Homework 1
AAA528, Fall 2018

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Due: 10/01, in Class

Problem 1 (50pts) A problem of covering m subjects with k teachers may be defined as follows. Let $T = \{T_1, \ldots, T_n\}$ be a set of teachers. Let $S = \{S_1, \ldots, S_m\}$ be a set of subjects. Each teacher $t \in T$ can teach some subset $S(t)$ of the subjects S (i.e., $S(t) \subseteq S$). Given a natural number $k < n$, is there a subset of size k of the teachers that together covers all m subjects, i.e., a subset $C \subseteq T$ such that $|C| = k$ and $(\bigcup_{t \in C} S(t)) = S$? Explain how to encode an instance of this problem into a propositional formula F. F should be satisfiable iff there is such a subset C.

Problem 2 (50pts) A solution to a graph coloring problem is an assignment of colors to vertices such that no two adjacent vertices have the same color. Formally, a finite graph $G = \langle V, E \rangle$ consists of vertices $V = \{v_1, \ldots, v_n\}$ and edges $E = \{(v_{i_1}, w_{i_1}), \ldots, (v_{i_k}, w_{i_k})\}$. The finite set of colors is given by $C = \{c_1, \ldots, c_m\}$. A problem instance is given by a graph and a set of colors; the problem is to assign each vertex $v \in V$ a $\mathit{color}(v) \in C$ such that for every edge $(v, w) \in E, \mathit{color}(v) \neq \mathit{color}(w)$. Clearly, not all instances have solutions. Explain how to encode an instance of a graph coloring problem into a propositional formula F. F should be satisfiable iff a graph coloring exists.