Final Exam
COSE215, Spring 2016

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Problem 1. (10pts) Compare finite automata (FA), pushdown automata, and Turing machines in terms of the languages they accept and machineries they have.

Problem 2. (10pts) Draw a Venn-diagram to illustrate the relationships between the following classes of languages:

- *RL*: the set of regular languages
- *CFL*: the set of context-free languages
- *FPDA*: the set languages that are accepted by some PDA by final state.
- *EPDA*: the set of languages accepted by some PDA by empty stack.
- *DPDA*: the set of languages accepted by some deterministic pushdown automata
- *R*: the set of recursive languages
- *RE*: the set of recursively-enumerable languages
- *D*: the set of decidable languages

Problem 3. (15pts)

1. (10pts) State the pumping lemma for context-free languages.
2. (5pts) What is the essential property of the PL for CFLs? Compare the property with that for regular languages.

Problem 4. (10pts) Explain the behaviors of the following Turing machines:

1. (5pts) $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_4\})$

$$\begin{array}{rcl} \delta(q_0, 1) & = & (q_0, 1, R) \\ \delta(q_0, 0) & = & (q_1, 1, R) \\ \delta(q_1, 1) & = & (q_1, 1, R) \\ \delta(q_1, B) & = & (q_2, B, L) \\ \delta(q_2, 1) & = & (q_3, 0, L) \\ \delta(q_3, 1) & = & (q_3, 1, L) \\ \delta(q_3, B) & = & (q_4, B, R) \end{array}$$

2. (5pts) $M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$

$$\begin{array}{rcl} \delta(q_0, 1) & = & (q_0, x, R) \\ \delta(q_0, B) & = & (q_1, B, L) \\ \delta(q_1, 1) & = & (q_1, 1, L) \\ \delta(q_1, x) & = & (q_2, 1, R) \\ \delta(q_2, 1) & = & (q_2, 1, R) \\ \delta(q_2, B) & = & (q_1, 1, L) \\ \delta(q_1, B) & = & (q_3, B, R) \end{array}$$

Problem 5. (25pts) Consider the diagonalization language:

$$L_d = \{w_i \mid w_i \notin L(M_i)\}$$

- (5pts) What does w_i mean in the above definition?
- (5pts) What does M_i mean in the above definition?
- (5pts) Explain what L_d means in English.
- (10pts) Prove that there is no Turing machine to accept L_d. Use the diagonalization method.

Problem 6. (30pts) True/False questions. (Do not answer when you are uncertain; A correct answer gets you 2 points but you lose 2 points for each wrong answer.)

1. We can use the pumping lemma for context-free languages to show that some language not to be regular.
2. $L = \{ww^R \mid w \in \{0,1\}^*\}$ is context-free.
3. $L = \{a^n b^n \mid n \geq 1\}$ is a deterministic context-free language.
4. $L = \{0^n 1^n 2^n \mid n \geq 1\}$ is context-free. .
5. $L = \{w \in \{a,b,c\}^* \mid n_a(w) = n_b(w) = n_c(w)\}$ is context-free.
6. $L = \{wcw \mid w \in \{0,1\}^+\}$ is context-free.
7. $L = \{w \mid |w| = 2k \text{ for some } k\}$ is a recursive language.
8. Pushdown automata with two stacks are as powerful as Turing machines.
9. It is possible to remove ambiguity from a context-free language.
10. A context-free grammar is said to be Chomsky Normal Form if its all productions are of the forms $A \to BC$, $A \to B$, and $A \to a$.
11. Context-free languages are closed under union, concatenation, and complementation.
12. There is a problem solvable by Turing machines with two tapes but unsolvable by Turing machines with a single tape.
13. The nondeterministic Turing machine is more powerful than the deterministic Turing machine.
14. The efficiency of the multitape Turing machine is the same as the standard Turing machines.
15. The concept of the universal Turing machine is essentially the same as the interpreter for programming languages.