COSE212: Programming Languages

Lecture 9 — Automatic Type Inference (1)

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Type Inference?

- $(\texttt{proc}\ (x)\ x)\ 1$:
- $\texttt{proc}\ (x)\ (x\ 1)$:
- $\texttt{proc}\ (x)\ (\texttt{proc}(y)\ x)$:

Automatic Type Inference

- A *static analysis* that automatically figures out types of expressions by observing how they are used.
- The analysis can *always* infer the types of any expression, for a carefully designed language.
  - If an expression has a type according to the type system, the analysis is guaranteed to find the type.
  - If the analysis finds a type for an expression, the expression is well-typed with they type according to the type system.
- The analysis consists of two steps:
  1. Generate type equations from the program.
  2. Solve the equations.

Generating Type Equations

For every subexpression and every variable,

- introduce type variables, and
  ex) $\texttt{proc}\ (f)\ \texttt{proc}\ (x)\ ((f\ 3) - (f\ x))$:

$$\underbrace{\texttt{proc}\ (\underbrace{f}_{t_f})\ \underbrace{\texttt{proc}\ (\underbrace{x}_{t_x})\ \underbrace{(\underbrace{(f\ 3)}_{t_3} - \underbrace{(f\ x)}_{t_2})}_{t_4}}_{t_1}}_{t_0}$$

- derive equations between the variables.

Deriving Equations from Typing Rules

- $$\frac{\Gamma \vdash E_1 : \mathsf{int} \qquad \Gamma \vdash E_2 : \mathsf{int}}{\Gamma \vdash E_1 + E_2 : \mathsf{int}}$$

$$t_{E_1} = \mathsf{int} \;\wedge\; t_{E_2} = \mathsf{int} \;\wedge\; t_{E_1+E_2} = \mathsf{int}$$

- $$\frac{\Gamma \vdash E : \mathsf{int}}{\Gamma \vdash \texttt{zero?}\ E : \mathsf{bool}}$$

$$t_E = \mathsf{int} \;\wedge\; t_{(\texttt{zero?}\ E)} = \mathsf{bool}$$

- $$\frac{\Gamma \vdash E_1 : \mathsf{bool} \qquad \Gamma \vdash E_2 : t \qquad \Gamma \vdash E_3 : t}{\texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 : t}$$

$$\begin{array}{rcl} t_{E_1} & = & \mathsf{bool} \wedge \\ t_{E_2} & = & t_{(\texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3)} \wedge \\ t_{E_3} & = & t_{(\texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3)} \end{array}$$

Deriving Equations from Typing Rules

- $$\frac{\Gamma \vdash E_1 : t_1 \to t_2 \qquad \Gamma \vdash E_2 : t_1}{\Gamma \vdash E_1\ E_2 : t_2}$$

$$t_{E_1} = t_{E_2} \to \ t_{(E_1\ \ E_2)}$$

- $$\frac{[x \mapsto t_1]\Gamma \vdash E : t_2}{\Gamma \vdash \texttt{proc}\ x\ E : t_1 \to t_2}$$

$$t_{(\texttt{proc}\ (x)\ \ E)} = t_x \to t_E$$

- $$\frac{\Gamma \vdash E_1 : t_1 \qquad [x \mapsto t_1]\Gamma \vdash E_2 : t_2}{\Gamma \vdash \texttt{let}\ x = E_1\ \texttt{in}\ E_2 : t_2}$$

Example 1

$$\underbrace{\texttt{proc}\ (\underbrace{f}_{t_f})\ \underbrace{\texttt{proc}\ (\underbrace{x}_{t_x})\ \underbrace{(\underbrace{(f\ 3)}_{t_3} - \underbrace{(f\ x)}_{t_2})}_{t_4}}_{t_1}}_{t_0}$$

Example 2

$$\texttt{proc}\ (f)\ (f\ 11)$$

Example 3

$$\texttt{if } x \texttt{ then } (x - \mathbf{1}) \texttt{ else } \mathbf{0}$$

Example 4

$$\texttt{proc}\ (f)\ (\texttt{zero?}\ (f\ f))$$