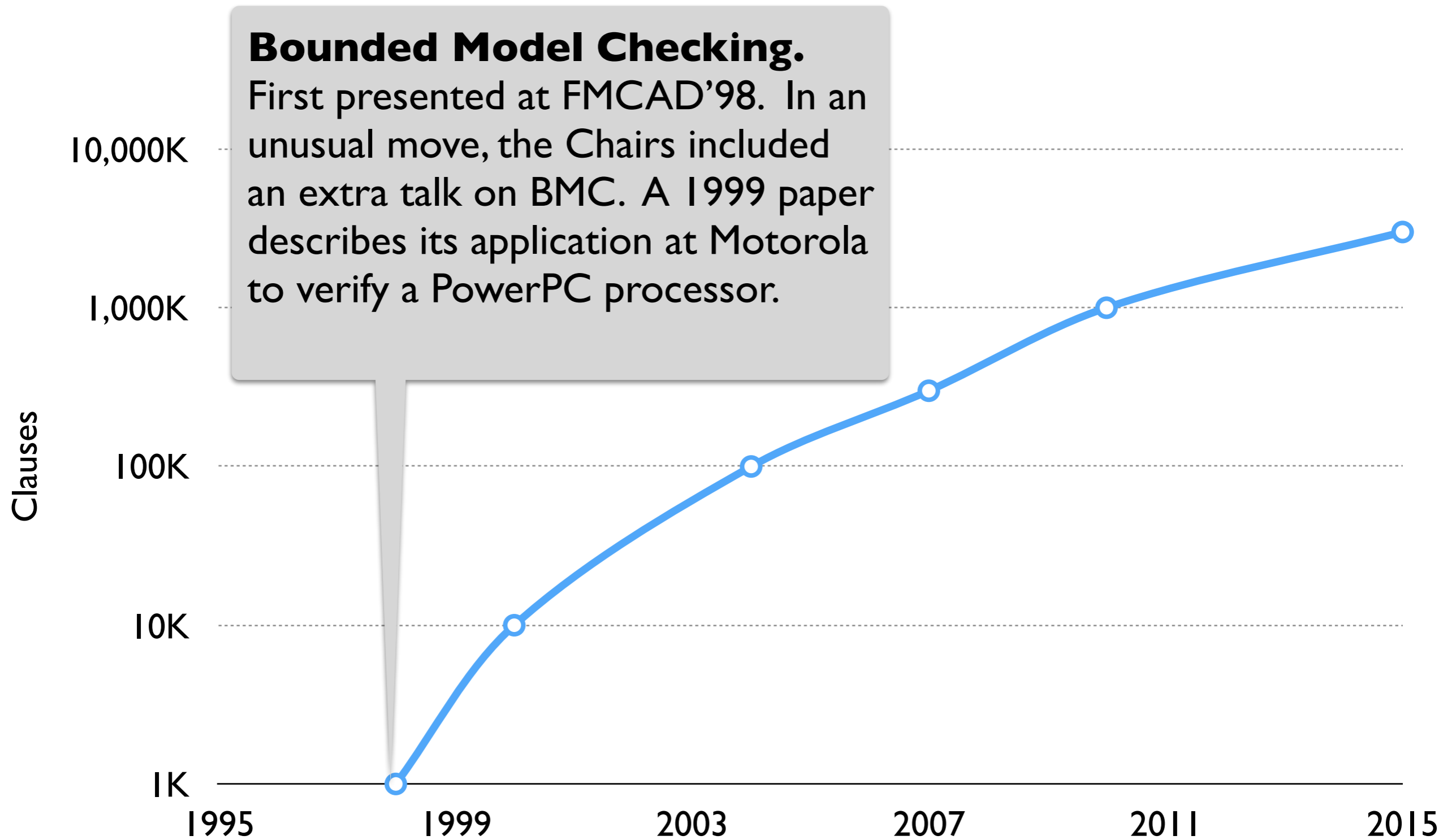


# A brief history of SAT solving and applications

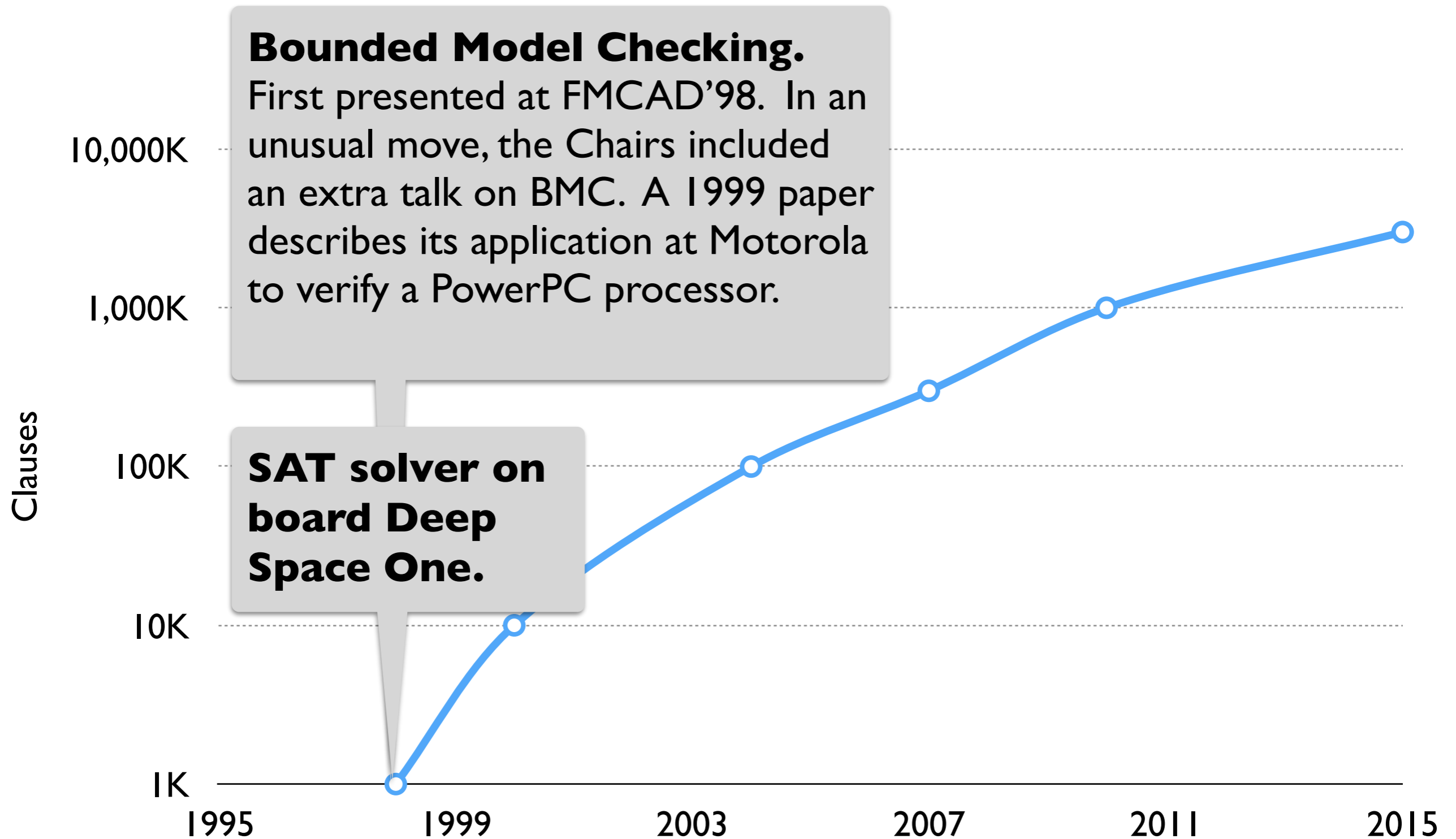


Based on a slide from Vijay Ganesh

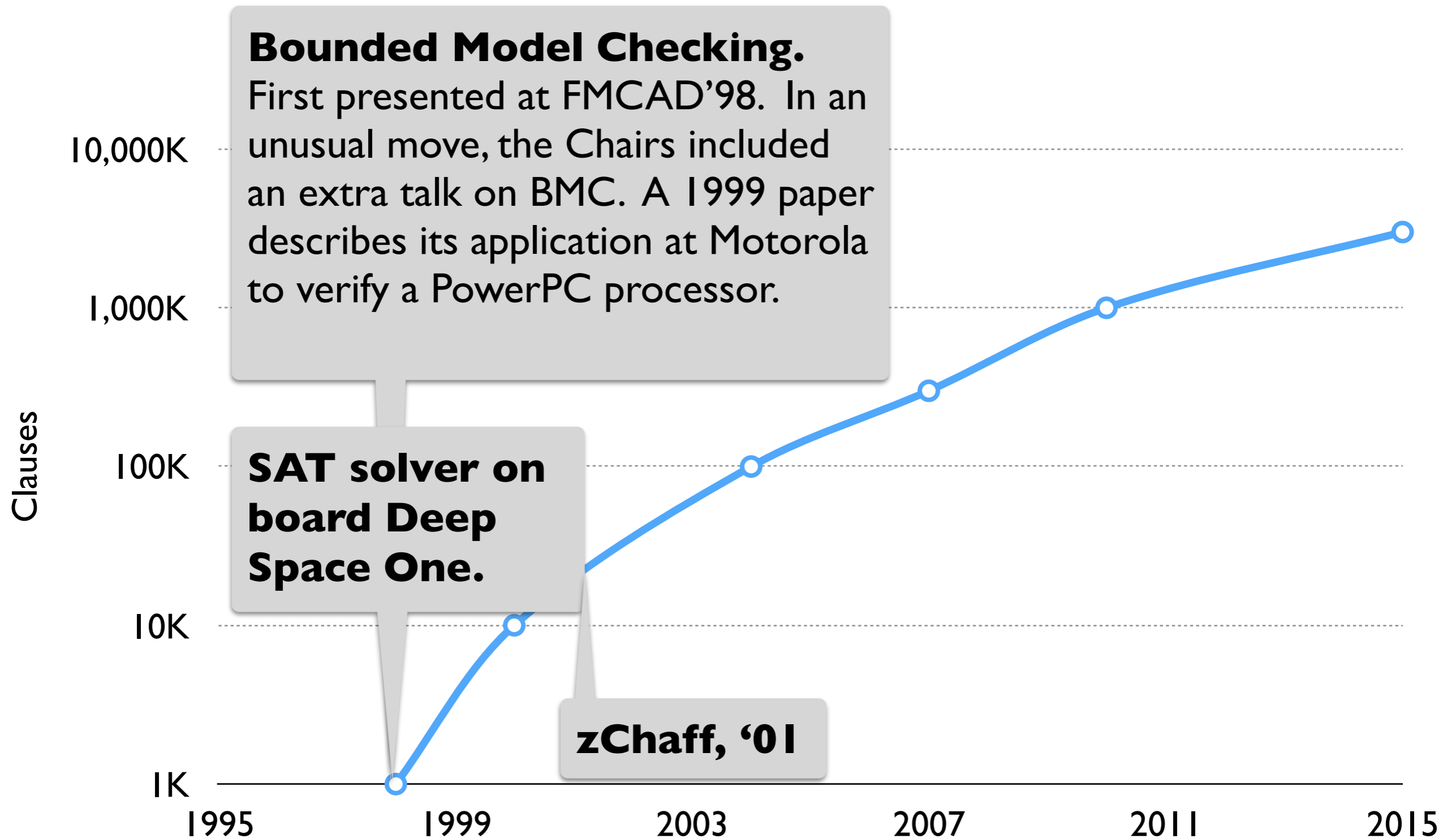
# A brief history of SAT solving and applications



