COSE215: Theory of Computation

Lecture 5 — Regular Expressions and Finite Automata

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2016 Spring

Equivalence between Regular Expressions and Finite Automata

Theorem (From RE to FA)

Every language defined by a regular expression is also defined by a finite automaton.

Theorem (From FA to RE)

Every language defined by some finite automata is also defined by a regular expression.

Conversion From Regular Expression to Finite Automata

Given a regular expression $\boldsymbol{R}$, we show that $\boldsymbol{L(R)}$ is accepted by an ϵ-NFA such that

- it has exactly one accepting state,
- no arcs into the initial state, and
- no arcs out of the accepting state.

Conversion from Regular Expression to Finite Automata

The proof is by structural induction on $\boldsymbol{R}$.

Base cases:

- $\boldsymbol{R} = \epsilon$:
- $\boldsymbol{R} = \emptyset$:
- $\boldsymbol{R} = \boldsymbol{a} (\in \boldsymbol{\Sigma})$:

From Regular Expression to Finite Automata

Inductive cases:

- $R = R_1 + R_2$:
- $R = R_1 R_2$:
- $R = R_1^*$:

Examples

- $\mathbf{0 \cdot 1^*}$:
- $\mathbf{(0 + 1) \cdot 0 \cdot 1}$:
- $\mathbf{(0 + 1)^* \cdot 1 \cdot (0 + 1)}$:

Memo

Memo