COSE215: Theory of Computation

Lecture 4 — ϵ-NFA

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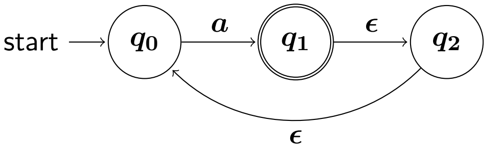
2019 Spring

NFA with ϵ-Transitions

NFAs with transitions on ϵ allowed.



$$M = (\{q_0, q_1, q_2\}, \{a\}, \delta, q_0, \{q_2\})$$

$$\delta(q_0, a) = \{q_1\} \qquad \delta(q_0, \epsilon) = \emptyset$$
$$\delta(q_1, a) = \emptyset \qquad \delta(q_1, \epsilon) = \{q_2\}$$
$$\delta(q_2, a) = \emptyset \qquad \delta(q_2, \epsilon) = \{q_0\}$$

NFA with ϵ-Transitions

Definition

An ϵ-NFA:

$$M = (Q, \Sigma, \delta, q_0, F)$$

where

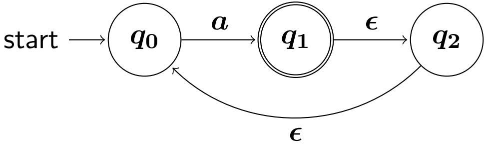
- Q: a finite set of *states*
- Σ: a finite set of *input symbols* (or input alphabet)
- $q_0 \in Q$: the *initial state*
- $F \subseteq Q$: a set of *final states*
- $\delta : Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$: *transition function*

Extended Transition Function

Informal definition of $\delta^* : Q \times \Sigma^* \to 2^Q$:

Definition

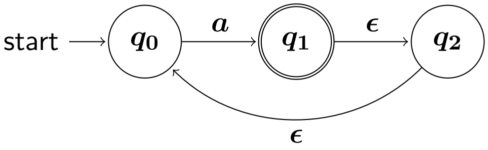
For an ϵ-NFA, the extended transition function is defined so that $\delta^*(q_i, w)$ contains q_j iff there is a path in the transition graph from q_i to q_j labeled by w.



- $\delta^*(q_1, a) =$
- $\delta^*(q_2, \epsilon) =$
- $\delta^*(q_2, aa) =$

Epsilon-Closures

$\text{ECLOSE}(q)$: the set of reachable states by ϵ-transitions.

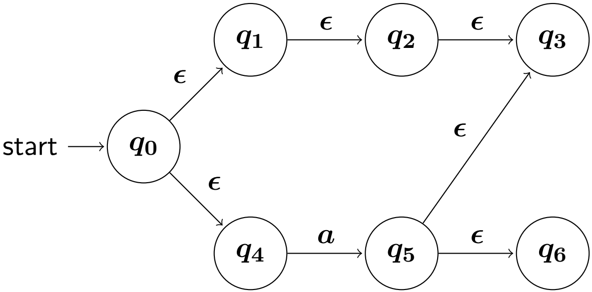


- $\text{ECLOSE}(q_0) =$
- $\text{ECLOSE}(q_1) =$
- $\text{ECLOSE}(q_2) =$

Defined recursively:

- (Basis): $\text{ECLOSE}(q)$ includes q
- (Induction): If state p is in $\text{ECLOSE}(q)$, and there is a transition from state p to state r labeled ϵ, then r is in $\text{ECLOSE}(q)$.

Example



$$\text{ECLOSE}(q_0) = \{ \qquad \}$$

$$\text{ECLOSE}(\{q_0, q_5\}) = \{ \qquad \}$$

Formal Definition of δ^*

$$\delta^* : Q \times \Sigma^* \to 2^Q$$

- (Basis)

$$\delta^*(q, \epsilon) = \textsc{EClose}(q)$$

- (Induction)

$$\delta^*(q, ua) = \textsc{EClose}(\bigcup_{s_i \in \delta^*(q,u)} \delta(s_i, a))$$

Language of ϵ-NFA

An ϵ-NFA $M = (Q, \Sigma, \delta, q_0, F)$ accepts a string w if

$$\delta^*(q_0, w) \cap F \neq \emptyset$$

and the language of automaton M is defined as follows:

$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset\}$$

From ϵ-NFA to DFA

Given an ϵ-NFA $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$, define a DFA:

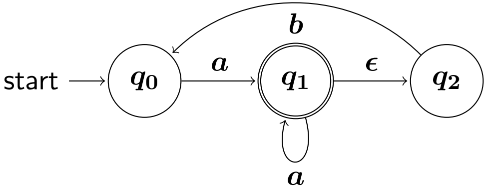
$$D = (Q_D, \Sigma, \delta_D, q_D, F_D)$$

- $Q_D = \{S \subseteq Q_E \mid S = \textsc{EClose}(S)\}$
- $q_D = \textsc{EClose}(q_0)$
- $F_D = \{S \in Q_D \mid S \cap F_E \neq \emptyset\}$
- For each $S \in Q_D$ and input symbol $a \in \Sigma$:

$$\delta_D(S, a) = \textsc{EClose}(\bigcup_{p \in S} \delta_E(p, a))$$

Exercise

Convert the following ϵ-NFA into an equivalent DFA.



Equivalence of ϵ-NFA and DFA

Theorem

A language L is accepted by some ϵ-NFA if and only if L is accepted by some DFA.

Proof.

- (If) Easy.
- (Only if) Exercise.

□