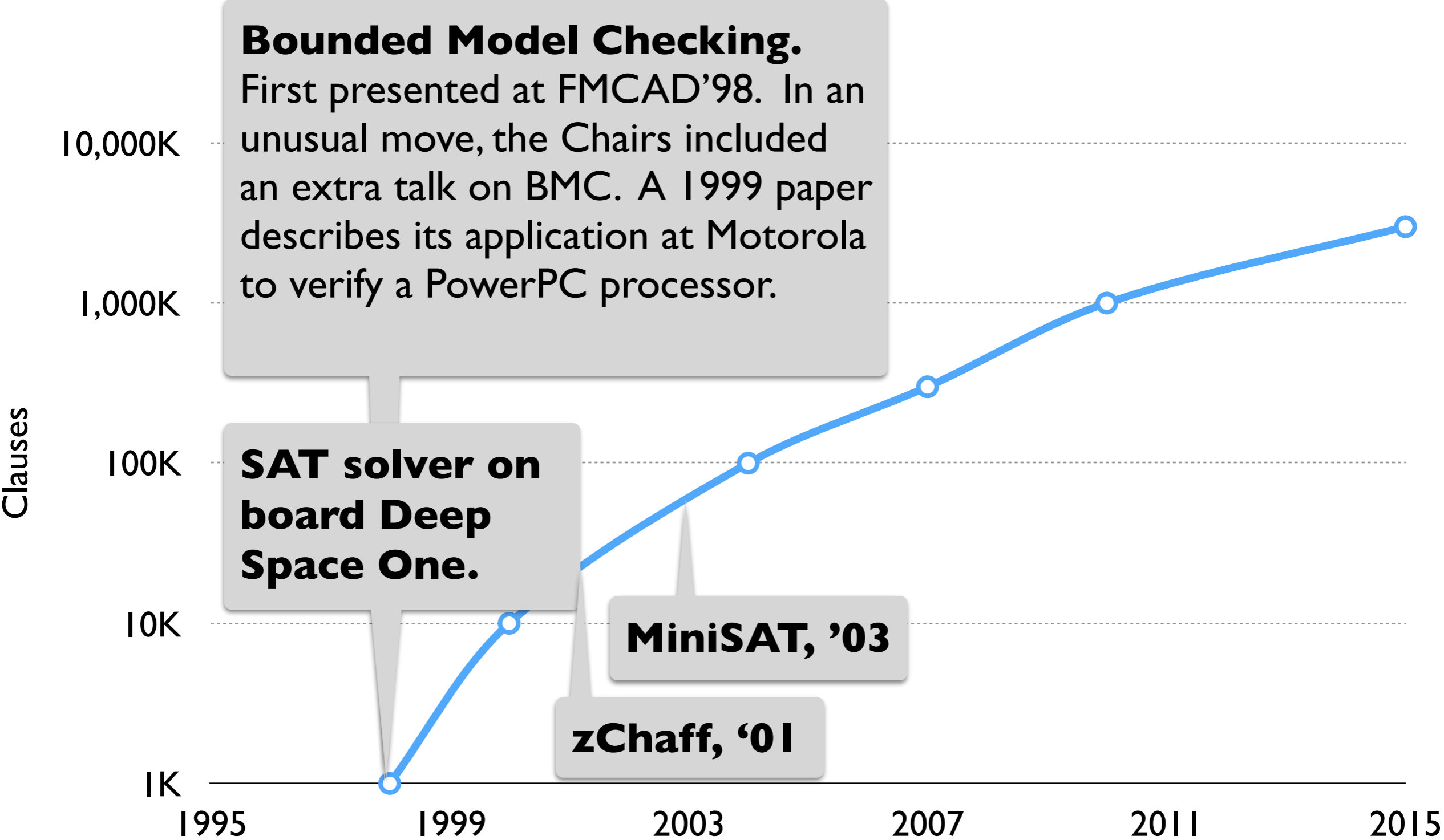
# A brief history of SAT solving and applications



# A brief history of SAT solving and applications

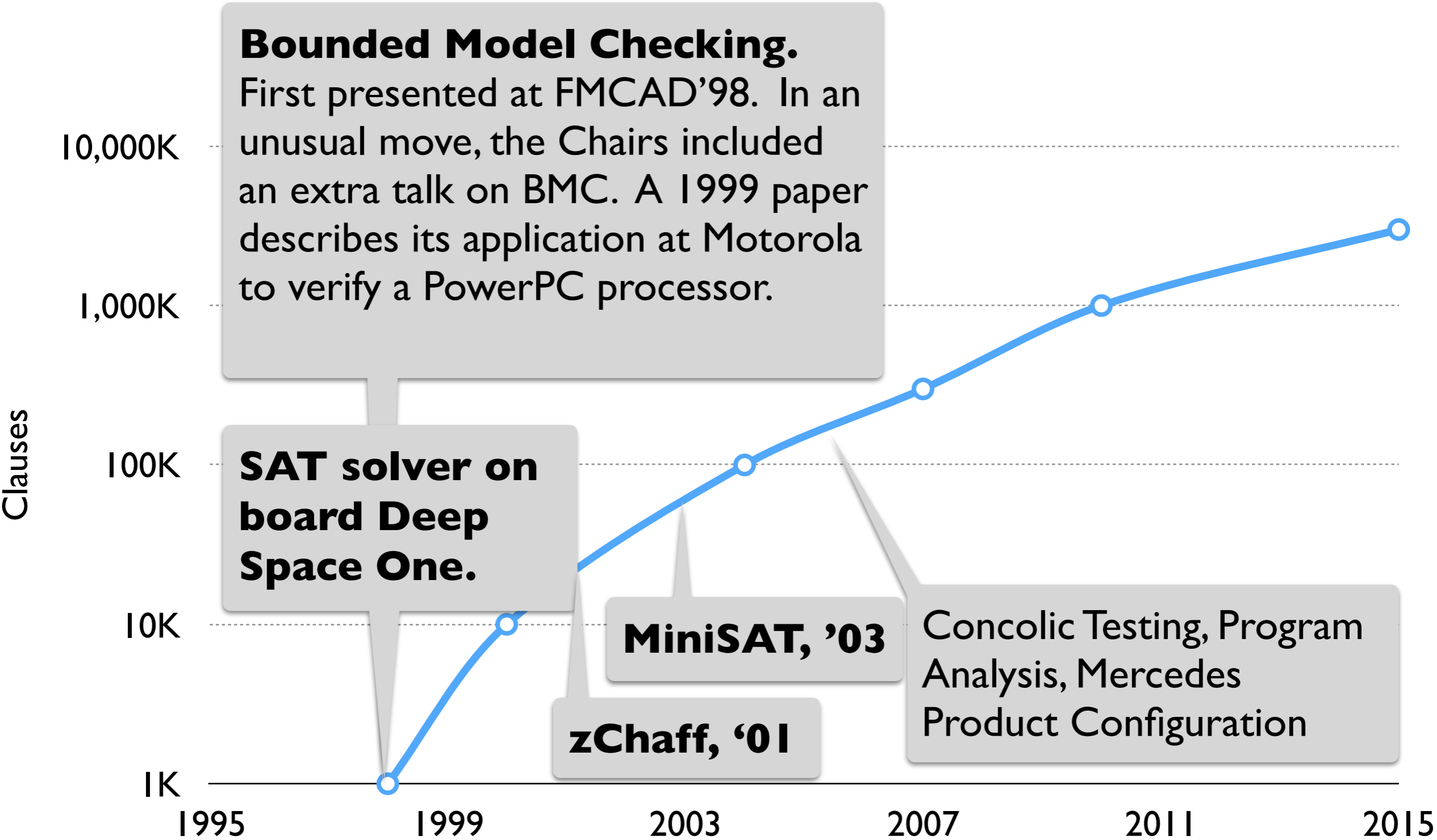


# A brief history of SAT solving and applications



Based on a slide from Vijay Ganesh

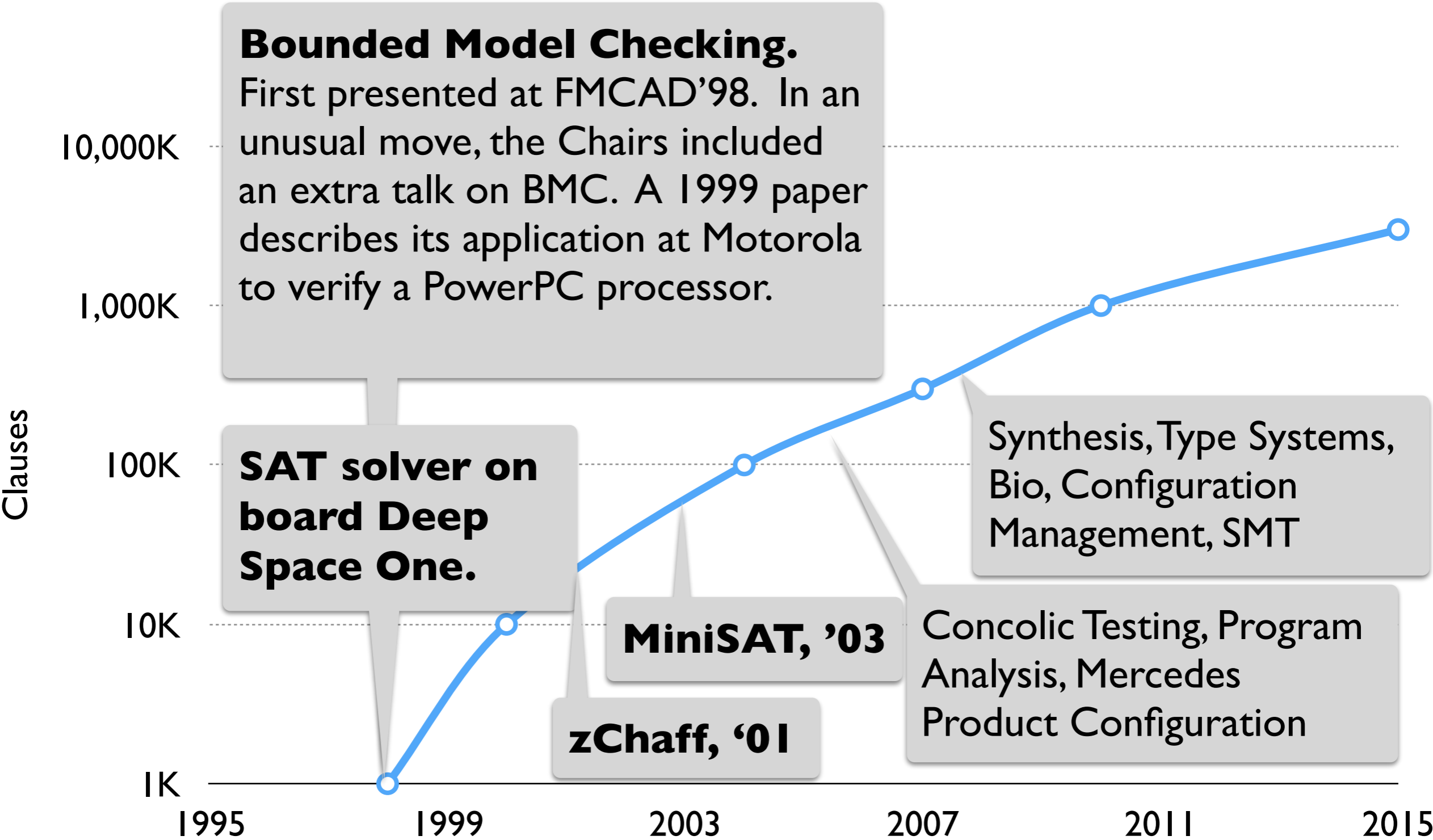
# A brief history of SAT solving and applications



Based on a slide from Vijay Ganesh



# A brief history of SAT solving and applications



Based on a slide from Vijay Ganesh

# **Bounded Model Checking (BMC) & Configuration Management**

# Bounded Model Checking (in general)

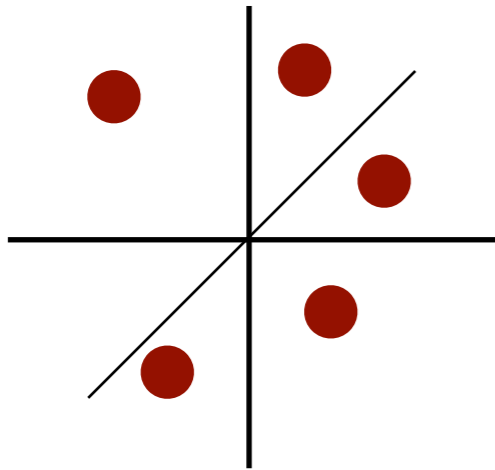
Given a system and a property, BMC checks if the property is satisfied by all executions of the system with  $\leq k$  steps, on all inputs of size  $\leq n$ .

# Bounded Model Checking (in general)

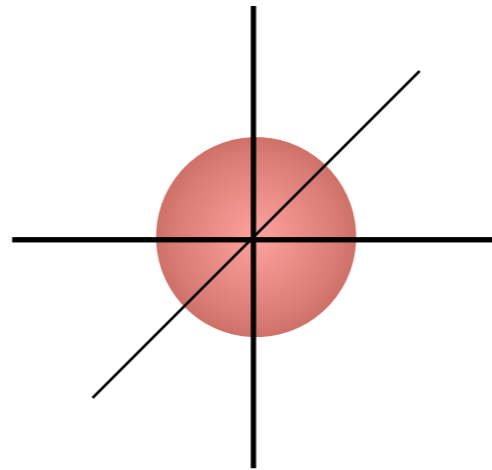
Given a system and a property, BMC checks if the property is satisfied by all executions of the system with  $\leq k$  steps, on all inputs of size  $\leq n$ .

