COSE215: Theory of Computation

Lecture 2 — Deterministic Finite Automata

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A Finite Automaton is a String Recognizer

- Deterministic Finite Automata (DFA)
- Nondeterministic Finite Automata (NFA)

Deterministic Finite Automata

Definition (DFA)

A *deterministic finite automaton* (or *DFA*):

$$M = (Q, \Sigma, \delta, q_0, F)$$

where

- Q: a finite set of *states*
- Σ: a finite set of *input symbols* (or input alphabet)
- $\delta : Q \times \Sigma \to Q$: a total function called *transition function*
- $q_0 \in Q$: the *initial state*
- $F \subseteq Q$: a set of *final states*

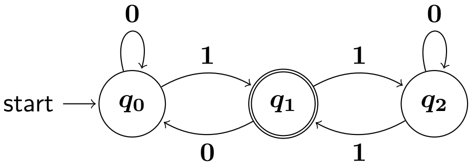
Example

Definition:

$$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$$

$$\delta(q_0, 0) = q_0, \quad \delta(q_0, 1) = q_1$$
$$\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_2$$
$$\delta(q_2, 0) = q_2, \quad \delta(q_2, 1) = q_1$$

Transition graph:



Transition table:

	0	1
$\rightarrow q_0$	q_0	q_1
$*q_1$	q_0	q_2
q_2	q_2	q_1

Exercise

Design a DFA that accepts the language:

$$\{x\mathbf{01}y \mid x \text{ and } y \text{ are any strings of } \mathbf{0}\text{'s and } \mathbf{1}\text{'s}\}$$

Extended Transition Function

Extend $\delta : Q \times \Sigma \rightarrow Q$ to input *strings*:

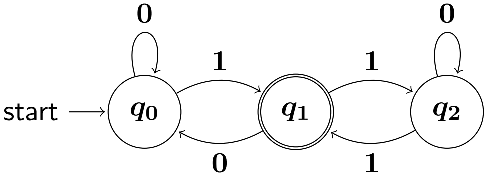
$$\delta^* : Q \times \Sigma^* \rightarrow Q$$

- (Basis) $s = \epsilon$:
$$\delta^*(q, \epsilon) = q$$
- (Induction) $s = wa$:
$$\delta^*(q, wa) = \delta(\delta^*(q, w), a)$$

Example

$$M = (\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_1\})$$

$$\begin{array}{ll} \delta(q_0, 0) = q_0, & \delta(q_0, 1) = q_1 \\ \delta(q_1, 0) = q_0, & \delta(q_1, 1) = q_2 \\ \delta(q_2, 0) = q_2, & \delta(q_2, 1) = q_1 \end{array}$$



- $\delta^*(q_0, 011) =$

Language of Automata

Definition

A DFA $M = (Q, \Sigma, \delta, q_0, F)$ accepts a string w if

$$\delta^*(q_0, w) \in F$$

and the *language* of automaton M, denoted $L(M)$, is defined as the set of all strings accepted by M:

$$L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}.$$

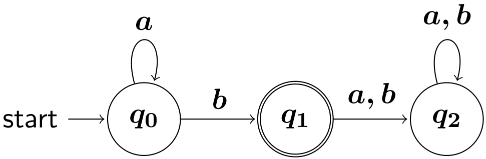
Definition

A language L is said to be *regular* iff there exists some DFA M such that

$$L = L(M)$$

Exercises

1. What is the language of the following automaton?



2. Design a DFA that accepts the language:

$$L(M) = \{abw \mid w \in \{a, b\}^*\}$$

3. Design a DFA that accepts strings ending with $\mathbf{01}$:

$$L = \{w01 \mid w \in \{0, 1\}^*\}$$