Homework 3
COSE215, Spring 2018

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Due: 5/24 (Thu), 14:00 (in class)

Problem 1 (20pts) Design context-free grammars for the following languages:

1. The language described by regular expression $0^*1(0+1)^*$
2. $L = \{a^n b^m \mid n \neq m-1\}$ (n and m are non-negative integers)
3. The language of all balanced round and square parentheses.

   $$L = \{\epsilon, (), [], ()[], ([]), [()], \ldots, ([]([][()])[]), \ldots\}$$

   Note that strings like ([)] that are not properly balanced do not belong to L.

Problem 2 (10pts) The following grammar generates prefix expressions with operands x and y and binary operators $+$, $-$, and $*$:

$$E \rightarrow +EE \mid *EE \mid -EE \mid x \mid y$$

1. Find leftmost and rightmost derivations, and a parse tree for the string $+*-xyxy$.
2. Is this grammar ambiguous or unambiguous? Explain.

Problem 3 (10pts) Design a PDA that accepts the following language:

$$L = \{0^n 1^n \mid n \geq 1\}$$

Problem 4 (10pts) Design a PDA that accepts the following language:

$$L = \{w \mid n_0(w) = n_1(w)\}$$

where $n_0(w)$ (resp., $n_1(w)$) denotes the number of 0 (respl, 1) in w.

Problem 5 (10pts) Design a deterministic PDA that accepts the language:

$$L = \{0^n 1^m \mid n \leq m\}$$

Problem 6 (20pts) Consider the following grammar:

$$\begin{array}{lcl} S & \rightarrow & ASB \mid \epsilon \\ A & \rightarrow & aAS \mid a \\ B & \rightarrow & SbS \mid A \mid bb \end{array}$$

1. Eliminate ϵ-productions.
2. Eliminate any unit productions in the resulting grammar.
3. Eliminate any useless symbols in the resulting grammar.
4. Put the resulting grammar into Chomsky Normal Form.

Problem 7 (20pts) Prove that the following languages are not context-free:

1. $L = \{a^n b^n c^i \mid i \leq n\}$
2. $L = \{0^p \mid p \text{ is a prime}\}$