We will focus on **safety properties** (i.e., making sure a bad state, such as an assertion violation, is not reached).

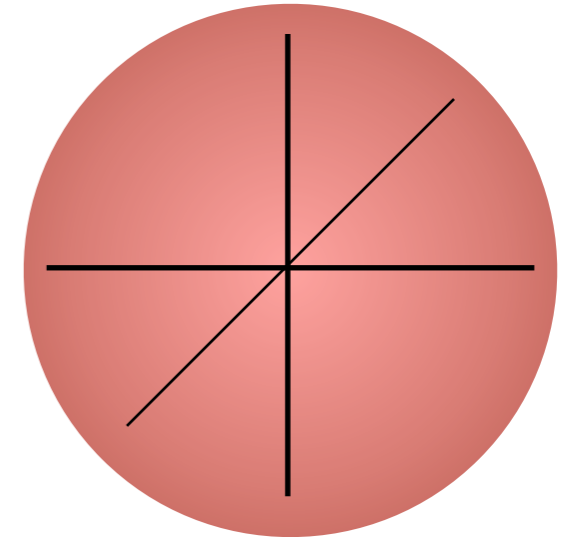
# Bounded Model Checking (in general)



Testing: checks a few executions of arbitrary size



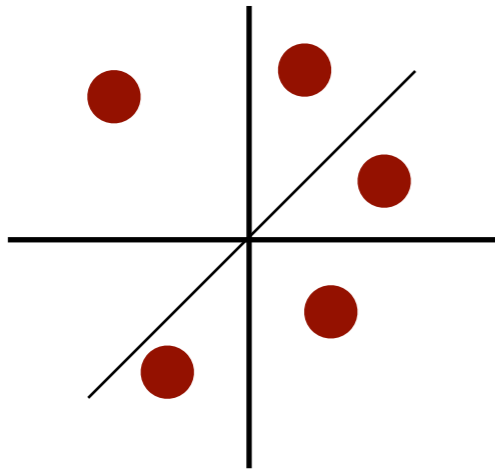
BMC: checks all executions of size  $\leq k$



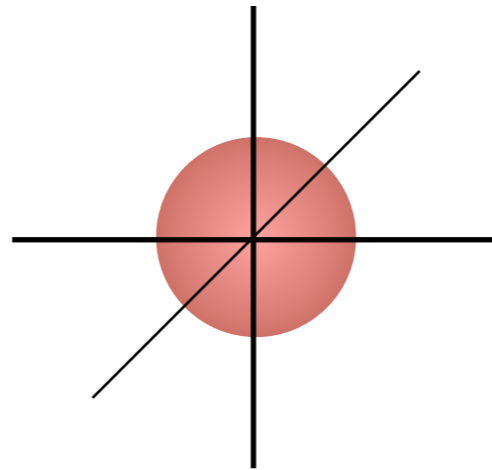
Verification: checks all executions of every size



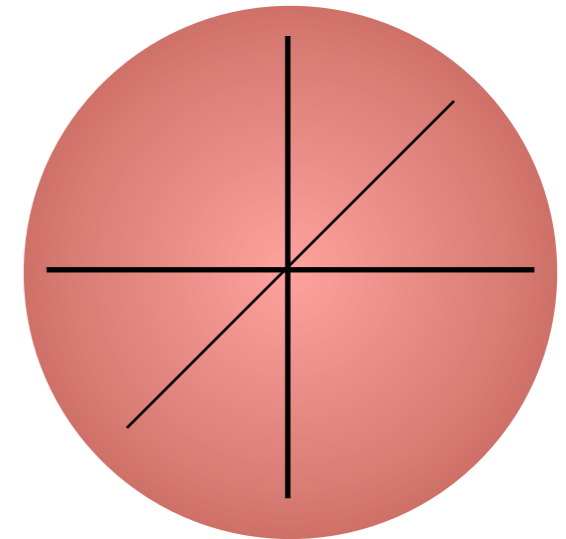
# Bounded Model Checking (in general)



Testing: checks a few executions of arbitrary size



BMC: checks all executions of size  $\leq k$



Verification: checks all executions of every size

low confidence  
low human labor

The **small scope hypothesis** says that many bugs can be triggered with small inputs and executions.

high confidence  
high human labor

# BMC by example



# BMC by example

```
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    }
    return year;
}
```

**The Zune Bug:** on December 31, 2008, all first generation Zune players from Microsoft became unresponsive because of this code. What's wrong?



# BMC by example

```
int daysToYear(int days) {  
    int year = 1980;  
    while (days > 365) {  
        if (isLeapYear(year)) {  
            if (days > 366) {  
                days -= 366;  
                year += 1;  
            }  
        } else {  
            days -= 365;  
            year += 1;  
        }  
    }  
    return year;  
}
```

Infinite loop triggered on the last day of every leap year.

# BMC by example

```
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
    }
    return year;
}
```

A desired safety property:  
the value of the days  
variable decreases in every  
loop iteration.

# BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
    }
    return year;
}
```

# BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
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                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

- Unwind all loops  $k$  times (e.g.,  $k=1$ ), and add an **unwinding assertion** after each.

# BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
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                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

- Unwind all loops  $k$  times (e.g.,  $k=1$ ), and add an **unwinding assertion** after each.
- If a CEX violates a program assertion, we have found a buggy behavior of length  $\leq k$ .

# BMC step 1 of 4: finitize loops & inline calls

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int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

- Unwind all loops  $k$  times (e.g.,  $k=1$ ), and add an **unwinding assertion** after each.
- If a CEX violates a program assertion, we have found a buggy behavior of length  $\leq k$ .
- If a CEX violates an unwinding assertion, the program has no buggy behavior of length  $\leq k$ , but it may have a longer one.

# BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

- Unwind all loops  $k$  times (e.g.,  $k=1$ ), and add an **unwinding assertion** after each.
- If a CEX violates a program assertion, we have found a buggy behavior of length  $\leq k$ .
- If a CEX violates an unwinding assertion, the program has no buggy behavior of length  $\leq k$ , but it may have a longer one.
- If there is no CEX, the program is correct for all  $k$ !

# BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

Assume call to isLeapYear is inlined (replaced with the procedure body). We'll keep it for readability.



## BMC step 2 of 4: eliminate side effects

```
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

## BMC step 2 of 4: eliminate side effects

```
int days;
int year = 1980;
if (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
        if (days > 366) {
            days = days - 366;
            year = year + 1;
        }
    } else {
        days = days - 365;
        year = year + 1;
    }
    assert days < oldDays;
    assert days <= 365;
}
return year;
```

## BMC step 2 of 4: eliminate side effects

```
int days;
int year = 1980;
if (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
        if (days > 366) {
            days = days - 366;
            year = year + 1;
        }
    } else {
        days = days - 365;
        year = year + 1;
    }
    assert days < oldDays;
    assert days <= 365;
}
return year;
```

Convert to **Static Single Assignment** (SSA) form:

- Replace each assignment to a variable  $v$  with a definition of a fresh variable  $v_i$ .
- Change uses of variables so that they refer to the correct definition (version).
- Make conditional dependences explicit with gated  $\phi$  nodes.

## BMC step 2 of 4: eliminate side effects

```
int days0;
int year0 = 1980;
if (days0 > 365) {
    int oldDays0 = days0;
    if (isLeapYear(year0)) {
        if (days0 > 366) {
            days1 = days0 - 366;
            year1 = year0 + 1;
        }
    } else {
        days3 = days0 - 365;
        year3 = year0 + 1;
    }
    assert days4 < oldDays0;
    assert days4 <= 365;
}
return year5;
```

Convert to **Static Single Assignment** (SSA) form:

- Replace each assignment to a variable  $v$  with a definition of a fresh variable  $v_i$ .
- Change uses of variables so that they refer to the correct definition (version).
- Make conditional dependences explicit with gated  $\phi$  nodes.

## BMC step 2 of 4: eliminate side effects

```
int days0;  
int year0 = 1980;  
boolean g0 = (days0 > 365);  
int oldDays0 = days0;  
boolean g1 = isLeapYear(year0);  
boolean g2 = days0 > 366;  
days1 = days0 - 366;  
year1 = year0 + 1;  
days2 = φ(g1 && g2, days1, days0);  
year2 = φ(g1 && g2, year1, year0);  
days3 = days0 - 365;  
year3 = year0 + 1;  
days4 = φ(g1, days2, days3);  
year4 = φ(g1, year2, year3);  
assert days4 < oldDays0;  
assert days4 <= 365;  
year5 = φ(g0, year4, year0);  
return year5;
```

Convert to **Static Single Assignment** (SSA) form:

- Replace each assignment to a variable  $v$  with a definition of a fresh variable  $v_i$ .
- Change uses of variables so that they refer to the correct definition (version).
- Make conditional dependences explicit with gated  $\phi$  nodes.

## BMC step 2 of 4: eliminate side effects

```
int days0;
int year0 = 1980;
if (days0 > 365) {
    int oldDays0 = days0;
    if (isLeapYear(year0)) {
        if (days0 > 366) {
            days1 = days0 - 366;
            year1 = year0 + 1;
        }
    } else {
        days3 = days0 - 365;
        year3 = year0 + 1;
    }
    assert days4 < oldDays0;
    assert days4 <= 365;
}
return year4;
```

```
int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 =  $\varphi$ (g1 && g2, days1, days0);
year2 =  $\varphi$ (g1 && g2, year1, year0);
days3 = days0 - 365;
year3 = year0 + 1;
days4 =  $\varphi$ (g1, days2, days3);
year4 =  $\varphi$ (g1, year2, year3);
assert days4 < oldDays0;
assert days4 <= 365;
year5 =  $\varphi$ (g0, year4, year0);
return year5;
```

## BMC step 3 of 4: convert into equations

```
int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 =  $\varphi$ (g1 && g2, days1, days0);
year2 =  $\varphi$ (g1 && g2, year1, year0);
days3 = days0 - 365;
year3 = year0 + 1;
days4 =  $\varphi$ (g1, days2, days3);
year4 =  $\varphi$ (g1, year2, year3);
assert days4 < oldDays0;
assert days4 <= 365;
year5 =  $\varphi$ (g0, year4, year0);
return year5;
```

## BMC step 3 of 4: convert into equations

```
year0 = 1980 ∧
g0 = (days0 > 365) ∧
oldDays0 = days0 ∧
g1 = isLeapYear(year0) ∧
g2 = days0 > 366 ∧
days1 = days0 - 366 ∧
year1 = year0 + 1 ∧
days2 = ite(g1 ∧ g2, days1, days0) ∧
year2 = ite(g1 ∧ g2, year1, year0) ∧
days3 = days0 - 365 ∧
year3 = year0 + 1 ∧
days4 = ite(g1, days2, days3) ∧
year4 = ite(g1, year2, year3) ∧
year5 = ite(g0, year4, year0) ∧
(¬(days4 < oldDays0) ∨
¬(days4 <= 365))
```

A solution to these equations is a sound

**counterexample:** an interpretation for all logical variables that satisfies the program semantics (for up to  $k$  unwindings) but violates at least one of the assertions.



## **BMC step 4 of 4: convert into CNF**

$$\text{year}_1 = \text{year}_0 + 1$$

# BMC step 4 of 4: convert into CNF

$$\text{year}_1 = \text{year}_0 + 1$$

$$\text{year}_0 = \underset{31 \ 30 \ 29}{000} \dots \underset{2 \ 1 \ 0}{000}$$

Represent numbers as  
arrays of bits ...

