Mid-term Exam
COSE215, Spring 2017

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Problem 1. (10pts) Design a DFA that accepts the following language:

1. (5pts) $L = \{w \in \{0,1\}^* \mid w \text{ has at most two 1s}\}$.
2. (5pts) $L = \{w_1 01 w_2 \mid w_1, w_2 \in \{0,1\}^*\}$.

Problem 2. (15pts) We can transform an NFA

$$N = (Q_N, \Sigma, \delta_N, q_0, F_N)$$

to an equivalent DFA D as follows:

$$D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$$

where

- $Q_D = 2^{Q_N}$
- $F_D = \{S \in Q_D \mid S \cap F_N \neq \emptyset\}$.
- For each $S \in Q_D$ and input symbol $a \in \Sigma$:

$$\delta_D(S, a) = \bigcup_{p \in S} \delta_N(p, a).$$

Then, we can prove $L(N) = L(D)$ by showing that

$$\forall w.\ \delta_D^*(\{q_0\}, w) = \delta_N^*(q_0, w).$$

Prove the claim by induction on the length of w.

Problem 3. (15pts) Recall that an ϵ-NFA is a five-tuple:

$$M = (Q, \Sigma, \delta, q_0, F)$$

where Q is a finite set of states, Σ is a finite set of input symbols, $\delta : Q \times (\Sigma \cup \{\epsilon\}) \to 2^Q$ is a transition function, $q_0 \in Q$ is the initial state, and $F \subseteq Q$ is a set of final states.

1. (5pts) Define $\text{ECLOSE} : 2^Q \to 2^Q$, which takes a set of states and returns the states reachable via ϵ-transitions.
2. (5pts) Define the extended transition function $\delta^* : Q \times \Sigma^* \to 2^Q$.
3. (5pts) Define the language of NFA:

$$L(M) = \{w \in \Sigma^* \mid \qquad\qquad\qquad\qquad \}$$

Problem 4. (10pts) Write regular expressions for the following languages:

1. (5pts) $\{a^n b^m \mid (n+m) \text{ is even}\}$
2. (5pts) $\{w \in \Sigma^* \mid \text{every } 0 \text{ in } w \text{ is followed by at least one } 1\}$

Problem 5. (10pts) Let D be a DFA. Let $R_{ij}^{(k)}$ be the regular expression whose language is the set of strings w such that w is the label of a path from state i to state j in D, and that path has no intermediate node whose number is greater than k. Define $R_{ij}^{(k)}$ when $k > 0$.

Problem 6. (10pts) Prove that $L = \{0^i \mid i \text{ is a prime}\}$ is non-regular.

Problem 7. (10pts) Consider the language:

$$L = \{w0^n \mid w \in \{a,b\} \text{ and } n_a(w) = n\}$$

where $n_a(w)$ is the number of a in w. Find a context-free grammar of the language.

Problem 8. (20pts) True/false questions. (Leave a blank when you are uncertain: a correct answer gets you 2 points but you lose 2 points for each wrong answer.)

1. $\emptyset^* = \emptyset$.
2. $(L^*)^* = L^*$ for any language L.
3. Every finite language is regular.
4. Every infinite language is context-free.
5. There is some language that can be specified by a context-free grammar but not by a regular expression.
6. There is a language that satisfies the pumping lemma conditions but is not regular.
7. The C programming language is regular.
8. Suppose D_1 and D_2 are any DFAs. We can always construct a DFA D such that $L(D) = L(D_1) \cap (\Sigma^* - L(D_2))$.
9. Some context-free languages are recognizable by finite automata.
10. Recall that regular expressions are constructed by the grammar:

$$e \to a \in \Sigma \mid \epsilon \mid \emptyset \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e^*$$

The set of all regular expressions is expressible by a regular expression.