

# COSE215: Theory of Computation

## Lecture 18 — Extensions of Turing Machines

Hakjoo Oh  
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# Final Exam

- 6/19 (Mon), 09:00–10:15 (in class)
- Do not be late.
- Coverage: lecture 12~

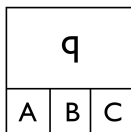
# Extensions

Extend the standard Turing machine with

- ① storage in the state
- ② multiple tracks
- ③ a stay-option
- ④ multiple tapes
- ⑤ non-determinism

## Storage in the state

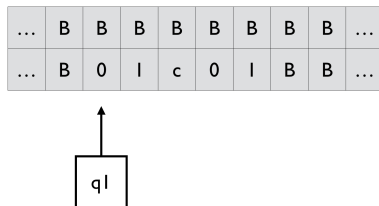
The finite control stores a finite amount of data:



### Example

A Turing machine that accepts  $01^* + 10^*$ .

# Multiple Tracks



## Example

A Turing machine that accepts  $L = \{w c w \mid w \in \{0, 1\}^+\}$ .

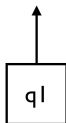
# Turing Machines with a Stay-Option

The tape head can be stationary:

$$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

ex)  $\delta(q_0, 0) = (q_1, 1, S)$



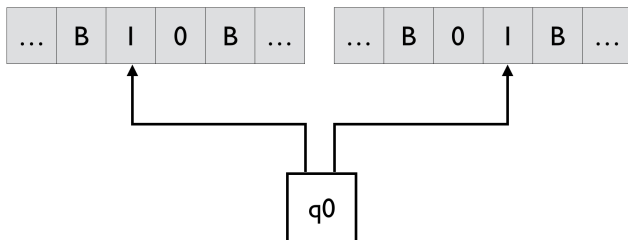
# Equivalence

- ① Is the TM with a stay-option is as powerful as the standard TM?
- ② Is the standard TM is as powerful as the TM with a stay-option?

# Multitape Turing Machines

Turing machine with

- multiple tapes
- each tape has its own tape head





# Multitape Turing Machines

$$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

$$\delta : Q \times \Gamma^n \rightarrow Q \times \Gamma^n \times \{L, R\}^n$$

Initially,

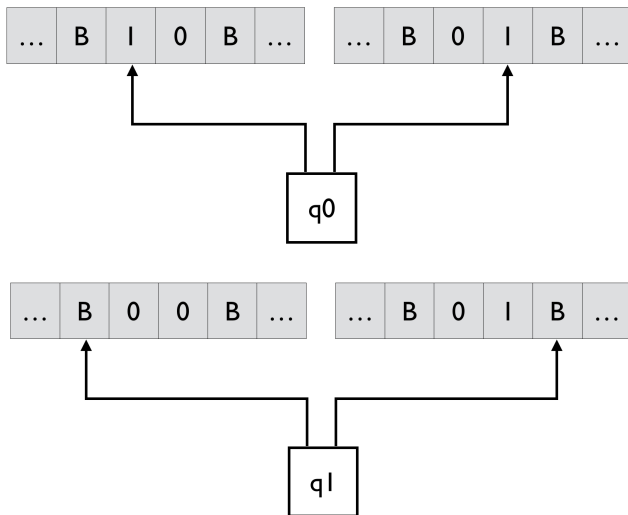
- 1 The input is placed on the first tape.
- 2 All other cells of all the tapes hold the blank.
- 3 The finite control is in the initial state.
- 4 The head of the first tape is at the left end of the input.
- 5 All other tape heads are at some arbitrary cell.

In one move, the multitape TM does the following:

- 1 The control enters a new state.
- 2 On each tape, a new tape symbol is written on the cell scanned.
- 3 Each of the tape heads makes a move independently of each other.

# Multitape Turing Machines

ex)  $\delta(q_0, 1, 1) = (q_1, 0, 1, L, R)$

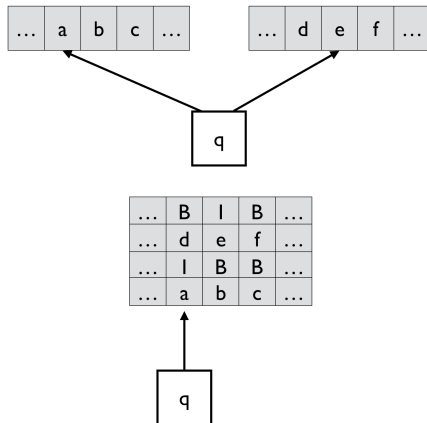


# Equivalence

To represent a MTM by a standard TM, we need to represent

- the contents of multiple tapes, and
- the positions of multiple tape heads.

Represent them by a tape with multiple tracks: e.g.,



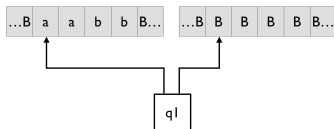
## cf) Efficiency

Although the expressiveness is the same, MTM can be more efficient than the standard TM.

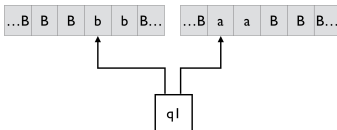
### Example

Design a multitape Turing machine that accepts  $L = \{a^n b^n \mid n \geq 1\}$ .

- In standard TM, repeated back-and-forth movements are required.
- In MTM, copy all  $a$ 's to tape 2



and then match  $b$ 's on tape 1 against  $a$ 's on tape 2



# Non-deterministic Turing Machines

$$(Q, \Sigma, \Gamma, \delta, q_0, B, F)$$
$$\delta : Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L, R\}}$$

- E.g.,  $\delta(q_0, a) = \{(q_1, b, R), (q_2, c, L)\}$
- A NTM accepts  $w$  if there is a sequence s.t.

$$q_0 w \vdash^* x_1 q_f x_2$$

with  $q_f \in F$ .

- Still, equivalent.

## Summary

No matter how we extend the standard Turing machines, the expressiveness remains the same.