# BMC step 4 of 4: convert into CNF

$$\text{year}_1 = \text{year}_0 + 1$$

$$\text{year}_0 = \underset{31}{0} \underset{30}{0} \underset{29}{0} \dots \underset{2}{0} \underset{1}{0} \underset{0}{0}$$

year<sub>0:31</sub>

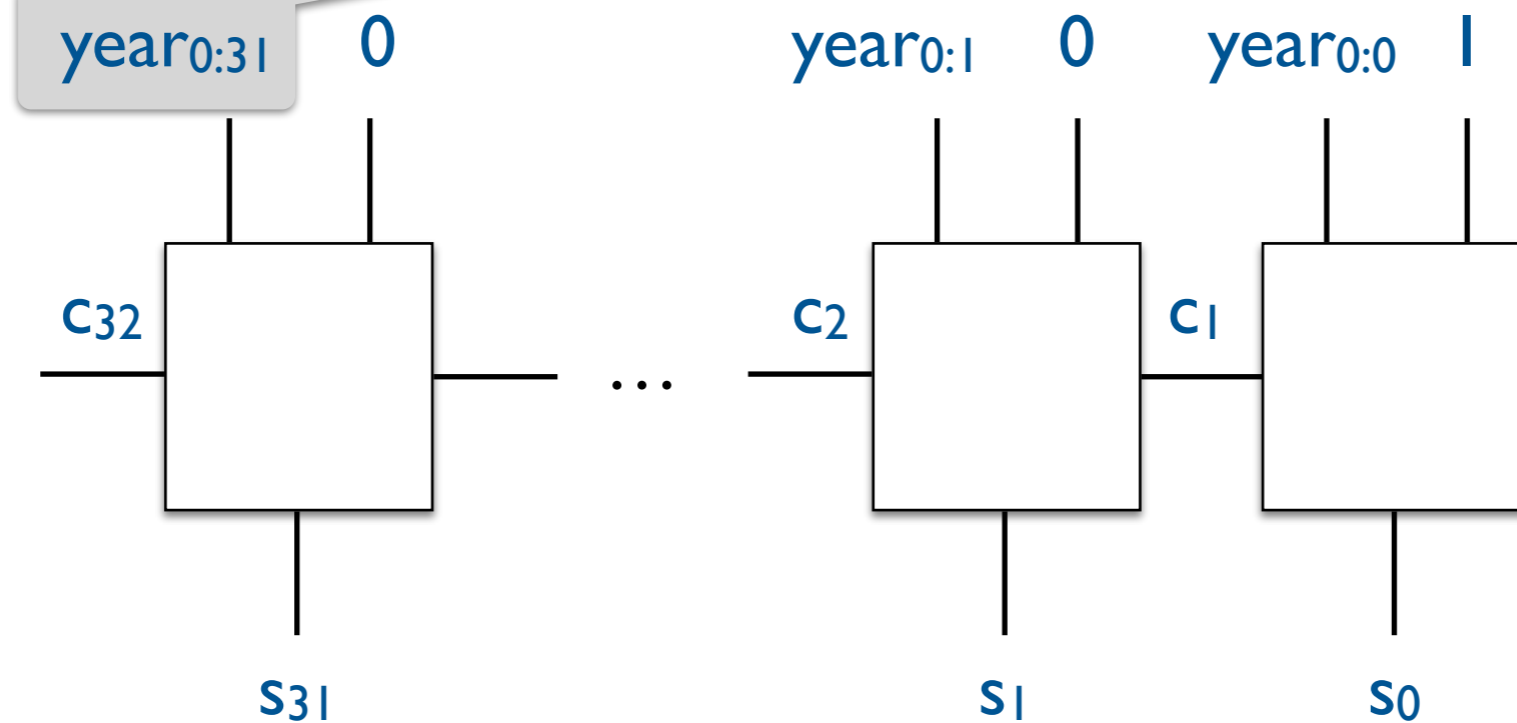
Represent numbers as arrays of bits, and create one propositional variable per bit for each number.

# BMC step 4 of 4: convert into CNF

$$\text{year}_1 = \text{year}_0 + 1$$

$$\text{year}_0 = \underset{31 \ 30 \ 29}{000} \dots \underset{2 \ 1 \ 0}{000}$$

Represent numbers as arrays of bits, and create one propositional variable per bit for each number.



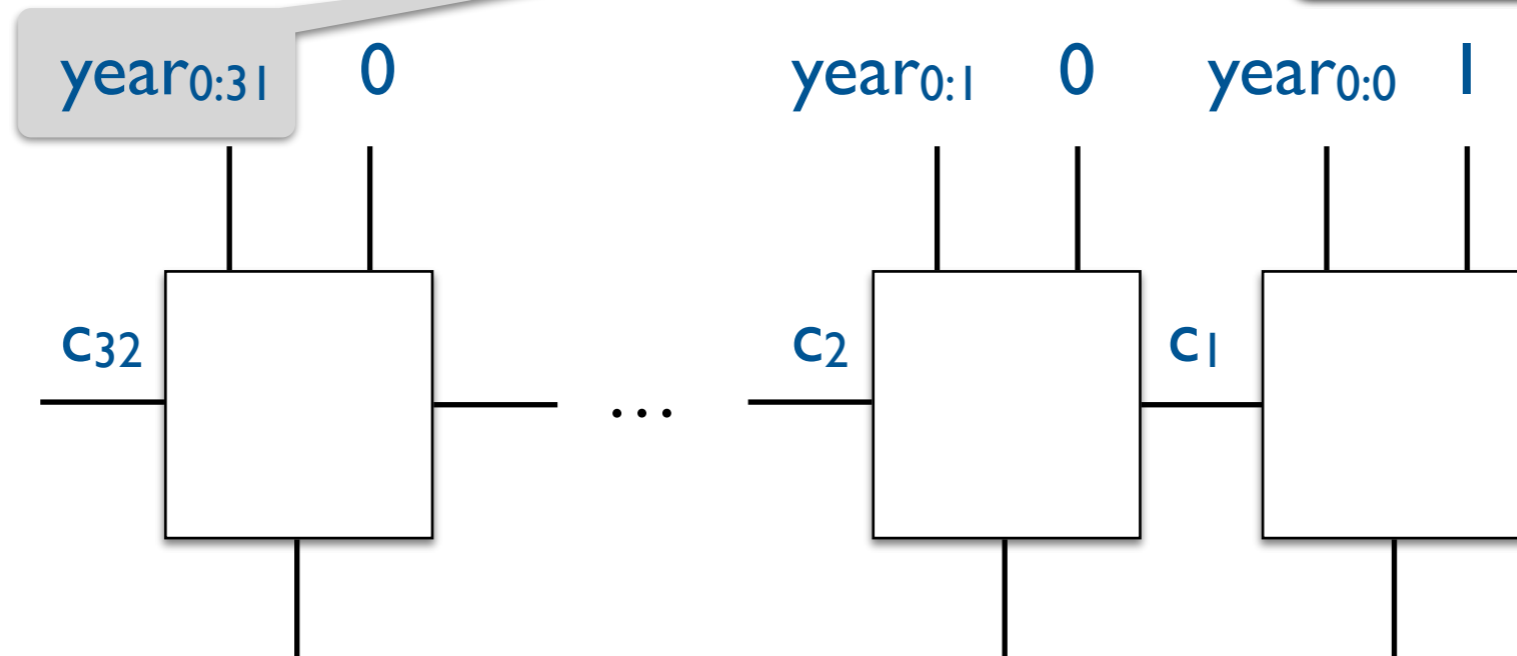
Construct an adder circuit for  $\text{year}_0 + 1$ .

# BMC step 4 of 4: convert into CNF

$$\text{year}_1 = \text{year}_0 + 1$$

$$\text{year}_0 = \underset{31}{000} \dots \underset{2}{000} \underset{1}{0} \underset{0}{1}$$

Represent numbers as arrays of bits, and create one propositional variable per bit for each number.



Construct an adder circuit for  $\text{year}_0 + 1$ .

$$\text{year}_{1:31} \iff s_{31} \wedge \dots \wedge \text{year}_{1:1} \iff s_1 \wedge s_0 \iff \text{year}_{1:0}$$

Introduce new clauses to constrain bits in  $\text{year}_1$  to match bits in the sum.

# BMC counterexample for k=1

```
int daysToYear(int days) {  
    int year = 1980;  
    while (days > 365) {  
        int oldDays = days;  
        if (isLeapYear(year)) {  
            if (days > 366) {  
                days -= 366;  
                year += 1;  
            }  
        } else {  
            days -= 365;  
            year += 1;  
        }  
        assert days < oldDays;  
    }  
    return year;  
}
```

**days = 366**

# **Bounded Model Checking (BMC) & Configuration Management**

# Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

- Decide if a new component can be added to the configuration.
- Add the component while optimizing some linear function.
- If the component cannot be added, find a way to add it by removing as few conflicting components from the current configuration as possible.

**maven**

eclipse





# Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

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**maven**

eclipse



SAT

# Configuration Management

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SAT

Pseudo-Boolean Constraints

# Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

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**maven**

eclipse

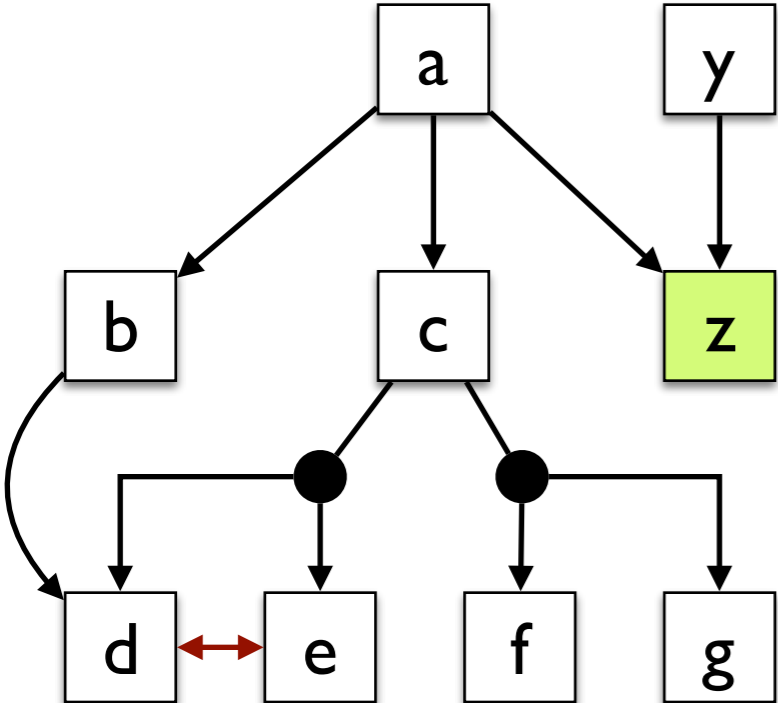


SAT

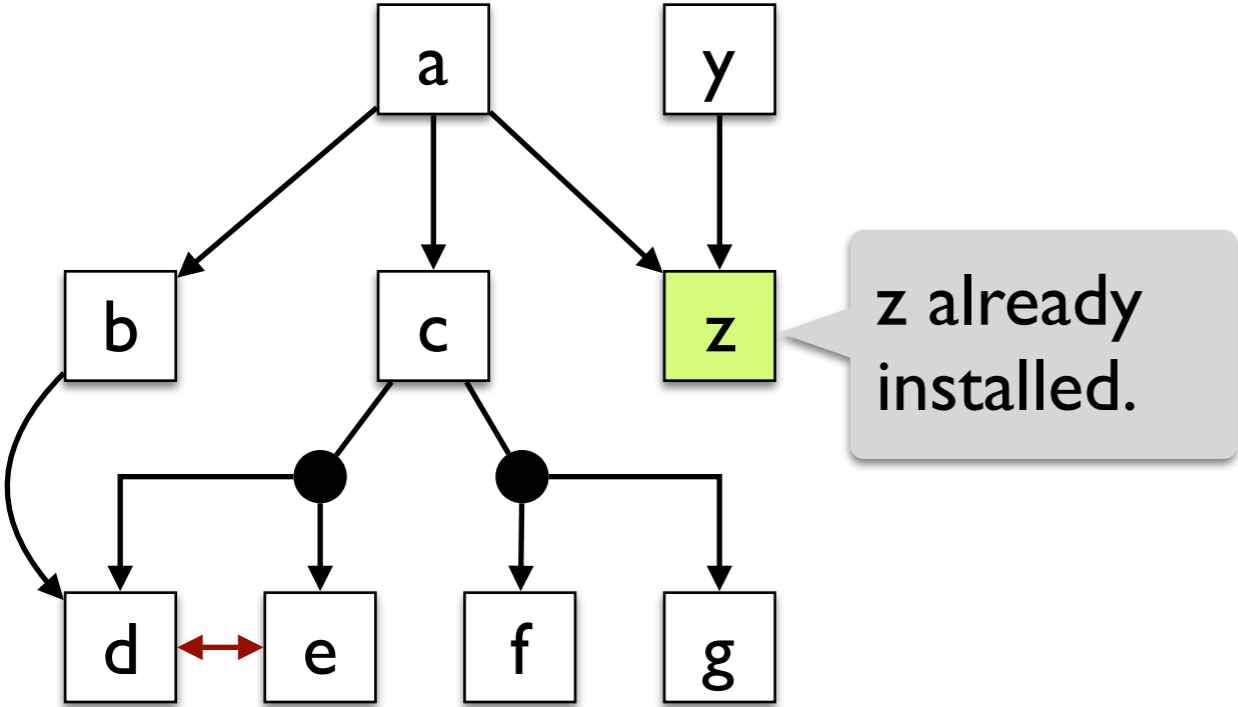
Pseudo-Boolean Constraints

Partial (Weighted) MaxSAT

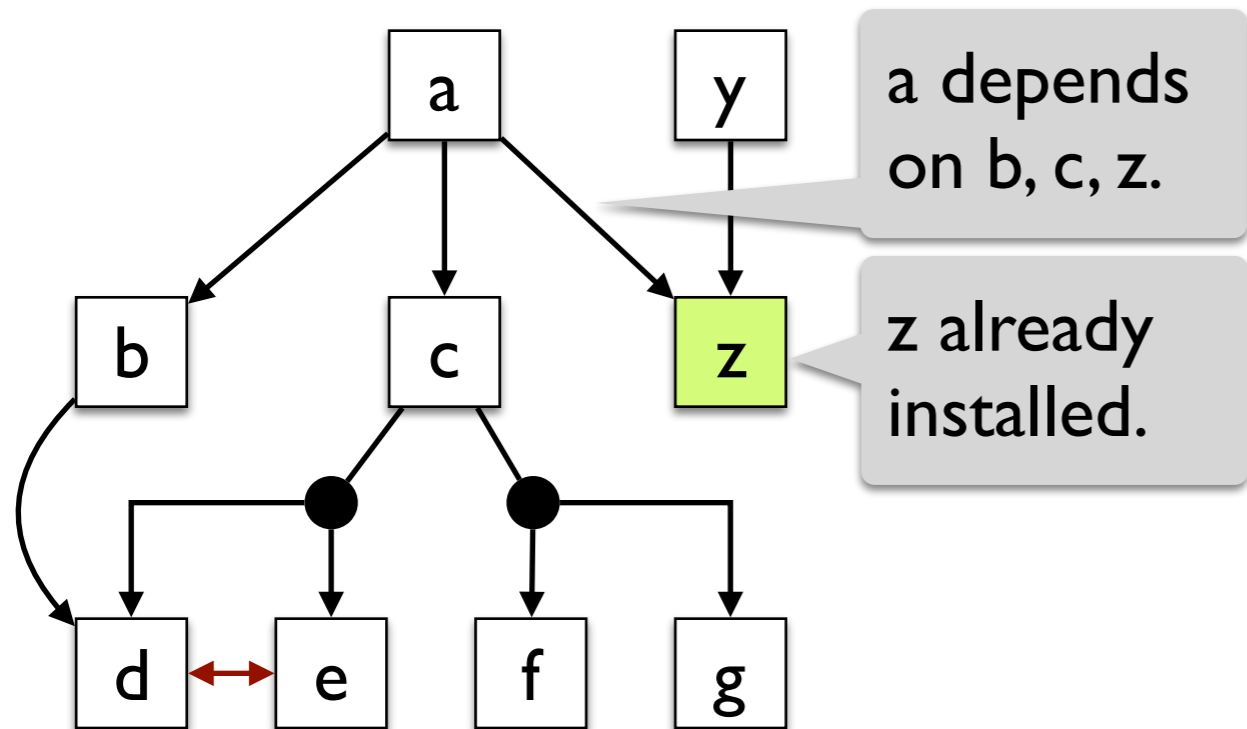
# Deciding if a component can be installed



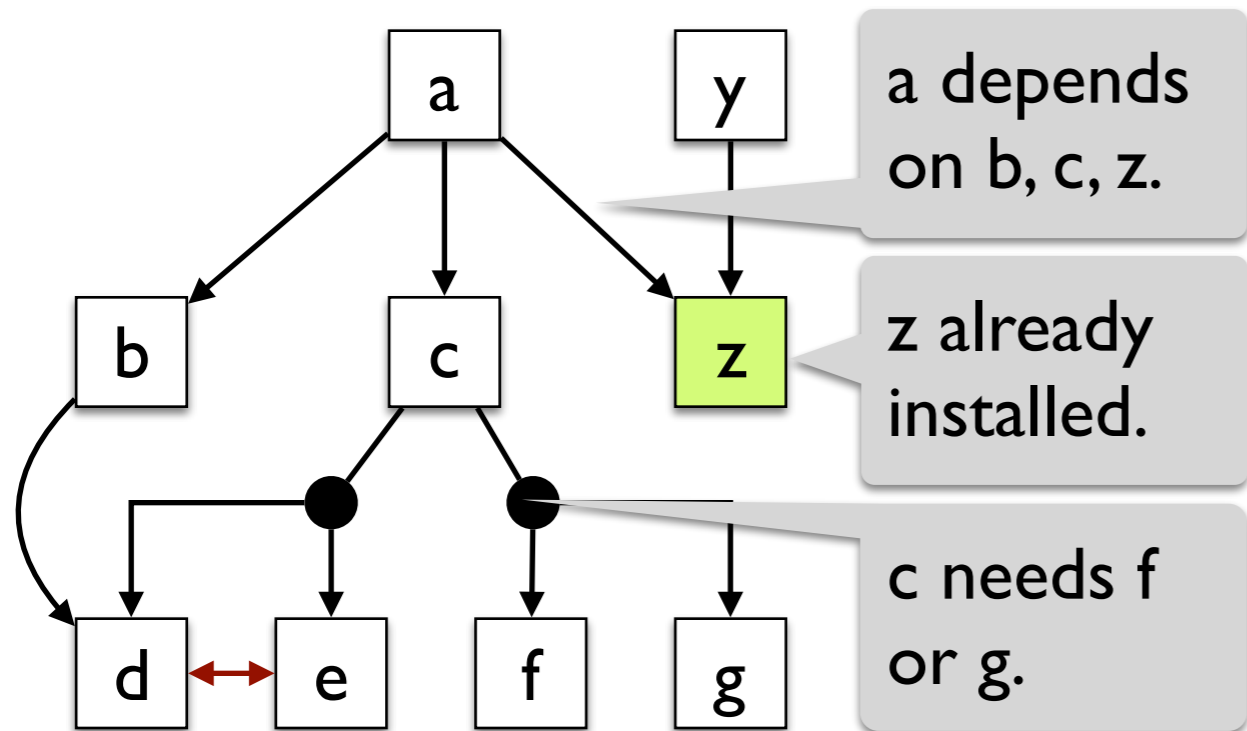
# Deciding if a component can be installed



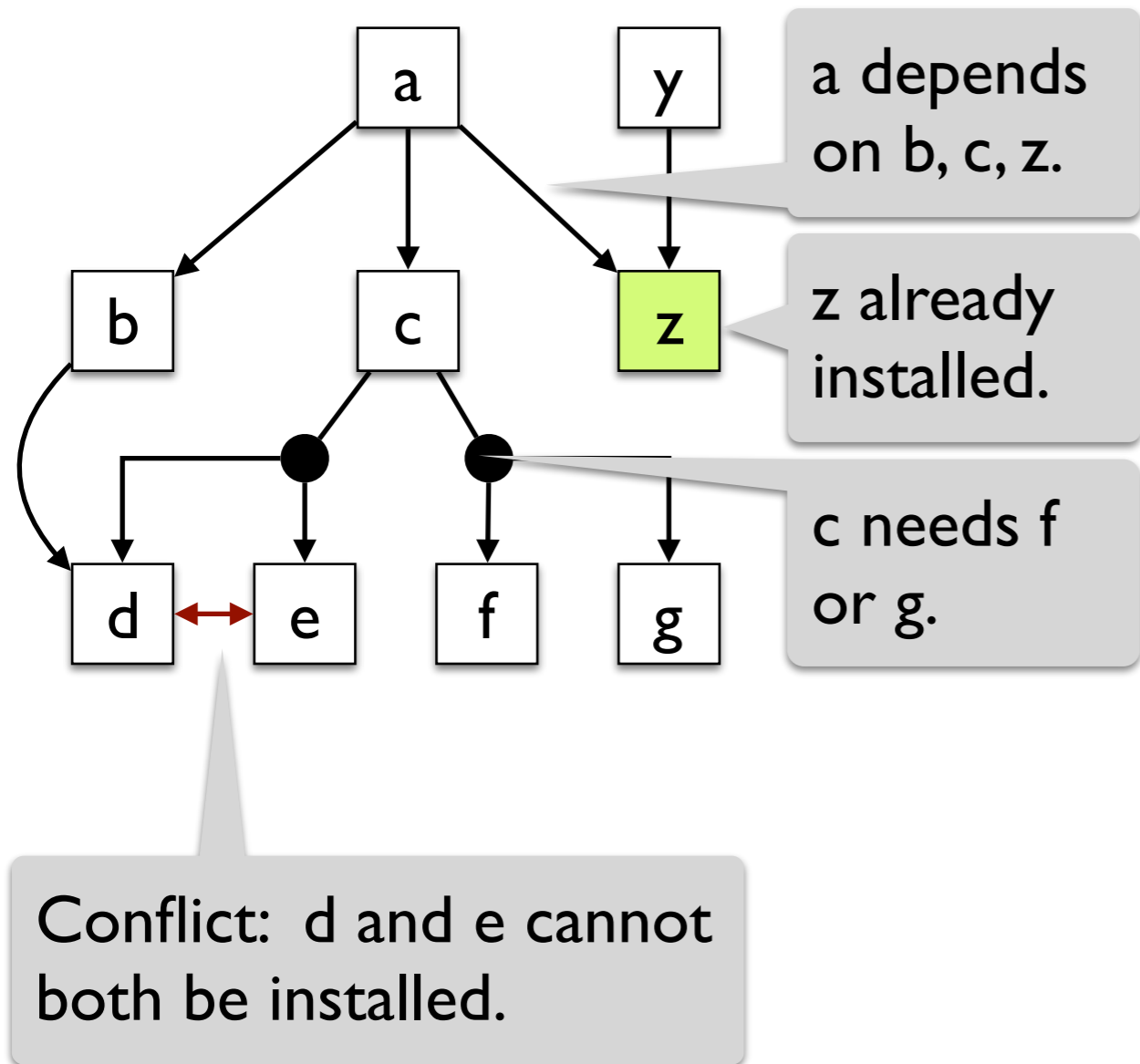
# Deciding if a component can be installed



# Deciding if a component can be installed

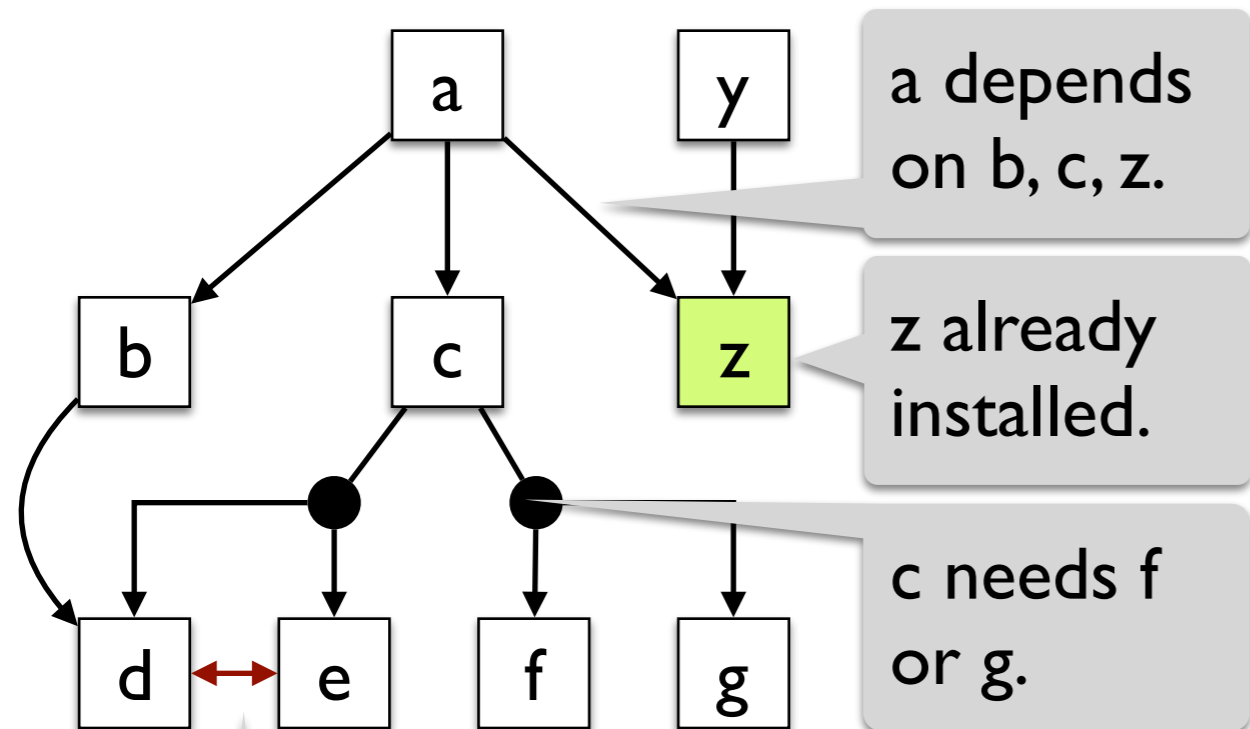


# Deciding if a component can be installed





# Deciding if a component can be installed



a depends on b, c, z.

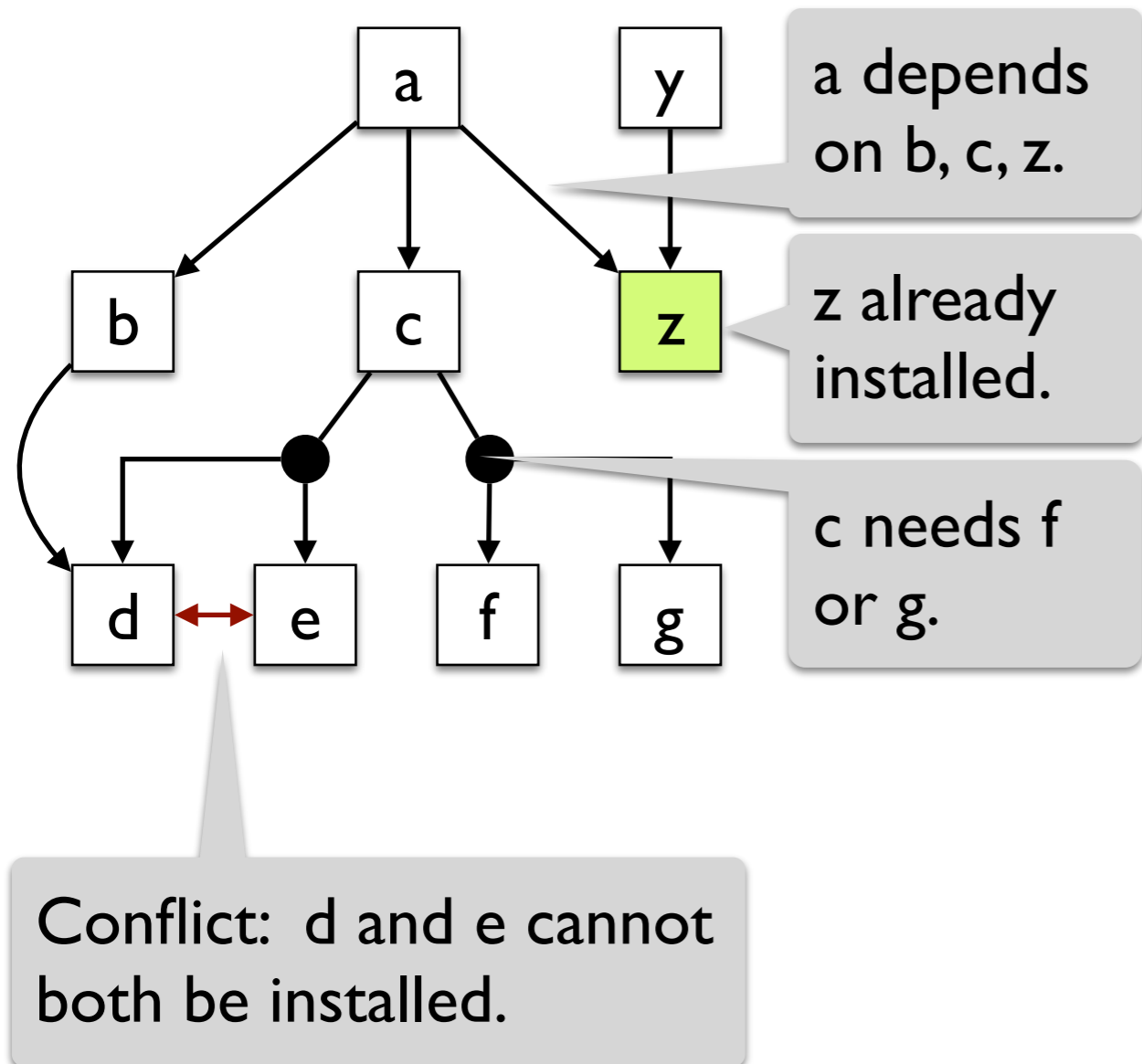
z already installed.

c needs f or g.

Conflict: d and e cannot both be installed.

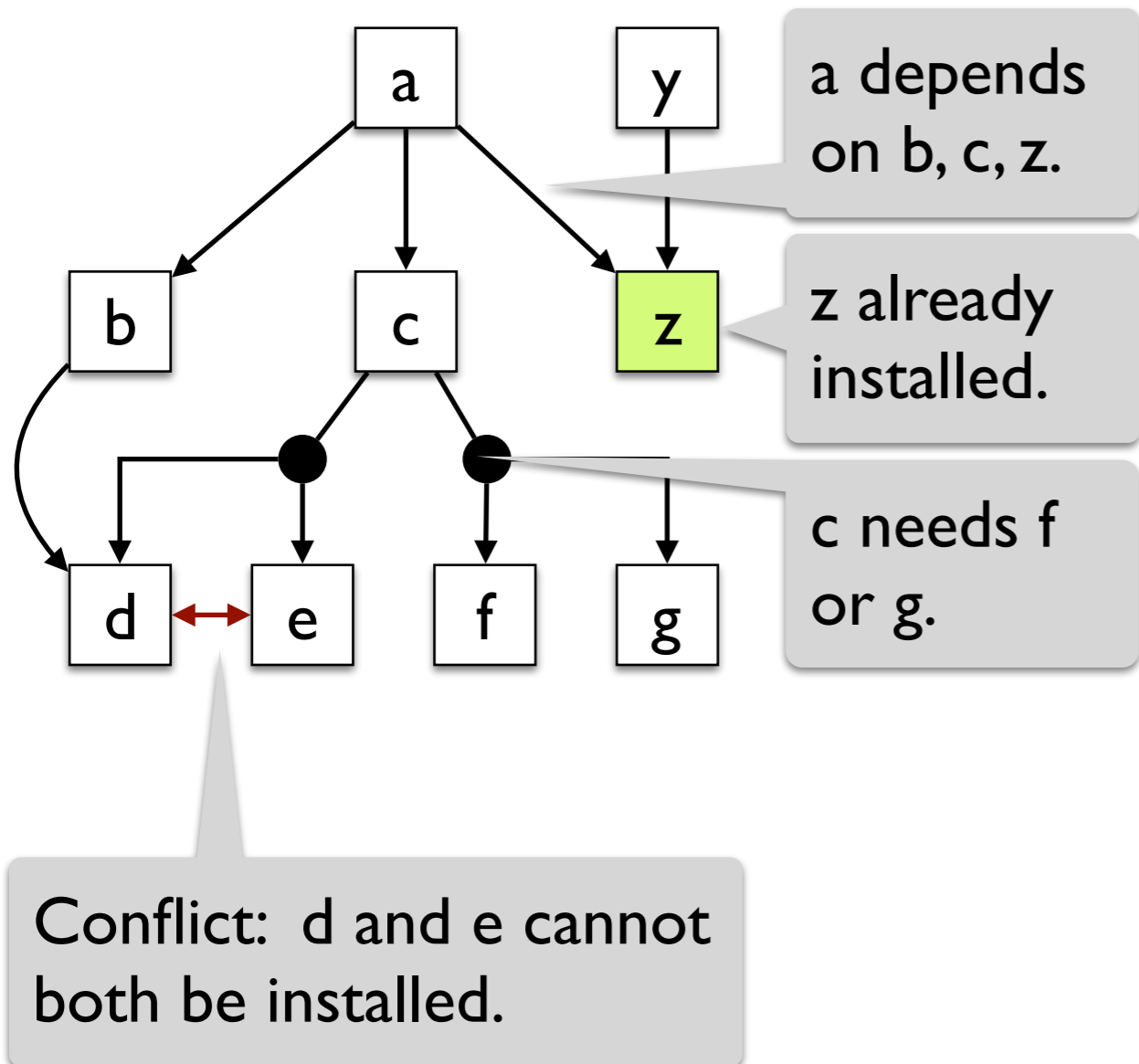
To install a, CNF constraints are:

# Deciding if a component can be installed



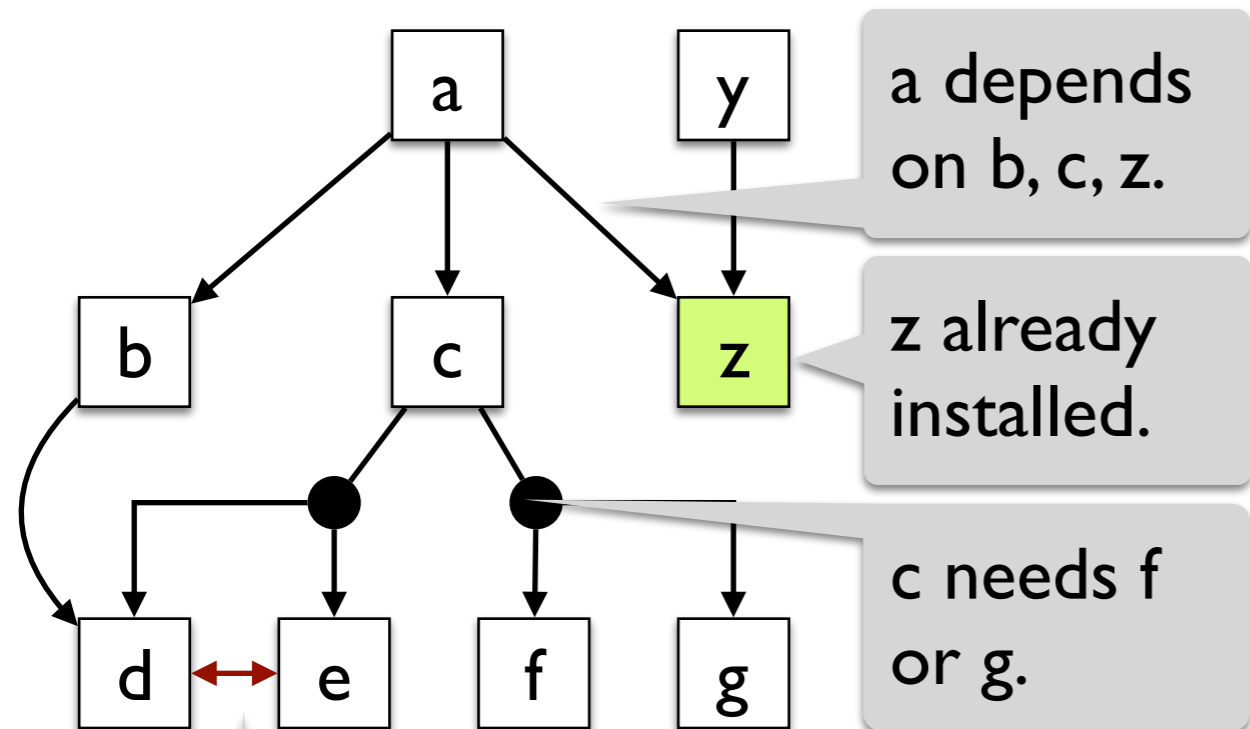
To install a, CNF constraints are:  
 $(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge$

# Deciding if a component can be installed



To install a, CNF constraints are:  
 $(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge$   
 $(\neg b \vee d) \wedge$

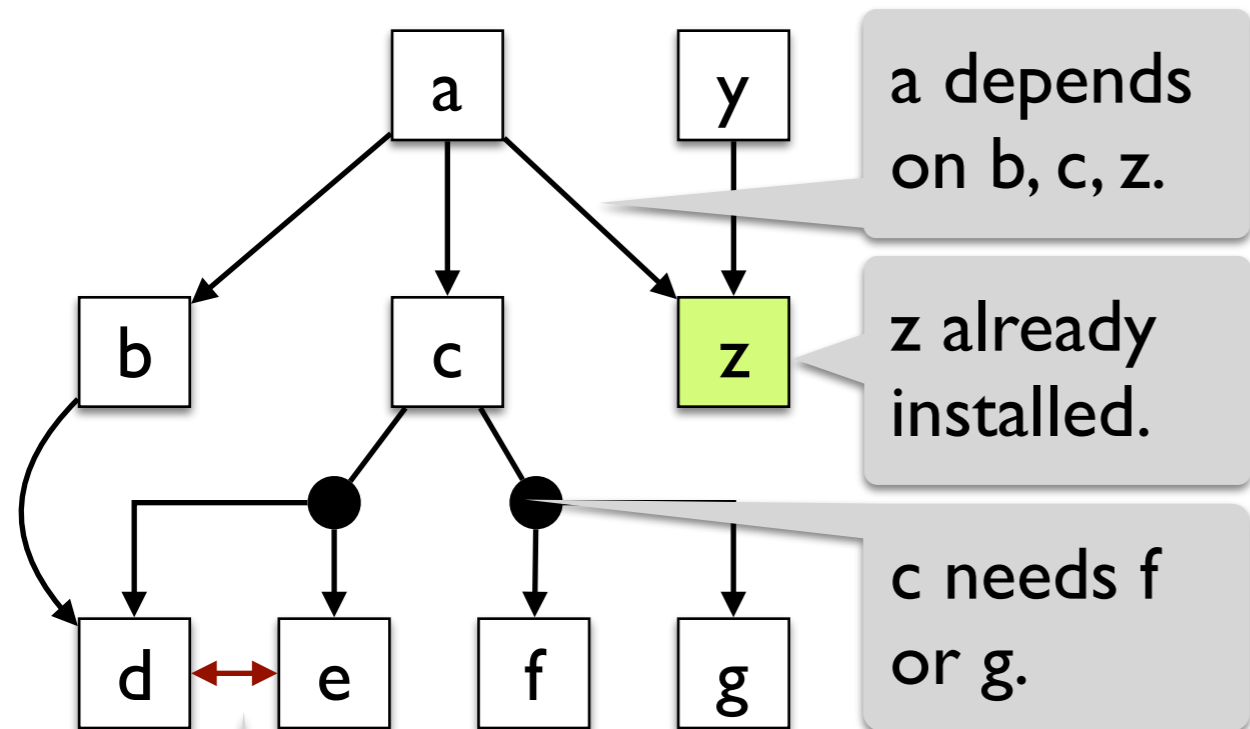
# Deciding if a component can be installed



To install a, CNF constraints are:

$$(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge$$
$$(\neg b \vee d) \wedge$$
$$(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge$$

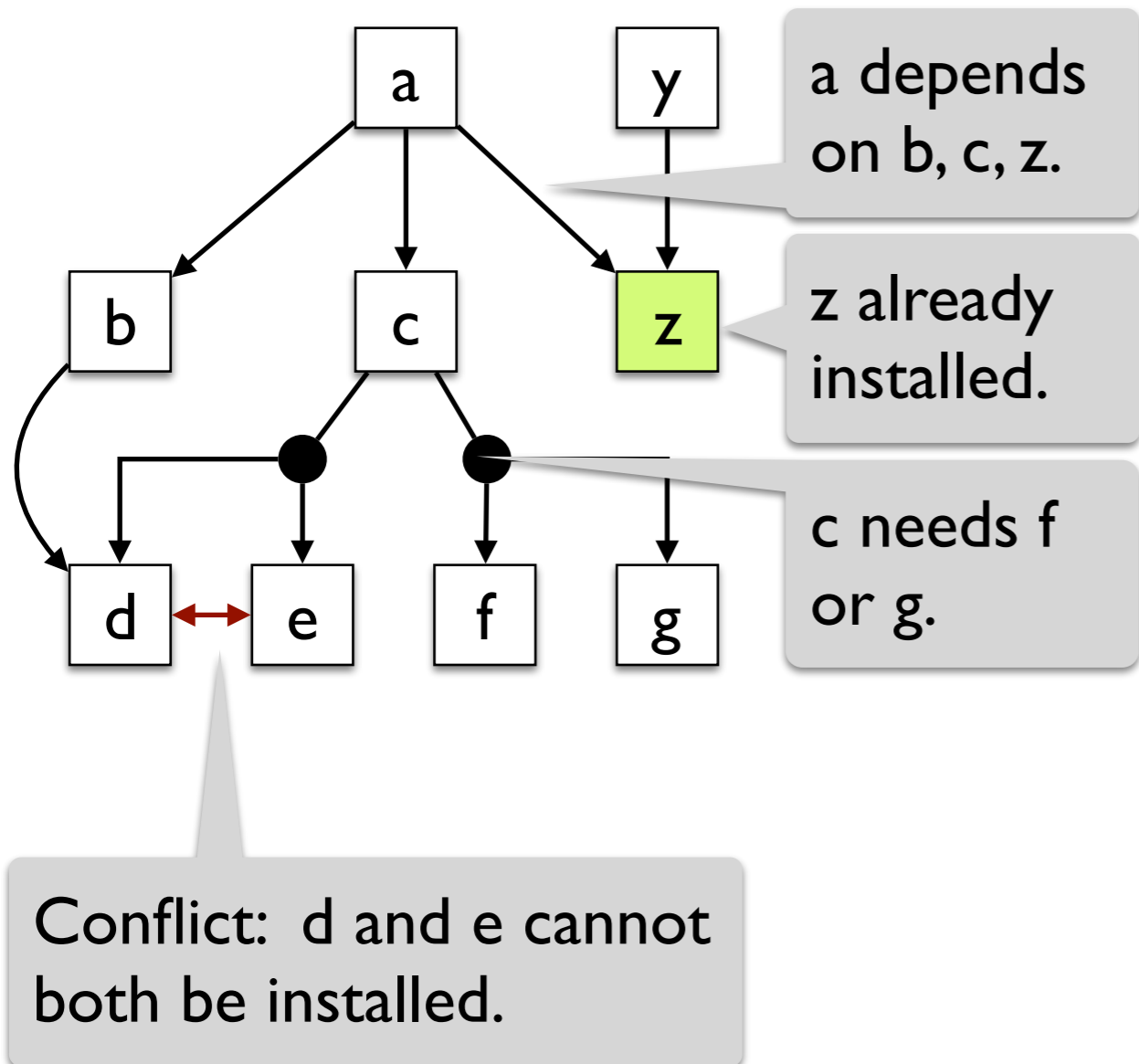
# Deciding if a component can be installed



To install a, CNF constraints are:

$$\begin{aligned} &(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge \\ &(\neg b \vee d) \wedge \\ &(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge \\ &(\neg d \vee \neg e) \wedge \end{aligned}$$

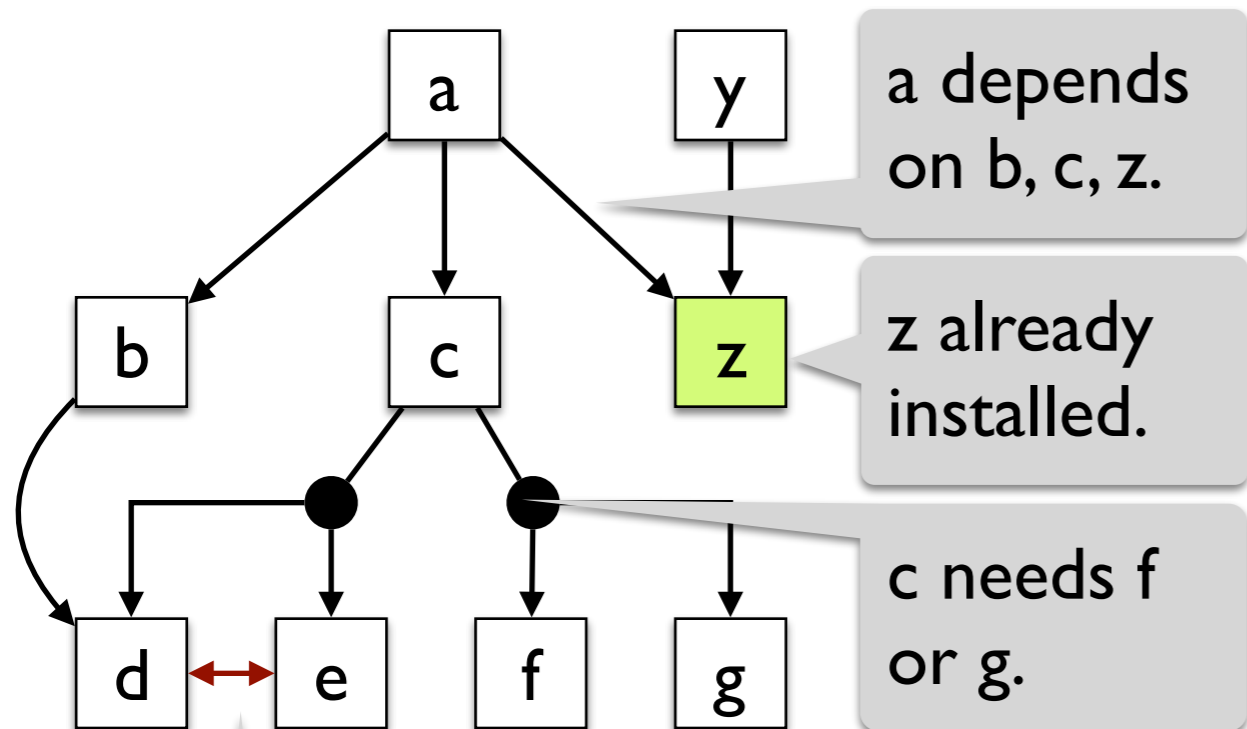
# Deciding if a component can be installed



To install a, CNF constraints are:

$$\begin{aligned} &(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge \\ &(\neg b \vee d) \wedge \\ &(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge \\ &(\neg d \vee \neg e) \wedge \\ &(\neg y \vee z) \wedge \end{aligned}$$

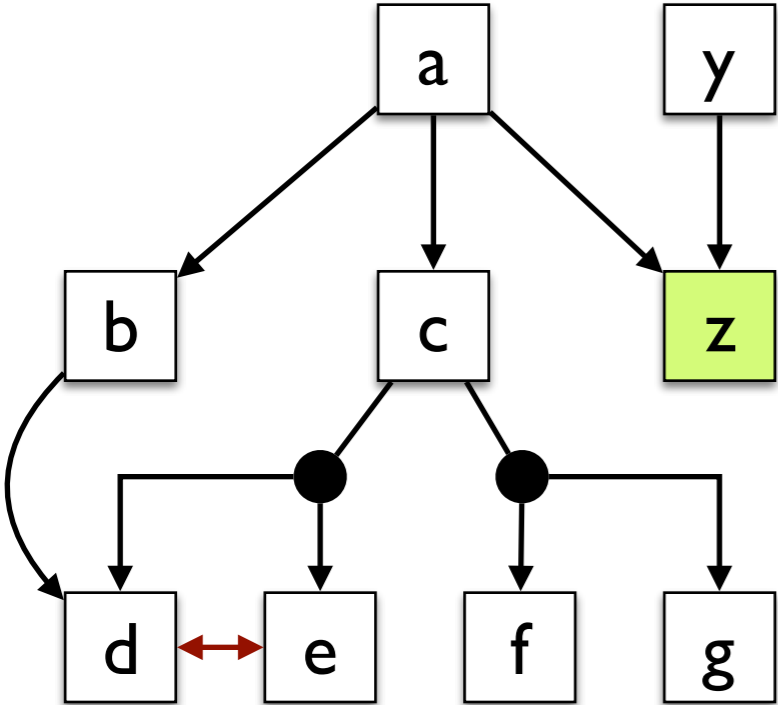
# Deciding if a component can be installed



To install a, CNF constraints are:

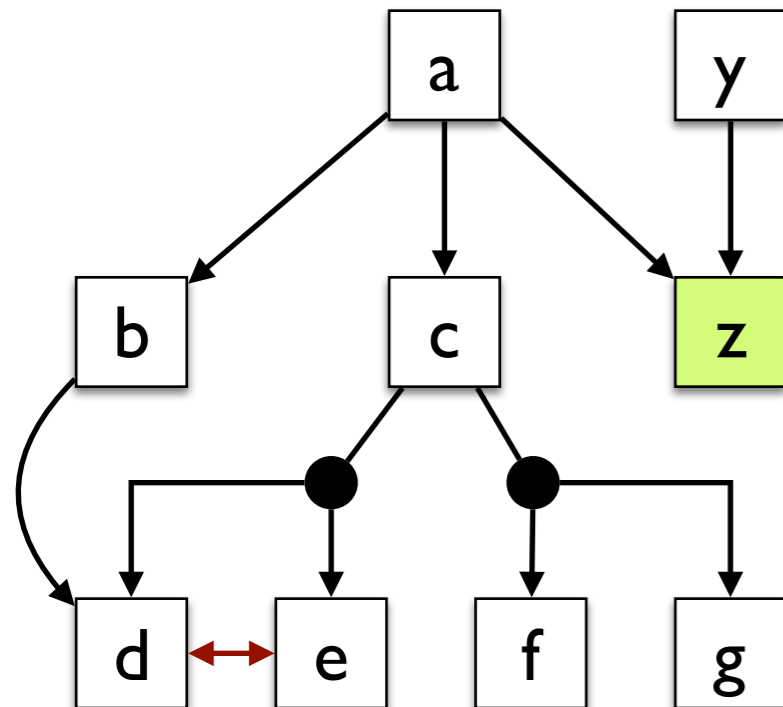
$$\begin{aligned} &(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge \\ &(\neg b \vee d) \wedge \\ &(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge \\ &(\neg d \vee \neg e) \wedge \\ &(\neg y \vee z) \wedge \\ &a \wedge z \end{aligned}$$

# Optimal installation





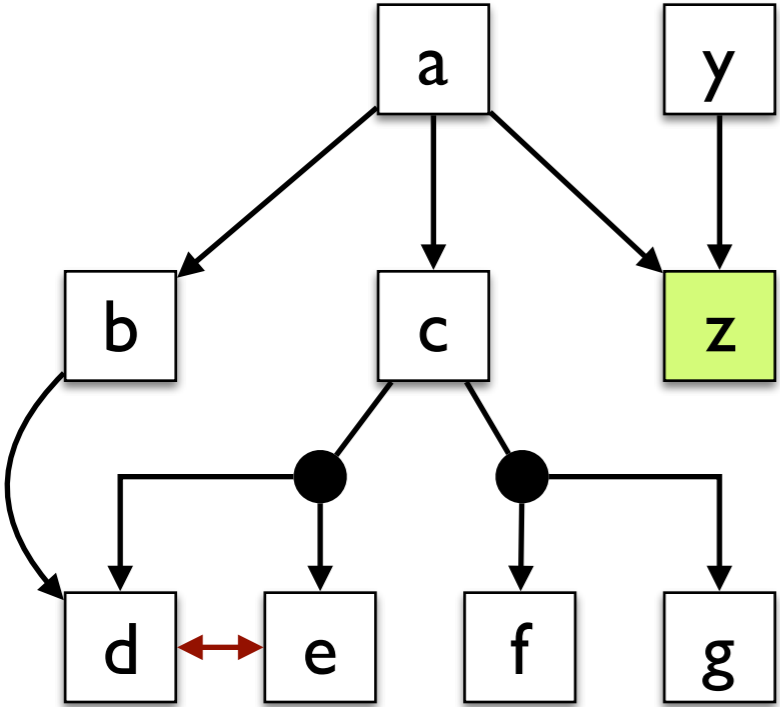
# Optimal installation



Pseudo-boolean solvers accept a linear function to minimize, in addition to a (weighted) CNF.

Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

# Optimal installation



Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

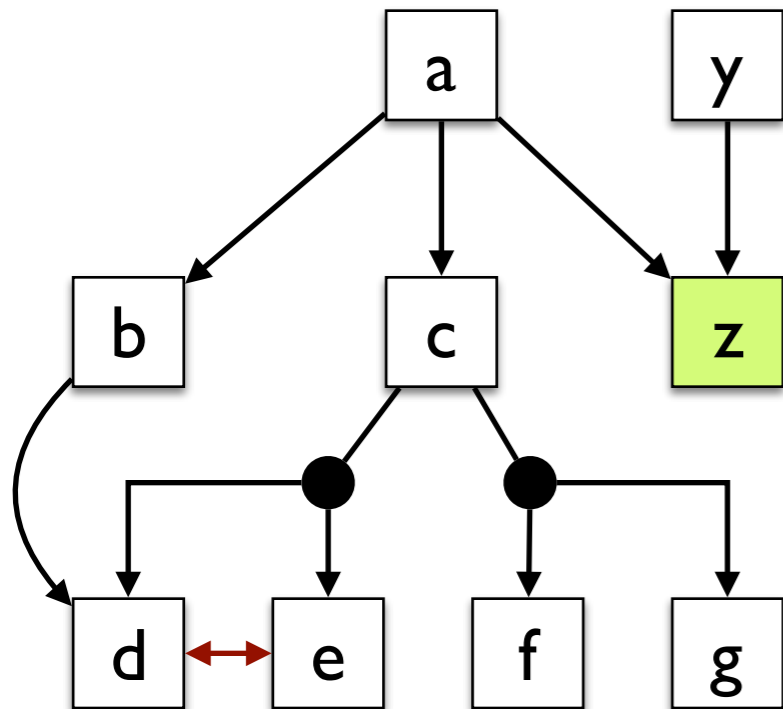
$$\min c_1x_1 + \dots + c_nx_n$$

$$a_{11}x_1 + \dots + a_{1n}x_n \geq b_1$$

...

$$a_{k1}x_1 + \dots + a_{kn}x_n \geq b_k$$

# Optimal installation



$$\mathbf{min} \ c_1x_1 + \dots + c_nx_n$$

$$a_{11}x_1 + \dots + a_{1n}x_n \geq b_1$$

...

$$a_{k1}x_1 + \dots + a_{kn}x_n \geq b_k$$

Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

$$\mathbf{min} \ a + b + c + d + e + 5f + 2g + y + 0z$$

$$(-a + b \geq 0) \wedge (-a + c \geq 0) \wedge (-a + z \geq 0) \wedge$$

$$(-b + d \geq 0) \wedge$$

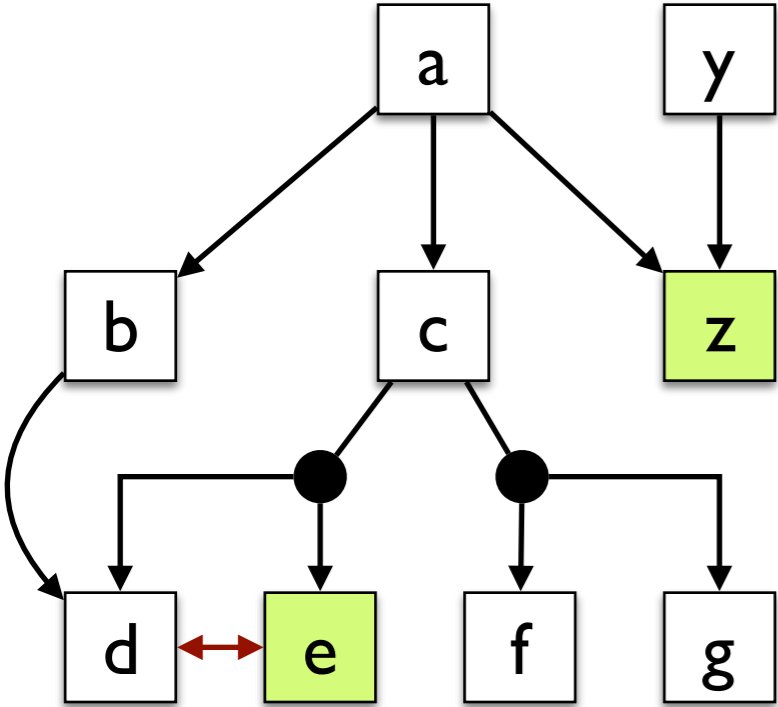
$$(-c + d + e \geq 0) \wedge (-c + f + g \geq 0) \wedge$$

$$(-d + -e \geq -1) \wedge$$

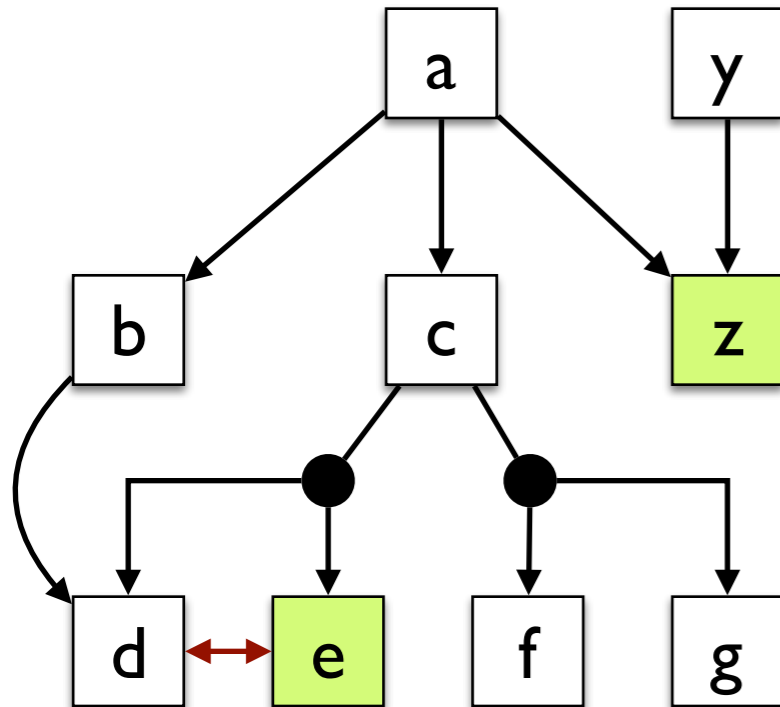
$$(-y + z \geq 0) \wedge$$

$$(a \geq 1) \wedge (z \geq 1)$$

# Installation in the presence of conflicts

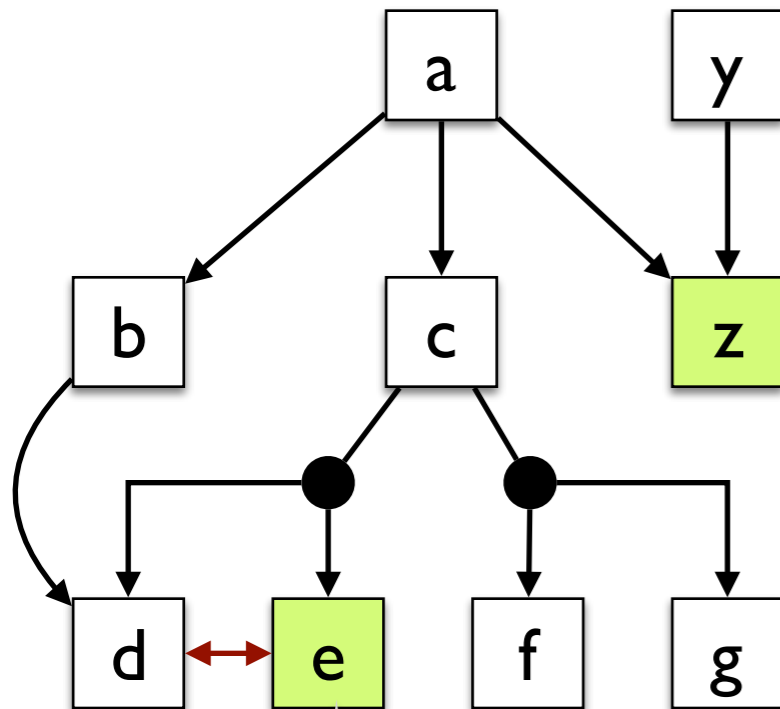


# Installation in the presence of conflicts



a cannot be installed because it requires b, which requires d, which conflicts with e.

# Installation in the presence of conflicts



To install a, while minimizing the number of removed components, Partial MaxSAT constraints are:

**hard:**  $(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge$   
 $(\neg b \vee d) \wedge$   
 $(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge$   
 $(\neg d \vee \neg e) \wedge (\neg y \vee z) \wedge a$

**soft:**  $e \wedge z$

Partial MaxSAT solver takes as input a set of **hard** clauses and a set of **soft** clauses, and it produces an assignment that satisfies all hard clauses and the greatest number of soft clauses.

# Summary

## Today

- SAT solvers have been used successfully in many applications & domains
- But reducing problems to SAT is a lot like programming in assembly ...
- We need higher-level logics!

## Next lecture

- On to richer logics: introduction to Satisfiability Modulo Theories (SMT)